

Arithmetic series

- 1 Find the sum of the first 10 terms of the following arithmetic sequences:
 - a $u_1 = 6$ and $d = 3$
 - b $u_1 = -18$ and $d = 4$
 - c $u_1 = 6$ and $u_{10} = 3.5$
 - d $u_6 = 2$ and $u_7 = 5$
 - e $u_4 = 4$ and $u_7 = 13$
- 2 Find the sum of the first 15 terms of the following arithmetic sequences:
 - a $2 + 6 + 10 + \dots$
 - b $-17 - 15 - 13 - \dots$
 - c $35 + 31 + 27 + \dots$
- 3 Find the sum of the 12 terms in the arithmetic series $12 + \dots + \frac{37}{4}$.
- 4 Find the sum of the first 20 terms of the sequence defined by $u_1 = \pi$ and $u_{20} = 20\pi$.
- 5 Find the sum of the first 40 terms of the following arithmetic sequences:
 - a $u_n = 6n$
 - b $u_n = 5 - 3n$
- 6 In an arithmetic sequence, the first term is 7, the common difference is 3 and the sum of the first n terms is 205. Find the value of n .
- 7 The first term of an arithmetic sequence is 6 and the 6th term is 26.
 - a Find the value of d , the common difference.
 - b Hence, find the sum of the first 13 terms.
- 8 Consider the arithmetic series: $-10 - 6 - 2 + \dots + 58$
 - a Find n , the number of terms in the series.
 - b Hence, find the sum of the series.
- 9 Consider the sum of odd natural numbers: $1 + 3 + 5 + \dots$
Show that the sum of the first n odd natural numbers is a perfect square.

- 10** The first term of an arithmetic sequence is u_1 and its common difference is d .
- a** Form an equation for u_1 in terms of d , given that the 8th term is $\frac{85}{3}$.
 - b** Form another equation for u_1 in terms of d , given that the 15th term is $\frac{113}{3}$.
 - c** Find the value of d .
 - d** Hence, find the value of u_1 .
 - e** Hence, find the sum of the first 25 terms of the sequence.
- 11** The first term of an arithmetic sequence is u_1 and its common difference is d .
- a** Form an equation for u_1 in terms of d , given that the 6th term is $\frac{62}{3}$.
 - b** Form another equation for u_1 in terms of d , given that the 15th term is $\frac{98}{3}$.
 - c** Find the value of d .
 - d** Hence, find the value of u_1 .
 - e** Find n , the number of terms required to give a sum of 750.
- 12** The first term of an arithmetic sequence is u_1 and its common difference is d .
- a** Form an equation for u_1 in terms of d , given that the 15th term is 48.
 - b** Form another equation for u_1 in terms of d , given that the sum of the first 6 terms is 12.
 - c** Find the value of d .
 - d** Hence, find the value of u_1 .
 - e** Hence, find the sum of the first 35 terms of the sequence.
- 13** The first term of an arithmetic sequence is u_1 and its common difference is d .
- a** Given that the sum of the first and third terms is zero, form an equation for u_1 in terms of d .
 - b** Form another equation for u_1 in terms of d , given that the sum of the first 6 terms is 63.
 - c** Find the value of d .
 - d** Hence, find the value of u_1 .

- 14** The first term of an arithmetic sequence is u_1 and its common difference is d .
- a** Form an equation for u_1 in terms of d , given that the sum of the first 8 terms is 60.
 - b** Form another equation for u_1 in terms of d , given that the sum of the first 13 terms is 260.
 - c** Find the value of d .
 - d** Hence, find the value of u_1 .
 - e** Hence, find an expression for the sum of n terms.
 - f** Find the smallest value of n such that the sum of the terms equals 1610.
- 15** The first term of an arithmetic sequence is u_1 and its common difference is d .
- a** Form an equation for u_1 in terms of d , given that the sum of the first 5 terms is 60.
 - b** The sum of the next 5 terms of the sequence is 185. Using this and the information from part a, form another equation for u_1 in terms of d .
 - c** Find the value of d .
 - d** Hence, find the value of u_1 .
 - e** Hence, find the sum of the 19th and 20th terms.
- 16** The sum of the first n terms of an arithmetic sequence is $S_n = 5n^2 - 10n$.
- a** Find the value of u_1 .
 - b** Find the value of the second term.
 - c** Hence, find the common difference, d .
 - d** Form a simplified expression for u_n .
- 17** In an arithmetic sequence, the sum of the first n terms is denoted by S_n .
If $S_{10} = 320$, and $S_9 = 261$, find u_{10} .
- 18** Consider the arithmetic sequence: $4, 1, -2, \dots$
- a** Write a simplified expression for the sum of the first n terms.
 - b** Find the sum of the sequence from the 17th to the 25th term, inclusive.
- 19** Find the sum of all integers between 20 and 50, inclusive, that are divisible by 6.
- 20** Consider an arithmetic sequence in which terms are ascending, and the common difference is d . The first of three consecutive terms in the sequence is y .
- a** Form an equation expressing d in terms of y , given that the sum of the three terms is -6 .
 - b** Form another equation relating y and d , given that the product of the three terms is 90.
 - c** Find y , the first term in the series.
 - d** Hence, state the value of the three terms.

Applications

- 21** Tobias starts his career with a monthly wage of \$3500. At the beginning of each year that follows, he receives a raise and his monthly wage for that year will be \$160 greater than the previous year.
- a** Find his yearly salary in the second year of his service.
 - b** Write down an expression for the total amount earned in m years.
 - c** Find the number of years, y , that it would take for him to earn a total of \$447 120.
- 22** A termite treatment will cost \$250 for the first half hour, \$245 for the second half hour, \$240 for the third half hour and so on. Calculate the cost of a treatment that takes 6 hours.
- 23** A telecommunications company sells 1700 mobile phones in the first month of its operation. The company plans to increase its sales by 150 mobile phones each month.
- a** Calculate the number of phones that the company plan to sell in the last month of the 4th year of its operation.
 - b** Calculate the number of phones that the company plan to sell in the entire 4-year period.
- 24** A ball starts rolling down a slope. It rolls 25 cm during the first second, 53 cm during the 2nd second, 81 cm during the 3rd second and so on.
At this rate, calculate the total distance it will have rolled after 10 seconds.
- 25** A worker at a factory is stacking cylindrical-shaped pipes which are stacked in layers. Each layer contains one pipe less than the layer below it. There are 4 pipes in the topmost layer, 5 pipes in the next layer, and so on. There are n layers in the stack.
- a** Form an expression for the number of pipes in the bottom layer.
 - b** Show that there are a total of $n \left(\frac{n+7}{2} \right)$ pipes in the stack.
- 26** For a sprint training exercise, a number of balls are placed in a straight line. The first ball is 2 m from the start, and there is a 3 m distance between each of the remaining consecutive balls. There are n balls placed out in the line.
Danielle must run from the start, collect the nearest ball and run back to the start, depositing the ball into a box. Then she must run back to collect the next ball, returning with it to the start. She continues this until all n balls have been collected.
- a** Write an expression for how far Danielle runs to collect and deposit the k th ball.
 - b** Write an expression for how far she runs to collect and deposit all n balls.
 - c** She wants to run 374 m in total to collect and deposit all the balls. Find n , the number of balls that would need to be placed in a line.

- 27** Adam is learning to drive. His first lesson is 24 minutes long, and each subsequent lesson is 3 minutes longer than the lesson before.
- a** How long will his 10th lesson be?
 - b** If Adam reaches 45.9 total hours of driving on his n th lesson, find n .
- 28** Caitlin starts training for a 4.85 km charity trail run by running every week for 28 weeks. She runs 1 km of the course in the first week and each week after that she runs 350 m more than the previous week, until she completes the whole course in one week. She then continues to run the whole course each week.
- a** Find the distance that she run in the 12th week, correct to two decimal places.
 - b** If she runs the full course for the first time in week n , find n .
 - c** Find the total distance that Caitlin runs in 28 weeks, correct to two decimal places.
- 29** A fishing trawler spends several days netting crabs. On the first day, it nets 580 kg of crabs, on the next day it nets 563 kg, and the amount netted continues to decrease by the same amount each day.
- a** Calculate the weight of crabs netted on the 14th day.
 - b** Calculate the total weight netted in the first 14 days.
 - c** If the daily weight netted first falls to 240 kg on day n , find n .
 - d** The trawler returns to port when it has netted a total weight of 9400 kg. Find the number of days the trawler spent at sea.