

IB DP Mathematics AASL – Prior-Learning Diagnostic Quiz

Name: _____

Calculator: Not permitted

Time Allowed: 60 mins

- Instructions: Show all working. Exact answers are expected where appropriate.
- Purpose: This diagnostic is designed to identify areas for review and practice rather than produce a formal course grade.

Q No.		Feedback/Reflection
1.	Calculate: $\frac{9}{12} - \frac{10}{12} + \frac{4}{12} = \frac{9-10+4}{12} = \frac{13-10}{12} = \frac{3}{12} = \frac{1}{4}$	1. Know this 2. Need to review 3. Need help with
2.	Write 0.0000564 in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$. 5.64×10^{-5}	1. Know this 2. Need to review 3. Need help with
3.	Write 7.2×10^5 in number form. 7200000 $= 720000$	1. Know this 2. Need to review 3. Need help with
4.	Simplify each expression fully:	
	(a) $\frac{x^7 \times x^{-2}}{x^3} = \frac{x^{7-2}}{x^3} = \frac{x^5}{x^3} = x^2$	1. Know this 2. Need to review 3. Need help with
	4. $(2x^3)^2 = (2)^2 \cdot (x^3)^2 = 4 \cdot x^6$	
	5. $\frac{a^{\frac{3}{2}}}{a^{\frac{1}{2}}} = a^{\frac{3}{2} - \frac{1}{2}} = a^{\frac{3-1}{2}} = a^{\frac{2}{2}} = a^1 \text{ or } a$	
	6. $(16x^8)^{\frac{1}{2}} = (16)^{\frac{1}{2}} \cdot (x^8)^{\frac{1}{2}} = \sqrt{16} \cdot x^{8 \times \frac{1}{2}} = 4x^4$	

5.	Simplify fully: $3\sqrt{12} - 2\sqrt{27} + \sqrt{75}$ $75 = 15 \times 5$ $\quad \quad \quad \uparrow$ $\quad \quad \quad 3 \times 5 \times 5$ $3\sqrt{4 \cdot 3} - 2\sqrt{9 \cdot 3} + \sqrt{25 \cdot 3}$ $3 \cdot 2\sqrt{3} - 2 \cdot 3\sqrt{3} + 5 \cdot \sqrt{3}$ $6\sqrt{3} - 6\sqrt{3} + 5\sqrt{3}$ $= 5\sqrt{3}$	1. Know this 2. Need to review 3. Need help with
6.	Rationalize the denominator: $\frac{4}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$ $= \frac{4(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$ $= \frac{4(\sqrt{3} + 1)}{\sqrt{3} \cdot \sqrt{3} + \sqrt{3} - \sqrt{3} - 1}$ $= \frac{4(\sqrt{3} + 1)}{3 - 1}$ $= \frac{4(\sqrt{3} + 1)}{2}$ $= 2(\sqrt{3} + 1)$	1. Know this 2. Need to review 3. Need help with
7.	Simplify and state any values that x cannot take: $\frac{x^2 - 9}{x(x - 3)}$ $= \frac{(x - 3)(x + 3)}{x(x - 3)}$ $= \frac{(x + 3)}{x}$	1. Know this 2. Need to review 3. Need help with
8.	Give your answer as a single fraction. $\frac{(x+1) \times 2}{(x+1) \times x} + \frac{3 \times x}{(x+1) \times x}$ $= \frac{(x+1) \cdot 2 + 3 \cdot x}{x(x+1)}$ $= \frac{2x + 2 + 3x}{x(x+1)}$ $= \frac{5x + 2}{x(x+1)}$	1. Know this 2. Need to review 3. Need help with

9.	A tank contains 200 liters of water. Water enters at a constant rate of 5 liters per minute. How much water is in the tank after 12 minutes? In 1 min = 5 L In 12 mins = 5×12 L = 60 L	1. Know this 2. Need to review 3. Need help with
10.	Make x the subject of the equation: $y = (x - 2)^2 + 5$ $y - 5 = (x - 2)^2$ $\pm \sqrt{y - 5} = x - 2$ $\Rightarrow x = 2 \pm \sqrt{y - 5}$	1. Know this 2. Need to review 3. Need help with
11.	Given that $A = \frac{1}{2} b \cdot h$ and $b = 3x + 2 = 3 \cdot 4 + 2 = 12 + 2 = 14$ and $h = x - 1 = 4 - 1 = 3$ Find A when $x = 4$. $A = \frac{1}{2} \cdot 14 \cdot 3$ $A = 21$	1. Know this 2. Need to review 3. Need help with

12. Solve:

$$2 \times \frac{m-2}{3} + \frac{m+1}{2} \times 3 = 5$$

$$\frac{2(m-2) + 3(m+1)}{6} = 5$$

$$2m - 4 + 3m + 3 = 6 \times 5$$

$$5m - 1 = 30$$

$$5m = 31$$

$$m = \frac{31}{5}$$

1. Know this
2. Need to review
3. Need help with

13. Solve:

$$2^{x-1} \times 2^{x+1} = 32$$

$$2^{x-1+x+1} = 32$$

$$2^{2x} = 2^5$$

$$\Rightarrow 2x = 5$$

$$x = \frac{5}{2}$$

1. Know this
2. Need to review
3. Need help with

14. Solve in inequality and represent your answer on a number line.

$$3 - 2x \leq 11$$

$$3 - 11 \leq 2x$$

$$-8 \leq 2x$$

$$2x \geq -8$$

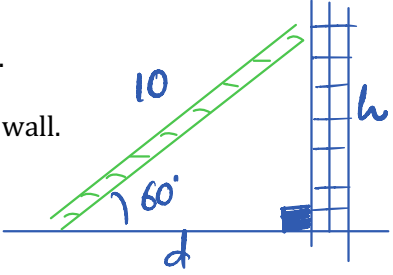
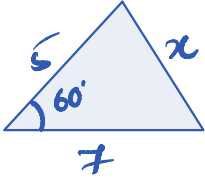
$$x \geq \frac{-8}{2}$$

$$x \geq -4$$



1. Know this
2. Need to review
3. Need help with

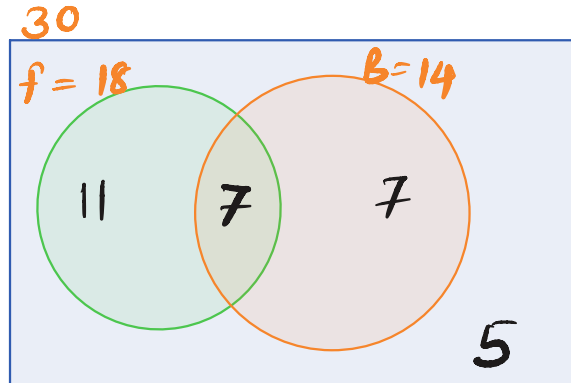
15.	A rectangle has a perimeter of 34 cm. Its length is 3 cm greater than its width. Find the dimensions of the rectangle. $2L + 2W = 34$ $2(L + W) = 34$ $L + W = 17$ ↓ $W + 3 + W = 17$ $2W + 3 = 17$  $L = W + 3$ $2W = 17 - 3$ $2W = 14$ $W = 7 \Rightarrow L = 10$	1. Know this 2. Need to review 3. Need help with
16.	Given that $f(x) = x^2 - 4x + 3$. Find: (a) $f(0) = 3$ (b) $f(a + 2) = (a + 2)^2 - 4(a + 2) + 3$ $= a^2 + 4a + 4 - 4a - 8 + 3$ $= a^2 - 1$ (c) the value of x for which $f(x) = 0$. $x^2 - 4x + 3 = 0$ $(x - 3)(x - 1) = 0$ $x = 3 \text{ or } x = 1$	1. Know this 2. Need to review 3. Need help with
17.	Describe the transformations that take $y = x^2$ to $y = (x - 3)^2 + 2$ Horizontal Translation by 3 units to right + Vertical Translation by 2 units up.	1. Know this 2. Need to review 3. Need help with

18.	For the function: $y = x^2 - 4 = a(x-h)^2 + k$ State: (a) the y-intercept when $x=0$ $y = -4$ (b) the x-intercepts when $y=0$ $x^2 - 4 = 0$ $\Rightarrow x^2 - 2^2 = 0$ $(x-2)(x+2) = 0 \Rightarrow x=2 \text{ or } x=-2$ (c) the coordinates of the vertex. $(0, -4)$	1. Know this 2. Need to review 3. Need help with
19.	A ladder 10 m long rests against a vertical wall. The ladder makes an angle of 60° with the ground. a) Find the height reached by the ladder. b) Find the horizontal distance from the wall. Give exact answers. a) $\sin 60^\circ = \frac{h}{10}$ $\Rightarrow h = 10 \times \sin 60^\circ$ $h = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ m}$ b) $\cos 60^\circ = \frac{d}{10}$ $\Rightarrow d = 10 \times \cos 60^\circ = 10 \times \frac{1}{2} = 5 \text{ m}$ 	1. Know this 2. Need to review 3. Need help with
20.	A triangle has sides 5 cm and 7 cm with an included angle of 60°. Find the length of the third side. Give an exact answer. $x^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \cdot \cos 60^\circ$ $x^2 = 25 + 49 - 70 \cdot \frac{1}{2}$ $x^2 = 25 + 49 - 35$ $x^2 = 39$ $x = \pm \sqrt{39}$ 	1. Know this 2. Need to review 3. Need help with

21. In a class of 30 students:

- 18 students play football.
- 14 students play basketball.
- 7 students play both football and basketball.

a) Draw a Venn diagram to represent this information.



b) How many students play only football? 11

c) How many students play only basketball? 7

d) How many students play neither sport? 5

e) Find the probability that a randomly selected student plays at least one of the two sports.

$$\frac{25}{30} \text{ or } \frac{5}{6}$$

f) Given that a student plays football, find the probability that they also play basketball.

$$\frac{7}{18}$$

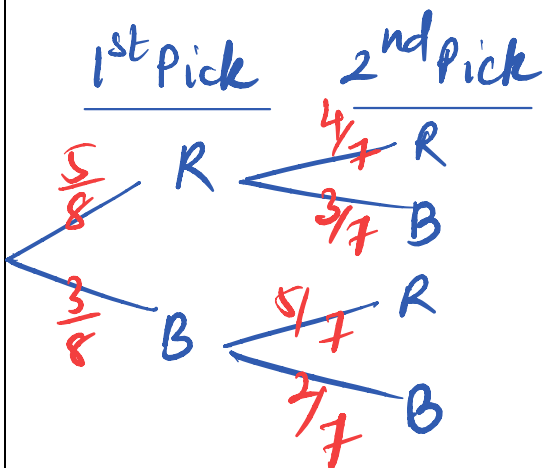
1. Know this

2. Need to review

3. Need help with

22. A bag contains 5 red balls and 3 blue balls. Two balls are selected without replacement.

a) Draw a tree diagram showing all possible outcomes.



b) Find the probability that both balls are red.

$$P(RR) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{10}{28} = \frac{5}{14}$$

c) Find the probability that the first ball is red, and the second ball is blue.

$$P(RB) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$$

d) Find the probability that the two balls are the same color.

$$P(RR + BB) = \frac{20}{56} + \frac{3}{8} \times \frac{2}{7} = \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \frac{13}{28}$$

e) Explain why the probability of drawing a red ball on the second selection depends on the outcome of the first selection.

As the events are dependent.

1. Know this
2. Need to review
3. Need help with

23. A solid is formed by joining a **cylinder** and a **cone**. The cone and cylinder have the same radius.

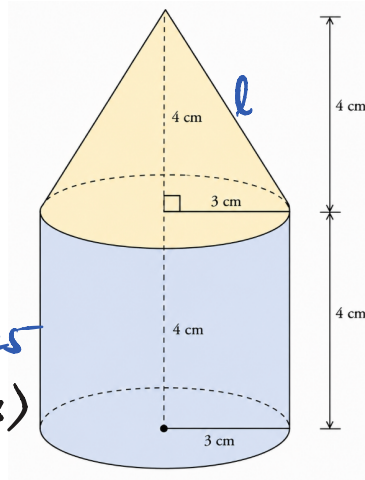
- Radius of the cylinder and cone: **3 cm**
- Height of the cylinder: **4 cm**
- Perpendicular height of the cone: **4 cm**

Use **exact values** in terms of π where appropriate.

- a) **Calculate** the slant height of the cone.

$$l^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$l = 5 \text{ cm} \text{ (no negative Ans)}$$



- b) **Calculate** the curved surface area of the cone.

$$\begin{aligned} \text{C.S.A of cone} &= \pi \cdot r \cdot l \\ &= \pi \cdot 3 \cdot 5 \\ &= 15\pi \text{ cm}^2 \end{aligned}$$

- c) **Calculate** the curved surface area of the cylinder.

$$\begin{aligned} \text{C.S.A of cylinder} &= 2\pi r h \\ &= 2 \cdot \pi \cdot 3 \cdot 4 = 24\pi \text{ cm}^2 \end{aligned}$$

- d) **Hence**, find the **total external surface area** of the composite solid.

Hint: Do not include the circular surface where the cone and cylinder are joined.

$$\begin{aligned} \text{T.S.A (cone + cylinder)} &= 15\pi + 24\pi + \pi(3)^2 \\ &= (15 + 24 + 9)\pi \\ &= 48\pi \text{ cm}^2 \end{aligned}$$

Area of the Base

- e) **Calculate** the volume of the cylinder.

$$\begin{aligned} V_{\text{cylinder}} &= \pi \cdot r^2 \cdot h \\ &= \pi \cdot (3)^2 \cdot 4 \\ &= 36\pi \text{ cm}^3 \end{aligned}$$

1. Know this
2. Need to review
3. Need help with

f) Calculate the volume of the cone.

$$\begin{aligned}V_{\text{cone}} &= \frac{1}{3} \pi r^2 h \\&= \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 4 \\&= 12\pi \text{ cm}^3\end{aligned}$$

g) Hence, find the total volume of the composite solid.

$$\begin{aligned}\text{T.V.} &= 36\pi + 12\pi \\&= 48\pi \text{ cm}^3\end{aligned}$$

h) The solid is made from a material with a density of 2 g/cm^3 . Find the mass of the solid.

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$\begin{aligned}M &= 2 \text{ g/cm}^3 \times 48\pi \text{ cm}^3 \\&= 96\pi \text{ g (units ??)}\end{aligned}$$

i) Find the ratio of the volume of the cone to the volume of the cylinder.

$$\text{Ratio} = \frac{V_{\text{cone}}}{V_{\text{cylinder}}} = \frac{12\pi}{36\pi} = \frac{1}{3}$$

24. The points $A(2, 1)$ and $B(6, 3)$ are joined by a line segment.

The line $2x - y - 2 = 0$ passes through A and B. $y = mx + c$ form

a) Find the gradient of AB.

$$m = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$$

b) Find the coordinates of the midpoint M of AB.

$$M = \left(\frac{2+6}{2}, \frac{1+3}{2} \right) = \left(\frac{8}{2}, \frac{4}{2} \right) = (4, 2)$$

c) Find the gradient of the perpendicular bisector of AB.

$$m_{\perp \text{ Bisector}} = -2$$

d) Hence, find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$

(As this line passes through M)

$$y = -2x + c$$

$c = 10$ $x = 4$ and $y = 2$

$$\Rightarrow 2 = -2(4) + c \Rightarrow y = -2x + 10 \text{ to find } c$$

e) The perpendicular bisector intersects the line $2x - y - 2 = 0$

Use the two equations to find the coordinates of the point of intersection between them.

Solving simultaneously,

$$y = 2x - 2$$

$$\Rightarrow x = 3$$

$$y = -2x + 10$$

$$\Rightarrow y = 2(3) - 2 = 6 - 2 = 4$$

$$\Rightarrow 2x - 2 = -2x + 10$$

$$2x + 2x = 10 + 2$$
$$4x = 12$$

$\therefore$ Point of intersection $(3, 4)$

f) Verify that your answer in part (e) is equidistant from A and B.

$$\sqrt{(3-2)^2 + (4-1)^2} = \sqrt{10}$$

$$\sqrt{(3-6)^2 + (4-3)^2} = \sqrt{10}$$

} equal
 $\therefore$ equidistant.

1. Know this

2. Need to review

3. Need help with