

Sequences

1 For each of the following sequences, find the indicated term:

- a Third term in the sequence: $2, -4, 6, -8, 16, \dots$
- b Fourth term in the sequence: $3, 3.5, 4, 4.5, 5, 5.5, \dots$
- c Fifth term in the sequence: $5, 4, 3, 2, 1, 0, -1, \dots$

2 If u_n describes the n th term of the following sequences, find the indicated term:

- a u_3 in the sequence: $4, -5, 6, -7, 8, \dots$
- b u_4 in the sequence: $200, 20, 2, 0.2, 0.02, \dots$
- c u_5 in the sequence: $1, 2.5, 4, 5.5, 7, 8.5, \dots$
- d $u_3 + u_5$ in the sequence: $6, -8, 9, -10, 11, \dots$
- e $2u_2 - u_4$ in the sequence: $9, 12, 15, 18, 21, \dots$
- f $-4(u_3 + u_4)$ in the sequence: $1, 4, 5, 9, 14, 23, \dots$

3 In the sequence $5, -5, 7, -7, 9, -9, \dots$, find n if $u_n = 7$.

4 For each of the following sequences:

- i Describe the recurring pattern in words.
 - ii Find the next number in the sequence.
- a $-1, 1, 3, 5, 7, \dots$
 - b $64, 32, 16, 8, 4, 2, \dots$
 - c $2, -4, 6, -8, 10, -12, \dots$

Recurrence relations

5 Determine whether the following are recurrence relations:

- a $\frac{S_n}{n} = n + 1$
- b $R_n = (3R_{n-1})^5 + R_{n-2}$
- c $u_n = u_{n-1} + 9$
- d $V_n = 3(n - 1)$

6 Consider the following sequence given in the table:

- a Write down the general rule for u_n in terms of n .
- b Use the general rule to find u_{20} .

n	1	2	3	4	...
u_n	1	4	9	16	...

7 Consider the following sequence given in the table:

a Write down the general rule for u_n in terms of n .

b Use the general rule to find u_{19} .

n	1	2	3	4	...
u_n	1	8	27	27	...

8 Write the recursive rule for each of the given description. Let u_n be the n th term.

a To find the next term, add 5 to the previous term.

b To find the next term, add the two previous terms.

c To find the next term, multiply the previous term by negative 1 and then add 7.

d To find the next term, subtract one and a half from the previous term.

e To find the next term, subtract 6 from the previous term.

f To find the next term, multiply the two previous terms.

g To find the next term, multiply the previous term by negative 5 and then subtract 3.

h To find the next term, add three and a quarter to the previous term.

9 For each of the following recursive rules, state the first 5 terms of the sequences in order:

a $u_n = 2u_{n-1}, u_1 = 2$

b $u_n = u_{n-1} + 6, u_1 = -8$

c $b_n = -b_{n-1} + 3, b_1 = 0$

10 For each of the recursive relations below, find the following.

i u_2

ii u_3

iii u_4

a $u_n = (-1)^{n+1} u_{n-1}, u_1 = 2$

b $u_{n+1} = (2u_n)^{n-1}, u_1 = 3$

c $u_n = n^{n+1} + u_{n-1}, u_1 = 3$

d $u_{n+1} = (4u_n)^{n-1} \times (n+1), u_1 = 5$

11 Consider the sequence which has a first term of 11 and a second term of 22, and subsequent terms are found by adding the two previous terms.

a Write the recursive rule for the sequence, letting u_n be the n th term.

b State the first five terms of the sequence.

Explicit rules

12 Determine whether the following are explicit relations:

a $b_n = b_{n-1}b_{n-2}$

b $d_n = n^2 + 3n + 8$

c $u_n = 8u_{n-1} + u_1$

d $c_n = 3n^2$

13 For each of the following explicit rules, state the first 5 terms of the sequences in order starting with $n = 1$:

a $b_n = 5n - 2$

b $s_n = n^2 + 6$

c $u_n = 2n^2 + n - 3$

14 For the following sequences:

i State the recursive rule that describes u_n in terms of u_{n-1} , and the initial term u_1 .

ii State the explicit rule that describes u_n in terms of n .

a A sequence starts with a first term of 1300 and each subsequent term increases by 2.5% of the previous term.

b A sequence starts with a first term of 44 and each term is 77 more than the previous term.

Technology

15 Find the indicated term in the following sequences:

a 21st term in the sequence: $-3, -6, -9, -12, -15, \dots$

b 53rd term in the sequence: $2, 3.5, 5, 6.5, 8, 9.5, \dots$

c 39th term in the sequence: $10, 8, 6, 4, 2, 0, -2, \dots$

16 Consider the sequence defined by $u_n = 2u_{n-1} + 3$, $u_1 = 12$. Calculate the sum of the first 25 terms.

17 For each of the following pairs of sequences, compare the first 10 terms of each sequence on a graph to determine which sequence is increasing faster:

a $u_{n+1} = u_n + 22$, $u_1 = 12$
 $A_{n+1} = A_n + 25$, $A_1 = 12$

b $u_{n+1} = 2.5u_n$, $u_1 = 5$
 $A_{n+1} = 3.5A_n$, $A_1 = 5$

- 18** For each pair of sequences, compare the first 10 terms of each sequence on a graph to determine which sequence is decreasing faster:
- a** $u_{n+1} = u_n - 5, u_1 = 4$ and $A_{n+1} = A_n - 10, A_1 = 12$
 - b** $u_{n+1} = 0.2u_n, u_1 = 20$ and $A_{n+1} = 0.6A_n, A_1 = 20$
- 19** For the explicit rule $u_n = 0.7n - 5$ which starts at $n = 1$, find the sum of the first 50 terms.
- 20** For the explicit rule $u_n = (0.2n)^2 + 5n$ which starts at $n = 1$, find the sum of the terms from the 20th to the 30th term inclusive.
- 21** Consider the sequence 3, 7, 11, 15, 19, 23, ...
- a** If u_n is the value of the n th term in the sequence, find u_{10} .
 - b** Find the first term greater than 100.
 - c** If S_n is the sum of the first n terms, find S_{10} .
 - d** Starting from $n = 1$, find the minimum number of terms for the sum to first exceed 500.
- 22** Consider the sequence 1, 2, 4, 8, 16, ...
- a** Find u_{13} .
 - b** Find the first term greater than 10 000.
 - c** If S_n is the sum of the first n terms, find S_{20} .
 - d** Starting from $n = 1$, find the greatest number of terms such that the sum is still less than 10 000 000.
- 23** Consider the sequence $u_n = u_{n-1} + 5, u_1 = 10$
- a** Find u_{40} .
 - b** Find the first term greater than or equal to 300.
 - c** If S_n is the sum of the first n terms, find S_{30} .
 - d** Starting from $n = 1$, find the minimum number of terms for the sum to be greater than 5000.

24 Consider the sequence $u_{n+1} = 1.5u_n$, $u_1 = 10$

- a** Find u_{10} to the nearest whole number.
- b** Find the first term greater than or equal to 1000. Round your answer to the nearest whole number.
- c** If S_n is the sum of the first n terms, find S_{18} . Round your answer to the nearest whole number.
- d** Starting from $n = 1$, find the minimum number of terms for the sum to be greater than 50 000.

25 Consider the sequence $u_n = 5 - 3n$, starting at $n = 1$:

- a** Find u_{30} .
- b** Find the first term less than -150 .
- c** If S_n is the sum of the first n terms, find S_{15} .
- d** Starting from $n = 1$, find the minimum number of terms required for the sum to be less than -600 .

26 Consider the sequence $u_n = 200 \times 0.5^{n-1}$

- a** Find u_8 .
- b** Find the first term less than 0.5.
- c** If S_n is the sum of the first n terms, find S_{10} .
- d** Starting from $n = 1$, find the minimum number of terms for the sum to first exceed 399.9.