Logarithmic equations

1 Solve for x in each of the following logarithmic equations:

a $\log_4 9x = 2$

b $9 \log x = 45$

c $\log_{64} x = \frac{1}{3}$

d $\log_2 (5x - 1) = 6$

e $\log_7 (10x + 2311) = 4$

f $\log_9 (5x - 32) = \log_9 8$

g $\log_7 (6x - 6) = \log_7 (3x - 2)$

h $\log_3 (5x - 2) = \log_3 (4x + 2)$

i $\log_{10} x + \log_{10} 9 = \log_{10} 45$

j $\log (x + 3) = \log x + \log 6$

k $\log_{10} 6 + \log_{10} x = \log_{10} (x + 8)$

l $\log (11x - 2) - \log (4x - 2) = \log 3$

m $\log (x + 30) - \log (x + 2) = \log x$

n $\log_9 (4x + 12) = \log_9 (x + 7) - \log_9 7$

2 Solve $\log_8 y = 4$ for y.

Exponential equations

3 Solve for the exact value of x in each of the following exponential equations:

a $3 (10^x) = 6$

b $\frac{1}{7} (2^x) = 3$

c $3 \left(5^{4x+5}\right) + 2 = 5$

d $2 \times 3^{5x} + 4 = 16$

4 Solve for the exact value of y in each of the following exponential equations:

a $2^{5y+2} = 3^{4-3y}$

b $3^{y+5} = 6^{2y}$

5 Solve for x in each of the following exponential equations, correct to two decimal places:

a $5^x = 9$

b $\left(\frac{1}{4}\right)^{x+3} = \sqrt{7}$

c $3^x = 2^{x+1}$

d $5^{x+4} = 25^{8x-4}$

6 Find the solution to $3^x = -2$.

7 Solve for y in each of the following exponential equations, correct to two decimal places:

a $12^y = 23$

b $3^{y+1} = 6$

c $4^{y-3} = 25$

d $4^{y+1} = 7$

e $7^{4y+3} = 6$

f $6^{y+(-3)} = 11^y$

g $6^{y+5} = 5^{y+2}$

8 Solve the following exponential equations:

a $9^y = 27$

b $9^{x+3} = 27^x$

c $\left(\sqrt{6}\right)^y = 36$

d $\left(\sqrt{2}\right)^k = 0.5$

e $3^{5x-10} = 1$

f $5^{-3x-1} = 3125$

g $5^{10x+33} = 125$

h $\dfrac{1}{3^{x-3}} = \sqrt[3]{9}$

i $\left(\dfrac{1}{9}\right)^{x+5} = 81$

j $\left(\dfrac{1}{8}\right)^{x-3} = 16^{4x-3}$

k $\dfrac{25^y}{5^{4-y}} = \sqrt{125}$

l $8^{x+5} = \dfrac{1}{32\sqrt{2}}$

m $30 \times 2^{x-6} = 15$

n $2^x \times 2^{x+3} = 32$

o $\left(2^2\right)^{x+7} = 2^3$

p $\left(2^4\right)^{2x-10} = 2^2$

q $81^{x-1} = 9^{3x+5}$

r $25^{x+1} = 125^{3x-4}$

s $a^{x-1} = a^4$

t $a^{x+1} = a^3\sqrt{a}$

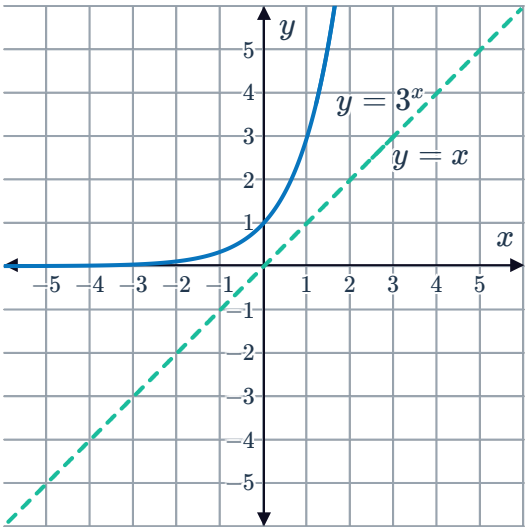
u $3^{x^2-3x} = 81$

v $27\left(2^x\right) = 6^x$

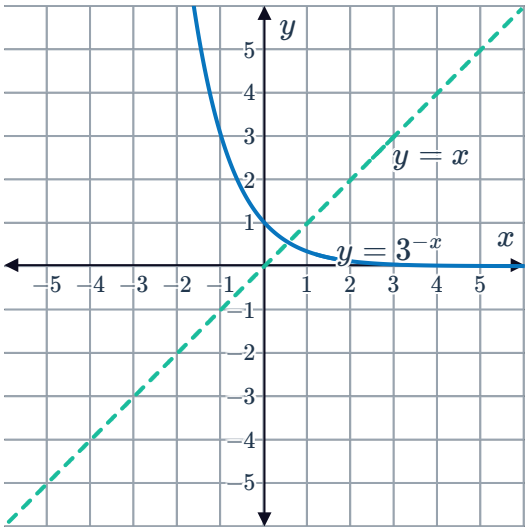
Inverse of exponential and logarithmic functions

9 Sketch the graph of the inverse of the following functions:

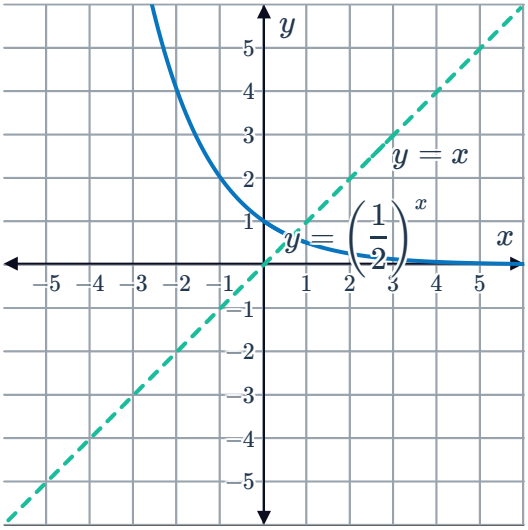
a



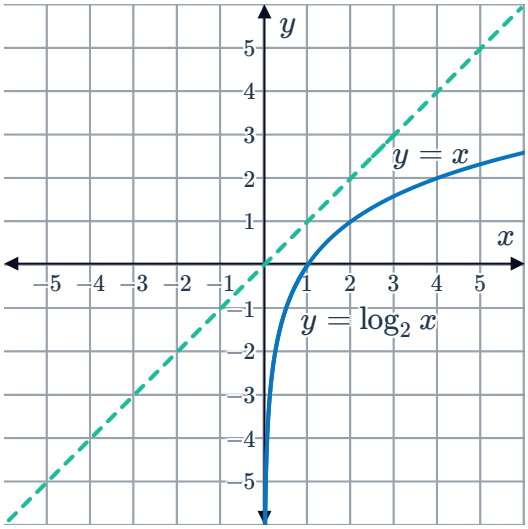
b



c



d



Applications

10 Find the x-coordinate of the point of intersection of the graphs of $y = 2^{5x}$ and $y = 4^{x-3}$.

11 Find the value of h, given the point $\left(h, \frac{1}{9}\right)$ lies on the curve $y = 3^{-x}$.

12 Given the points $(3, n)$, $(k, 16)$ and $\left(m, \frac{1}{4}\right)$ all lie on the curve with equation $y = 2^x$, find the value of:

a n **b** k **c** m

13 A certain type of cell splits in two every hour and each cell produced also splits in two each hour. The total number of cells after t hours is given by:

$$N(t) = 2^t$$

Find the time when the number of cells will reach the following amounts:

a 32 **b** 1024 **c** 4096

14 The frequency $f\ (\text{Hz})$ of the nth key of an 88-key piano is given by $f\ (n) = 440\left(2^{\frac{1}{12}}\right)^{n-49}$.

a Find the frequency of the forty-ninth key.

b Find the frequency of the 40th key to the nearest whole number.

c Find the value of n that corresponds to the key with a frequency of $1760\ \text{Hz}$.