ANSWERS

EXERCISE 1A

1 a 3^7 b x^9 c x^{5+n} d t^{12}
e 7^4 f x^4 g t^{6-x} h t^{3m-1}
i 5^6 j t^{12} k y^{3m} l a^{12m}

2 a 11^2 b 2^5 c 3^4 d 2^4
e 5^4 f 7^{t+2} g 3^{a-2} h 2^{3p-2}
i 7^2 j 3^{2-x} k 5^{4t} l 2^{3k-12}
m 2^{2a-b} n 2^{3x-4y} o 5^{2x+4} p 3^6

3 a x^2y^2 b a^3b^3 c $x^2y^2z^2$ d $27b^3$ e $625a^4$
f $100\,000x^5y^5$ g $\dfrac{p^2}{q^2}$ h $\dfrac{m^3}{n^3}$ i $\dfrac{x^4}{81}$
j $\dfrac{125}{z^3}$ k $\dfrac{16a^4}{b^4}$ l $\dfrac{27x^3}{64y^3}$

4 a $8b^5$ b a^2b^2 c $6a^4b^2$ d $\dfrac{x^2y}{3}$
e $\dfrac{a^6}{125b^3}$ f $\dfrac{8r^2}{5}$ g $16c^4d^3$ h $\dfrac{5k^2}{16}$

5 a 1 b $\frac{1}{6}$ c $\frac{1}{4}$ d 1 e 16 f $\frac{1}{16}$
g 125 h $\frac{1}{125}$ i 49 j $\frac{1}{49}$ k 1000 l $\frac{1}{1000}$

6 a 1 b 1 c $2,\ t \neq 0$ d $1,\ t \neq 0$
e 1 f 3 g $\frac{1}{25}$ h $\frac{1}{16}$ i $\dfrac{1}{x^5}$ j $\frac{8}{3}$
k $\frac{3}{2}$ l 5 m 3 n $\frac{4}{5}$ o $\frac{11}{3}$ p 9
q $\frac{27}{8}$ r $\frac{8}{27}$ s $\frac{25}{16}$ t $\frac{4}{25}$

7 a $\dfrac{1}{3b}$ b $\dfrac{3}{b}$ c $\dfrac{7}{a}$ d $\dfrac{1}{7a}$ e t^2
f $\dfrac{y}{3x}$ g $\dfrac{1}{25t^2}$ h $\dfrac{t^2}{5}$ i $\dfrac{x}{y}$ j $\dfrac{1}{xy}$
k $\dfrac{x}{y^3}$ l $\dfrac{1}{x^3y^3}$ m $\dfrac{1}{3pq}$ n $\dfrac{3}{pq}$ o $\dfrac{3p}{q}$
p x^3y^5 q $\dfrac{125y^9}{x^6}$ r $\dfrac{4d^6}{c^2}$ s $\dfrac{r^6t^2}{9}$ t $\dfrac{125}{8p^3q^6}$

8 a
$$\begin{aligned}\frac{1}{a^{-n}} &= \frac{a^0}{a^{-n}}\\ &= a^{0-(-n)}\\ &= a^{0+n}\\ &= a^n\end{aligned}$$
b
$$\begin{aligned}\left(\frac{a}{b}\right)^{-n} &= \left[\left(\frac{a}{b}\right)^{-1}\right]^n\\ &= \left(\frac{b}{a}\right)^n\\ &= \frac{b^n}{a^n}\end{aligned}$$

9 a $\mathrm{m\,s^{-1}}$ b $\mathrm{m^3\,h^{-1}}$ c $\mathrm{cm^2\,s^{-1}}$ d $\mathrm{cm^3\,min^{-1}}$
e $\mathrm{g\,s^{-1}}$ f $\mathrm{kg\,m\,s^{-1}}$ g $\mathrm{m\,s^{-2}}$

10 $2^{125} = (2^5)^{25} = 32^{25}$ and $3^{75} = (3^3)^{25} = 27^{25}$
As $27^{25} < 32^{25}$, 3^{75} is smaller.

11 The indices have a common factor of 6.
$2^{90} = (2^{15})^6 = 32\,768^6$
$3^{60} = (3^{10})^6 = 59\,049^6$
$5^{36} = (5^6)^6 = 15\,625^6$
$10^{24} = (10^4)^6 = 10\,000^6$
Order: $10^{24},\ 5^{36},\ 2^{90},\ 3^{60}$

EXERCISE 1B

1 a 4 b $\frac{1}{4}$ c 5 d $\frac{1}{5}$ e 2 f $\frac{1}{2}$
g -2 h $-\frac{1}{2}$ i 3 j $\frac{1}{3}$ k 2 l $\frac{1}{2}$

2 a not possible b -1 c $-\frac{1}{3}$ d not possible

3 a $10^{\frac{1}{2}}$ b $10^{-\frac{1}{2}}$ c $15^{\frac{1}{3}}$ d $15^{-\frac{1}{3}}$
e $19^{\frac{1}{4}}$ f $19^{-\frac{1}{4}}$ g $13^{\frac{1}{5}}$ h $13^{-\frac{1}{5}}$

4 a 4 b 8 c 32 d 32 e 8 f 27
g $\frac{1}{27}$ h $\frac{1}{2}$ i 2 j 4 k 8 l $\frac{1}{8}$
m $\frac{1}{4}$ n $\frac{1}{81}$ o $\frac{1}{125}$

5 a $2^{\frac{3}{2}}$ b $2^{\frac{5}{3}}$ c $2^{\frac{1}{2}}$ d $2^{\frac{4}{3}}$ e $2^{-\frac{3}{4}}$ f $2^{-\frac{4}{3}}$
g $2^{-\frac{3}{7}}$ h $2^{-\frac{6}{5}}$ i $2^{\frac{7}{2}}$ j $2^{\frac{9}{2}}$ k $2^{\frac{1}{3}}$ l $2^{\frac{3}{2}}$

6 a $3^{\frac{2}{3}}$ b $3^{\frac{3}{2}}$ c $3^{-\frac{3}{4}}$ d $3^{-\frac{4}{5}}$ e $3^{\frac{5}{2}}$ f $3^{\frac{5}{2}}$
g $3^{\frac{9}{5}}$ h $3^{-\frac{2}{3}}$

7 a $5^{\frac{2}{3}}$ b $2^{\frac{5}{4}}$ c $5^{\frac{3}{5}}$ d $11^{\frac{2}{7}}$ e $7^{-\frac{2}{3}}$ f $2^{\frac{6}{5}}$
g $5^{-\frac{4}{7}}$ h $3^{-\frac{5}{6}}$ i $2^{\frac{11}{2}}$ j $5^{\frac{7}{2}}$ k $13^{\frac{1}{3}}$ l $3^{\frac{5}{2}}$

8 a 125 b 9 c 128 d ≈ 2.41
e ≈ 2.68 f ≈ 91.2 g ≈ 1.93 h ≈ 849
i ≈ 2.13 j ≈ 9.16 k 0.0313 l $\approx 0.004\,12$
m ≈ 0.339 n ≈ 0.182 o ≈ 0.215

9 3

EXERCISE 1C

1 a 2.3×10^2 b 5.39×10^4 c 3.61×10^{-2}
d 6.8×10^{-3} e 3.26×10^0 f 5.821×10^{-1}
g 3.61×10^8 h 1.674×10^{-6}

2 a 2300 b 0.023 c $564\,000$
d $0.000\,793\,1$ e 9.97 f $60\,400\,000$
g 0.4215 h $0.000\,000\,036\,21$

3 a $\approx 4 \times 10^6$ red blood cells b 8×10^{-4} m
c 6.38×10^6 m d 4.3252×10^{19} arrangements

4 a $6\,990\,000$ m b 0.018 cm
c $32\,000\,000$ bacteria d $0.000\,008\,2$ t

5 a 6×10^{10} b 2.8×10^9 c 5.6×10^{-8}
d 5.4×10^{-6} e 9×10^{10} f 1.6×10^{15}
g 1.6×10^{-11} h 1.25×10^{-7} i 1.2×10^0
j 2.88×10^7 k 2.5×10^{-4} l 4×10^6

6 a 2×10^3 b 3×10^{-2} c 2×10^8
d 1×10^{-19} e 3.2×10^3 f 8×10^8

7 a 1000 times b i 5×10^{-21} ii $100\,000$ times
c 5×10^{10} times

8 a $\approx 4.01 \times 10^{13}$ b $\approx 2.59 \times 10^{12}$ c $\approx 7.08 \times 10^{-9}$
d $\approx 4.87 \times 10^{-11}$ e $\approx 8.01 \times 10^6$ f $\approx 3.55 \times 10^{-9}$
g $\approx 1.57 \times 10^{13}$ h $\approx 6.55 \times 10^{-22}$

9 a 5.84×10^7 b i 1.60×10^5 ii 5.84×10^8

10 **a** **i** 2.88×10^8 km **ii** $\approx 7.01 \times 10^{10}$ km **b** 0.9 s
c ≈ 26.6 times **d** **i** Microbe C **ii** ≈ 32.9 times

11 As land areas are rounded to 2 significant figures, the answers should be rounded to 2 significant figures.
a 9.1×10^6 km^2
b Quebec > British Columbia > Ontario > Alberta > Saskatchewan > Manitoba > Newfoundland and Labrador > New Brunswick > Nova Scotia > Prince Edward Island
c **i** ≈ 2.5 times **ii** ≈ 330 times **d** $\approx 0.58\%$

REVIEW SET 1A

1 **a** k^8 **b** p^5 **c** m^{48}

2 **a** $9w^2$ **b** $8x^6y^3$ **c** $\dfrac{a^6}{b^6}$ **d** $\dfrac{1}{125n^3}$

3 **a** $\frac{1}{7}$ **b** $\frac{3}{4}$ **c** $\frac{10}{11}$ **d** $\frac{16}{49}$

4 **a** 5.9×10^4 **b** 9×10^{-3} **c** 6.085×10^6 **d** 7.71×10^{-6}

5 **a** 7 **b** $\frac{1}{4}$ **c** 625 **d** $\frac{1}{9}$

6 **a** 623 000 **b** 0.000 300 8 **c** 4.597

7 **a** $2^{\frac{4}{5}}$ **b** $3^{-\frac{2}{3}}$ **c** $5^{\frac{7}{2}}$ **d** $2^{\frac{11}{2}}$

8 **a** ≈ 2.71 **b** ≈ 0.464 **c** ≈ 17.8 **d** ≈ 0.313

9 **a** $\dfrac{1}{m^2n^2}$ **b** $\dfrac{m}{n^2}$ **c** $\dfrac{125y^6}{x^3}$

10 **a** 1.8×10^{12} **b** 4×10^6 **c** 1.5×10^{-8} **d** 1.25×10^{-8}

11 **a** **i** $\approx 1.079 \times 10^{12}$ m **ii** $\approx 2.590 \times 10^{13}$ m
iii $\approx 9.461 \times 10^{15}$ m
b **i** $\approx 3.336 \times 10^{-9}$ s **ii** $\approx 3.336 \times 10^{-11}$ s
iii $\approx 3.336 \times 10^{-12}$ s
c $\approx 8.71 \times 10^5$ times faster

REVIEW SET 1B

1 **a** 2^6 **b** 5^{2x-3} **c** 7^{k+7}

2 **a** $15c^7$ **b** $7x^3y$ **c** $9p^2q^6$ **3** 81

4 **a** $\dfrac{b^2}{a^2}$ **b** $\dfrac{8a^3b^6}{27}$

5 **a** **i** $13^{\frac{1}{3}}$ **ii** $40^{-\frac{1}{5}}$ **b** ≈ 0.478

6 **a** 2.1×10^{15} **b** 6×10^{-12} **c** 4×10^{10}

7 **a** $2^{\frac{3}{4}}$ **b** $2^{-\frac{4}{5}}$ **c** $2^{-\frac{1}{2}}$ **8** 70 times

9 7^{20} as $2^{60} = (2^3)^{20} = 8^{20}$ and $8^{20} > 7^{20}$

10 **a** **i** 1.21×10^5 km **ii** 1.21×10^{10} cm
b 6.05×10^6 m
c Mercury < Mars < Venus < Earth < Neptune < Uranus < Saturn < Jupiter
d **i** ≈ 10.5 times larger **ii** 20.6 times larger

11 **a** 7^9 **b** As $(\sqrt[5]{7} \times \sqrt[4]{7})^{20} = 7^9$ then $\sqrt[5]{7} \times \sqrt[4]{7} = 7^{\frac{9}{20}}$ **d** $\sqrt[3]{11} \times \sqrt[5]{11} = 11^{\frac{3+5}{3\times5}} = 11^{\frac{8}{15}}$

EXERCISE 2A

1 **a** $8 \in P$ **b** $k \notin S$ **c** $14 \notin$ {odd numbers}
d $n(Y) = 9$

2 **a** true **b** true **c** true **d** true
e false **f** false **g** true **h** true

3 **a** rational **b** rational **c** rational **d** rational
e irrational **f** neither **g** rational **h** rational
i neither **j** rational

4 **a** **i** $A = \{1, 2, 3, 6\}$ **ii** finite **iii** $n(A) = 4$
b **i** $B = \{6, 12, 18, 24, ...\}$ **ii** infinite
c **i** $C = \{1, 17\}$ **ii** finite **iii** $n(C) = 2$
d **i** $D = \{17, 34, 51, 68, ...\}$ **ii** infinite
e **i** $E = \{2, 3, 5, 7, 11, 13, 17, 19\}$ **ii** finite
iii $n(E) = 8$
f **i** $F = \{12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28\}$
ii finite **iii** $n(F) = 13$

5 **a** $0.\dot{7} = \frac{7}{9}$ **b** $0.\overline{41} = \frac{41}{99}$ **c** $0.\overline{324} = \frac{12}{37}$

6 0.527 can be written as $\frac{527}{1000}$, where 527 and 1000 are integers

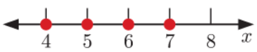
7 $0.\dot{9} = 1$ and $1 \in \mathbb{Z}$

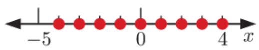
8 **Note:** There may be other answers.
a $\sqrt{2}, -\sqrt{2}$ are irrational, but $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational.
b $\sqrt{2}, \sqrt{50}$ are irrational, but $\sqrt{2} \times \sqrt{50} = \sqrt{100} = 10$ which is rational.

EXERCISE 2B

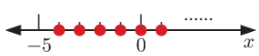
1 **a** The set of all real x such that x is greater than 4.
b The set of all integers x such that x is less than or equal to 5.
c The set of all real y such that y lies between 0 and 8.
d The set of all integers x such that x lies between 1 and 4 including 1 and 4.
e The set of all real t such that t lies between 2 and 7.
f The set of all real n such that n is less than or equal to 3 or n is greater than 6.

2 **a** $\{x \mid x > 3\}$ **b** $\{x \mid 2 < x \leqslant 5\}$
c $\{x \mid x \leqslant -1 \text{ or } x \geqslant 2\}$ **d** $\{x \mid -1 \leqslant x < 5,\ x \in \mathbb{Z}\}$
e $\{x \mid 0 \leqslant x \leqslant 6,\ x \in \mathbb{N}\}$ **f** $\{x \mid x < 0\}$

3 **a**


b


c


d


e


f


4 **a** $\{x \mid x > 7\}$ **b** $\{x \mid -8 < x < 15,\ x \in \mathbb{Z}\}$
c $\{x \mid 4 \leqslant x < 6,\ x \in \mathbb{Q}\}$

EXERCISE 2C

1 **a** $A \subseteq B$ **b** $A \nsubseteq B$ **c** $A \subseteq B$ **d** $A \nsubseteq B$
e $A \subseteq B$

2 **a** true **b** true **c** false **d** true **e** false **f** true

3 $B \nsubseteq A$, $C \subseteq A$

4 **a** false **b** true **c** false **d** true

5 **a** $A' = \{1, 3, 4, 7, 8, 9\}$ **b** $B' = \{1, 4, 6, 8, 9\}$
c $C' = \{2, 4, 6, 8\}$ **d** $D' = \{1, 2, 3, 5, 6, 7, 9\}$
e $E' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (or $E' = U$)
f $F' = \{3, 4, 5, 6, 7, 8, 9\}$

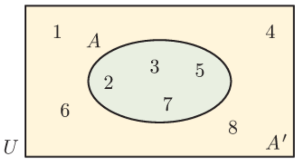
6 **a** $P' = \{$A, B, D, E, G, H, I, K, L, N, O, Q, R, S, T, V, W, X$\}$
b $Q' = \{$vowels$\}$
c $R' = \{$B, C, D, E, F, G, J, K, M, N, Q, R, U, V, W, X, Y, Z$\}$
d $S' = \{$A, B, C, D, E, F, G, H, I, J$\}$

7 **a** $A' = \{1, 2, 3, 4, 13, 14, 15\}$
$B' = \{1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15\}$

b **i** false **ii** true **iii** true **iv** false

EXERCISE 2D

1 **a**

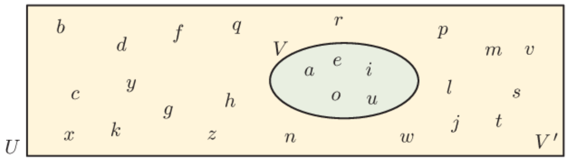


$A = \{2, 3, 5, 7\}$

b $A' = \{1, 4, 6, 8\}$

c **i** $n(A) = 4$ **ii** $n(A') = 4$ **iii** $n(U) = 8$

2 **a**



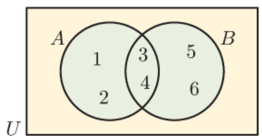
b $V' = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

c **i** $n(V) = 5$ **ii** $n(V') = 21$ **iii** $n(U) = 26$

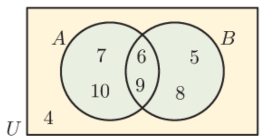
3 **a** **i** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
ii $N = \{3, 8\}$ **iii** $M = \{1, 3, 4, 7, 8\}$

b $n(N) = 2$, $n(M) = 5$ **c** No, $N \subseteq M$.

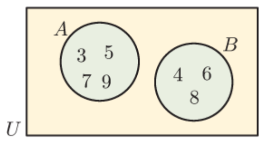
4 **a**



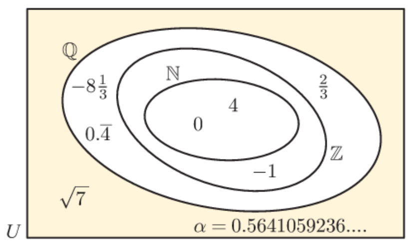
b



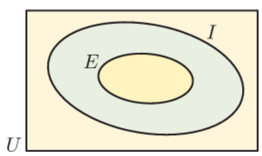
c



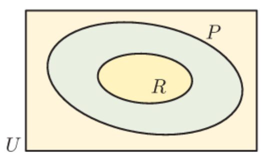
5 **a**, **b**, **c**



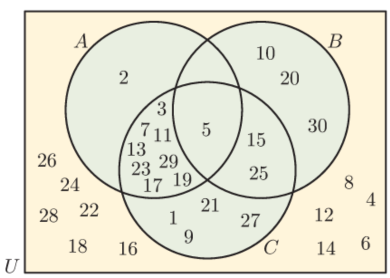
6 **a**



b



7



EXERCISE 2E.1

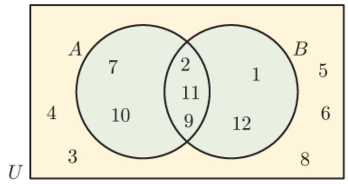
1 **a** **i** $C = \{1, 3, 7, 9\}$ **ii** $D = \{1, 2, 5\}$
iii $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ **iv** $C \cap D = \{1\}$
v $C \cup D = \{1, 2, 3, 5, 7, 9\}$

b **i** $n(C) = 4$ **ii** $n(D) = 3$ **iii** $n(U) = 9$
iv $n(C \cap D) = 1$ **v** $n(C \cup D) = 6$

2 **a** **i** $A = \{2, 7\}$ **ii** $B = \{1, 2, 4, 6, 7\}$
iii $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ **iv** $A \cap B = \{2, 7\}$
v $A \cup B = \{1, 2, 4, 6, 7\}$

b **i** $n(A) = 2$ **ii** $n(B) = 5$ **iii** $n(U) = 8$
iv $n(A \cap B) = 2$ **v** $n(A \cup B) = 5$

3 **a**



b **i** $A \cap B = \{2, 9, 11\}$
ii $A \cup B = \{1, 2, 7, 9, 10, 11, 12\}$
iii $B' = \{3, 4, 5, 6, 7, 8, 10\}$

c **i** $n(A) = 5$ **ii** $n(B') = 7$
iii $n(A \cap B) = 3$ **iv** $n(A \cup B) = 7$

4 **a** $A \cap B = \{1, 3, 9\}$
b $A \cup B = \{1, 2, 3, 4, 6, 7, 9, 12, 18, 21, 36, 63\}$

5 **a** $X \cap Y = \{\text{B, M, T, Z}\}$
b $X \cup Y = \{\text{A, B, C, D, M, N, P, R, T, W, Z}\}$

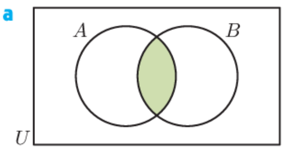
6 **a** **i** $n(A) = 8$ **ii** $n(B) = 10$
iii $n(A \cap B) = 3$ **iv** $n(A \cup B) = 15$

b $n(A) + n(B) - n(A \cap B) = 8 + 10 - 3$
$= 15 = n(A \cup B)$

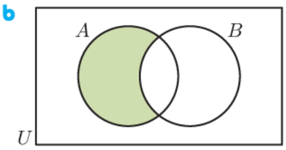
7 **a** $\varnothing$ **b** U **c** $\varnothing$

EXERCISE 2E.2

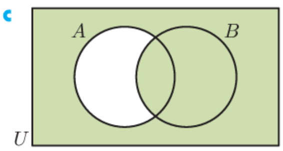
1 **a**



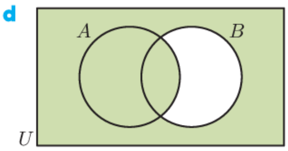
b



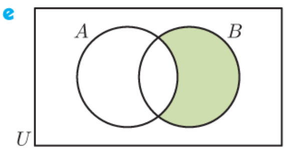
c



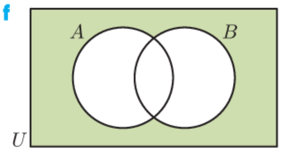
d



e

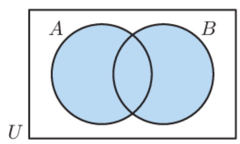


f



2 **a** in A but not in B
b the complement of 'in exactly one of A or B'

3 **a** **i**



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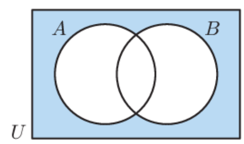

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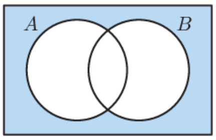
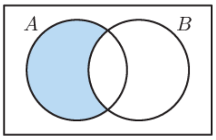
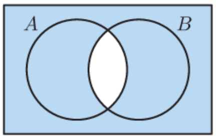
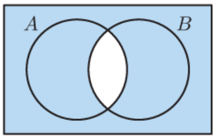
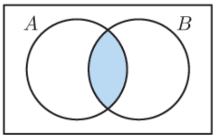


iv


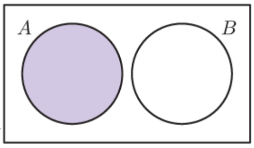
v


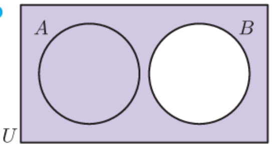
vi


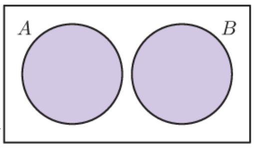
vii


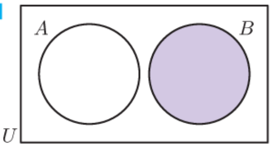
b **i** Since **a v** and **a vi** have the same regions shaded, $(A \cap B)' = A' \cup B'$.

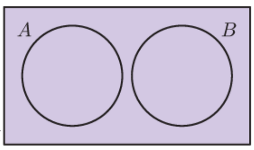
ii Since **a ii** and **a iii** have the same regions shaded, $(A \cup B)' = A' \cap B'$.

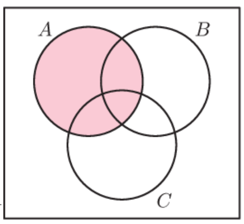
4 **a**


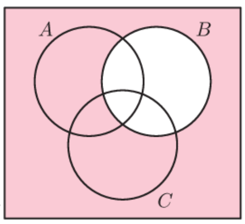
b


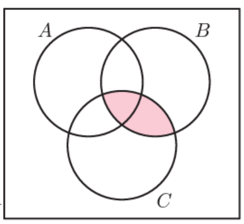
c


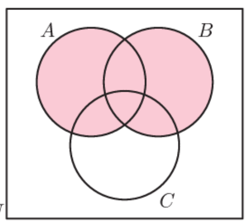
d


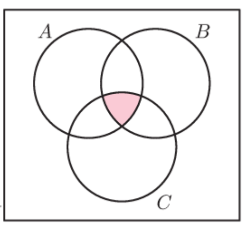
e


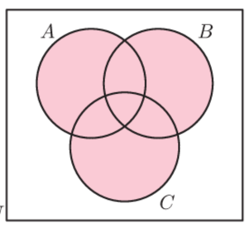
5 **a** **i**


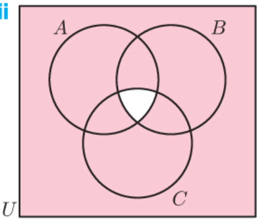
ii


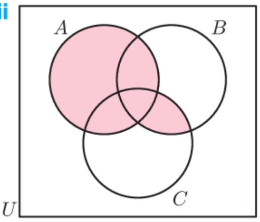
iii


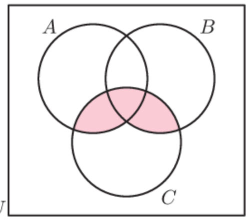
iv


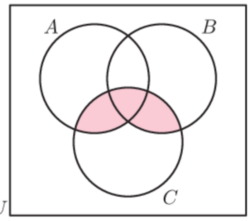
v


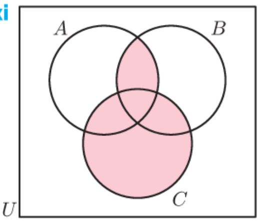
vi


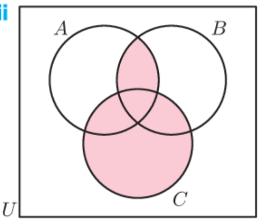
vii


viii


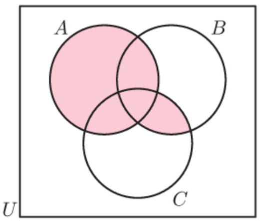
ix


x


xi


xii


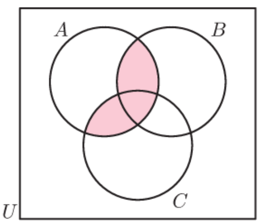
b **i** $(A \cup B) \cap (A \cup C)$ can be illustrated by:



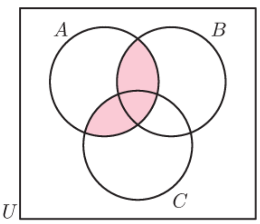
This is identical to the Venn diagram in **a viii**, which illustrates $A \cup (B \cap C)$.

Hence
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

ii $A \cap (B \cup C)$:



$(A \cap B) \cup (A \cap C)$:



The Venn diagrams have the same regions shaded, so $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

EXERCISE 2F

1 **a** 10 **b** 9 **c** 14 **d** 4 **e** 7 **f** 2

2 **a** **i** $n(A) = a + b$ **ii** $n(B) = b + c$

iii $n(A \cap B) = b$ **iv** $n(A \cup B) = a + b + c$

b **i**
$$\begin{aligned} n(A) + n(B) - n(A \cap B) &= a + b + b + c - b \\ &= a + b + c \\ &= n(A \cup B) \end{aligned}$$

ii
$$\begin{aligned} & n(A) + n(B) - n(A \cup B) \\ &= a + b + b + c - (a + b + c) \\ &= a + 2b + c - a - b - c \\ &= b \\ &= n(A \cap B) \end{aligned}$$

iii If A and B are disjoint then $n(A\cap B)=b=0$.
So, $n(A\cup B)=a+b+c$
$=a+c$
and $n(A)+n(B)=a+b+b+c$
$=a+c$
$=n(A\cup B)$

3 **a** $n(A)-n(A\cap B)=(a+b)-b$
$=a$
$\therefore\ n(A\cap B')=n(A)-n(A\cap B)$

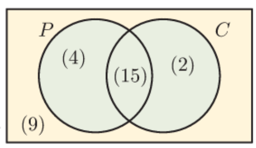
b $n(U)-n(A'\cap B)=(a+b+c+d)-c$
$=a+b+d$
$\therefore\ n(A\cup B')=n(U)-n(A'\cap B)$

4 **a** 17 **b** 5 **5** **a** 7 **b** 10

EXERCISE 2G

1 **a** 18 **b** 2 **c** 17 **d** 12

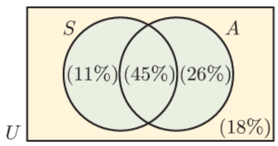
2 **a** 75 **b** 9 **c** 24 **d** 42

3


a 21 **b** 4 **c** 6 **d** 9

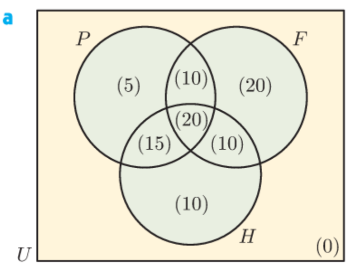
4 **a** 20 **b** 32 **c** 25 **d** 13 **5** **a** 10 **b** 5

6 **a** 18 **b** 38 **7** **a** 15 **b** 14 **c** 8

8 **a**


b **i** 45% **ii** 82% **iii** 11% **iv** 37%

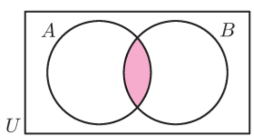
9 11 violin players **10** 43% **11** 19 places

12 **a**


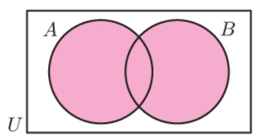
b 15 students **c** 55 students

13 The number who participate in all three sports must be less than or equal to 30.

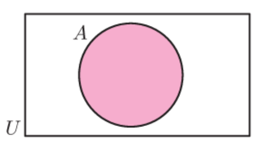
EXERCISE 2H

1 **a**


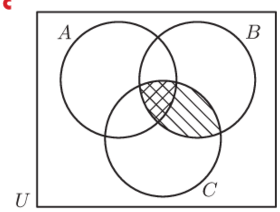
The shaded region could be either $A\cap B$ or $B\cap A$.
$\therefore\ A\cap B=B\cap A$



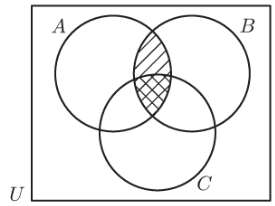
The shaded region could be either $A\cup B$ or $B\cup A$.
$\therefore\ A\cup B=B\cup A$

b


The shaded region is either $A\cap A$, A, or $A\cup A$.
$\therefore\ A\cap A=A$
and $A\cup A=A$

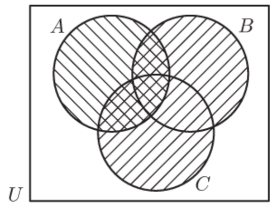
c


represents $B\cap C$
represents $A\cap(B\cap C)$

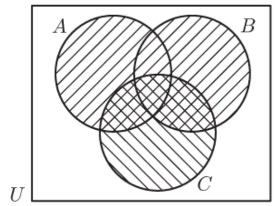


represents $A\cap B$
represents $(A\cap B)\cap C$

Region shaded is the same in each case.
$\therefore\ A\cap(B\cap C)=(A\cap B)\cap C$

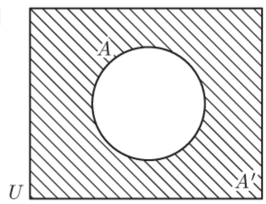


represents A
represents $B\cup C$
whole shaded region represents $A\cup(B\cup C)$



represents C
represents $A\cup B$
whole shaded region represents $(A\cup B)\cup C$

Total shaded region is the same in each case.

d


represents A
represents A'
A' is the region outside A and is shaded.
$(A')'$ is the region not in A' and is unshaded.
$\therefore\ (A')'=A$

2 **a** $A\cup(B\cup A')$
$=A\cup(A'\cup B)$ {commutative law}
$=(A\cup A')\cup B$ {associative law}
$=U\cup B$ {complement law}
$=U$ {domination law}

b $A\cap(B\cap A')$
$=A\cap(A'\cap B)$ {commutative law}
$=(A\cap A')\cap B$ {associative law}
$=\varnothing\cap B$ {complement law}
$=\varnothing$ {domination law}

c $A\cup(B\cap A')$
$=(A\cup B)\cap(A\cup A')$ {distributive law}
$=(A\cup B)\cap U$ {complement law}
$=A\cup B$ {identity law}

d $(A'\cup B')'$
$=((A\cap B)')'$ {De Morgan's law}
$=A\cap B$ {involution law}

e $(A\cup B)\cap(A'\cap B')$
$=(A\cup B)\cap(A\cup B)'$ {De Morgan's law}
$=\varnothing$ {complement law}

f $(A\cup B)\cap(C\cup D)$
$=((A\cup B)\cap C)\cup((A\cup B)\cap D)$ {distributive law}
$=(A\cap C)\cup(B\cap C)\cup(A\cap D)\cup(B\cap D)$
{distributive law}
$=(A\cap C)\cup(A\cap D)\cup(B\cap C)\cup(B\cap D)$
{commutative law}

REVIEW SET 2A

1 1.3 can be written as $\frac{13}{10}$, and 13 and 10 are integers

2 false

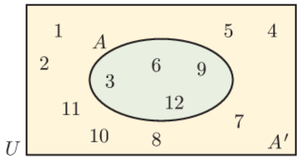
3 **a** yes **b** $P = \{23, 29, 31, 37\}$, $n(P) = 4$

4 S is the set of all real t such that t lies between -1 and 3, including -1.

5 $\{x \mid 0 < x \leqslant 5\}$ **6** **a** $P \not\subseteq Q$ **b** $P \subseteq Q$

7 **a** $A' = \{1, 2, 4, 5, 6, 8, 10\}$ **b** $B' = \{1, 2, 3, 5, 7\}$

8 **a**

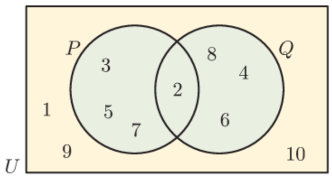


b $A' = \{1, 2, 4, 5, 7, 8, 10, 11\}$ **c** $n(A') = 8$

9 **a** False, as $0 \in \mathbb{N}$, but $0 \notin \mathbb{Z}^+$.
b False, as $\frac{1}{2} \in \mathbb{Q}$, but $\frac{1}{2} \notin \mathbb{Z}$.

10 **a** **i** $A = \{1, 2, 3, 4, 5\}$ **ii** $B = \{1, 2, 7\}$
iii $U = \{1, 2, 3, 4, 5, 6, 7\}$
iv $A \cup B = \{1, 2, 3, 4, 5, 7\}$ **v** $A \cap B = \{1, 2\}$
b **i** $n(A) = 5$ **ii** $n(B) = 3$ **iii** $n(A \cup B) = 6$

11 **a**



b **i** $P \cap Q = \{2\}$ **ii** $P \cup Q = \{2, 3, 4, 5, 6, 7, 8\}$
iii $Q' = \{1, 3, 5, 7, 9, 10\}$
c **i** $n(P') = 6$ **ii** $n(P \cap Q) = 1$ **iii** $n(P \cup Q) = 7$
d yes

12 **a** The shaded region is the complement of X, that is, everything that is not in X.
b The shaded region represents 'in exactly one of X or Y but not both'.
c The shaded region represents everything in X or in neither set.

13 **a** 11 **b** 14 **c** 21 **d** 2 **14** 200 families

REVIEW SET 2B

1 No **2** $0.\overline{51} = \frac{17}{33}$, and 17 and 33 are integers.

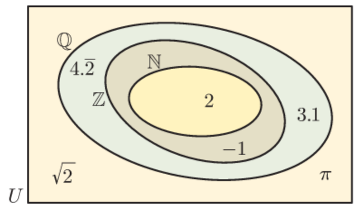
3

4 **a** **i** $A = \{1, 3, 5, 15\}$ **ii** finite **iii** 4
b **i** $B = \{8, 16, 24, 32, ...\}$ **ii** infinite
c **i** $C = \{31, 33, 35, 37, 39, 41, 43, 45, 47, 49\}$
ii finite **iii** 10
d **i** $D = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
ii finite **iii** 10

5 $Q \subseteq P$, $R \not\subseteq P$

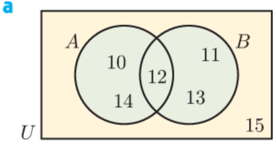
6 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
a $A = \{2, 3, 5, 7, 11\}$ **b** $A' = \{1, 4, 6, 8, 9, 10, 12\}$
c $n(A) = 5$ **d** $n(A') = 7$ **e** $n(U) = 12$

7

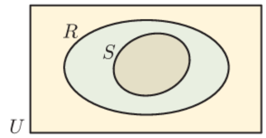


U is the set of all real numbers, $\mathbb{R}$.

8 **a**

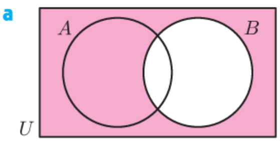


b

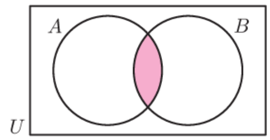


9 **a** $A \cap B = \{1, 2, 3, 6\}$
b $A \cup B = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$

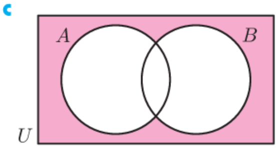
10 **a**



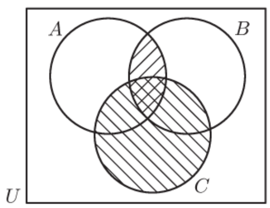
b

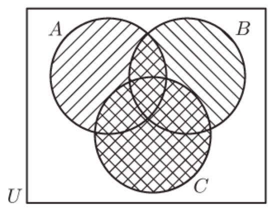


c



11





represents $A \cap B$
represents C
whole shaded region represents $(A \cap B) \cup C$

represents $A \cup C$
represents $B \cup C$
represents $(A \cup C) \cap (B \cup C)$

Area shaded is the same in each case.

12 **a** 18 **b** 4

13 **a** 65% **b** 35% **c** 22% **d** 28% **e** 9%

14
$$\begin{aligned} A \cap (B \cup A') &= (A \cap B) \cup (A \cap A') && \{\text{distributive law}\} \\ &= (A \cap B) \cup \varnothing && \{\text{complement law}\} \\ &= A \cap B && \{\text{identity law}\} \end{aligned}$$

EXERCISE 3A

1 **a** $6x + 15$ **b** $4x^2 - 12x$ **c** $-6 - 2x$
d $-3x^2 - 3xy$ **e** $2x^3 - 2x$ **f** $x^3 - x$
g $a^2b - ab^2$ **h** $x^3 - 3x^2$ **i** $3a^2 + 9a + 3$
j $5x^2 - 15x + 10$ **k** $-8c^2 + 12c + 28$
l $6a^3 - 10a^2 + 2a$

2 **a** $7x - 14$ **b** $-5x - 6$ **c** $4x - x^2$ **d** $2x^3$
e $a^2 + b^2$ **f** $-x^3 + 9x^2 - 12x$

3 **a** $x^2 + 7x + 10$ **b** $x^2 + x - 12$ **c** $x^2 + 2x - 15$
d $x^2 - 12x + 20$ **e** $2x^2 - 5x - 3$ **f** $6x^2 - 23x + 20$

g $2x^2 - xy - y^2$ **h** $-2x^2 - 7x - 3$
i $-x^2 - x - 2xy - 2y$

4 **a** $x^2 + 5x - 18$ **b** $2x^2 + 7x - 31$ **c** $x^2 - 8x - 12$
d $-7t + 2$ **e** $2x^2 + 3$ **f** $10x^2 + 53x - 96$

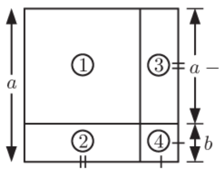
5 **a** $x^2 - 49$ **b** $9 - a^2$ **c** $25 - x^2$ **d** $4x^2 - 1$
e $16 - 9y^2$ **f** $9x^2 - 16z^2$

6 **a** 27 **b** $22p^2 + p - 4$ **c** $5y^2 + 8z^2$ **d** $8x^4$

7 **a** $x^2 + 10x + 25$ **b** $4x^2 + 12x + 9$
c $x^2 + 14x + 49$ **d** $9x^2 + 24x + 16$
e $x^4 + 10x^2 + 25$ **f** $9x^4 + 12x^2 + 4$
g $25x^2 + 30xy + 9y^2$ **h** $4x^4 + 28x^2y + 49y^2$
i $x^6 + 16x^4 + 64x^2$

8 **a** $x^2 - 6x + 9$ **b** $x^2 - 4x + 4$
c $9x^2 - 6x + 1$ **d** $25p^2 - 60p + 36$
e $4x^2 - 20xy + 25y^2$ **f** $a^2b^2 - 4ab + 4$
g $x^4 - 10x^2 + 25$ **h** $16x^4 - 24x^2y + 9y^2$
i $x^4 + 2x^2y^2 + y^4$

9



Hint:
Area ①
$= \text{total area} - 2 \times \text{area}$ ②
$-$ area ④

10 **a** $2x^2 + 14x + 85$ **b** $5x^2 + 18x - 8$ **c** $19x + 74$
d $p^4 - 7p^2 - 10p + 41$ **e** $9x^4 - 10x^2 + 8x - 3$
f $y^4 - x^5 + 2x^3y + 9xy^2 + 25x^2$

EXERCISE 3B

1 **a** $x^3 + 3x^2 + 6x + 8$ **b** $x^3 + 5x^2 + 3x - 9$
c $x^3 + 5x^2 + 7x + 3$ **d** $2x^3 + x^2 - 6x - 5$
e $2x^3 + 7x^2 + 8x + 3$ **f** $2x^3 - 9x^2 + 4x + 15$
g $3x^3 + 14x^2 - x + 20$ **h** $8x^3 - 14x^2 + 7x - 1$

2 **a** $x^3 + 9x^2 + 26x + 24$ **b** $x^3 - x^2 - 14x + 24$
c $x^3 - 10x^2 + 31x - 30$ **d** $2x^3 + x^2 - 12x + 9$
e $12x^3 + 11x^2 - 2x - 1$ **f** $-3x^3 + 26x^2 - 33x - 14$
g $-3x^3 + 16x^2 - 12x - 16$ **h** $x^3 + 9x^2 + 27x + 27$
i $x^3 - 6x^2 + 12x - 8$

3 **a** $2 \times 2 = 4$ terms **b** $3 \times 2 = 6$ terms
c $2 \times 3 = 6$ terms **d** $3 \times 3 = 9$ terms
e $4 \times 2 = 8$ terms **f** $4 \times 3 = 12$ terms

4 **a** $x^4 + 2x^3 + x^2 + 8x + 3$
b $2x^4 + 7x^3 - 2x^2 - 5x + 2$
c $6x^4 - 7x^3 - 8x^2 + 13x - 4$
d $x^4 - x^3 - 19x^2 + 49x - 30$

EXERCISE 3C

1 **a** $x^3 + 3x^2 + 3x + 1$ **b** $x^3 + 9x^2 + 27x + 27$
c $x^3 + 15x^2 + 75x + 125$ **d** $x^3 + 3x^2y + 3xy^2 + y^3$
e $x^3 - 3x^2 + 3x - 1$ **f** $x^3 - 15x^2 + 75x - 125$
g $x^3 - 12x^2 + 48x - 64$ **h** $x^3 - 3x^2y + 3xy^2 - y^3$
i $8 + 12y + 6y^2 + y^3$ **j** $8x^3 + 12x^2 + 6x + 1$
k $27x^3 + 27x^2 + 9x + 1$
l $8y^3 + 36xy^2 + 54x^2y + 27x^3$
m $8 - 12y + 6y^2 - y^3$ **n** $8x^3 - 12x^2 + 6x - 1$
o $27x^3 - 27x^2 + 9x - 1$
p $8y^3 - 36xy^2 + 54x^2y - 27x^3$

3 **a** $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
b $x^4 + 4x^3 + 6x^2 + 4x + 1$
c $x^4 + 8x^3 + 24x^2 + 32x + 16$
d $x^4 + 12x^3 + 54x^2 + 108x + 81$
e $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
f $x^4 - 4x^3 + 6x^2 - 4x + 1$
g $x^4 - 8x^3 + 24x^2 - 32x + 16$
h $16x^4 - 32x^3 + 24x^2 - 8x + 1$

4 **a** 1 5 10 10 5 1
1 6 15 20 15 6 1
We start and finish each row with a 1. The other entries are obtained by adding the two adjacent numbers in the row above.

b **i** $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
ii $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
iii $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$
iv $a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$

c **i** $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$
ii When $x = 1$, $(x-2)^5 = (-1)^5 = -1$
and $1^5 - 10 \times 1^4 + 40 \times 1^3 - 80 \times 1^2 + 80 \times 1 - 32$
$= 1 - 10 + 40 - 80 + 80 - 32$
$= -1$ ✓

EXERCISE 3D

1 **a** $x(x-5)$ **b** $2x(x+3)$ **c** $2x(2-y)$
d $3b(a-2)$ **e** $2x^2(1+4x)$ **f** $6x(2-x)$
g $x^2(x+1)$ **h** $3ab(b-3a)$

2 **a** $(x+5)(3+x)$ **b** $(b+3)(a-5)$ **c** $(x+4)(x+1)$
d $(x+2)(2x+5)$ **e** $(c-d)(a+b)$ **f** $(2+y)(y-1)$
g $(x-1)(ab+c)$ **h** $(x+2)(a-1)$ **i** $(x-3)(x-2)$
j $(x+5)(x+8)$ **k** $2x(x-2)$
l $(x+y)(x+y+1)(x+y-1)$

3 **a** $(x+4)(x-4)$ **b** $(8+x)(8-x)$
c $(3x+1)(3x-1)$ **d** $(7+2x)(7-2x)$
e $(y+2x)(y-2x)$ **f** $(2a+5b)(2a-5b)$
g $(9x+4y)(9x-4y)$ **h** $(2x^2+y)(2x^2-y)$
i $(3ab+4)(3ab-4)$ **j** $(x+5)(x+1)$
k $3(3x+2)(x-2)$ **l** $3(x-3)(x-1)$

4 **a** $2(x+2)(x-2)$ **b** $3(y+3)(y-3)$
c $2(1+3x)(1-3x)$ **d** $x(2+3x)(2-3x)$
e $ab(a+b)(a-b)$ **f** $2(5+xy)(5-xy)$
g $b(3b+2)(3b-2)$ **h** $x(x^2+y^2)(x+y)(x-y)$

5 **a** $(x+2)^2$ **b** $(x-5)^2$ **c** $(3x+5)^2$ **d** $(x-4)^2$
e $(2x+7)^2$ **f** $(x-10)^2$

6 **a** $-(3x-1)^2$ **b** $3(x+3)^2$ **c** $-2(3x-1)^2$
d $2(x+5)(x-5)$ **e** $2(x-4)^2$ **f** $-3(x+3)^2$

EXERCISE 3E

1 **a** $(a+1)(2+b)$ **b** $(c+d)(a+4)$ **c** $(a+2)(b+3)$
d $(n+3)(m+p)$ **e** $(x+5)(2y-1)$ **f** $(3-c)(2a+b)$

2 **a** $(x+4)(x+2)$ **b** $(x+3)(x+7)$
c $(x+5)(x+4)$ **d** $(2x+1)(x+3)$
e $(3x+2)(x+4)$ **f** $(5x+3)(4x+1)$

3 **a** $(x-4)(x+5)$ **b** $(x-7)(x+2)$ **c** $(x-3)(x-2)$
d $(x-5)(x-3)$ **e** $(x+7)(x-8)$ **f** $(2x+1)(x-3)$
g $(3x+2)(x-4)$ **h** $(4x-3)(x-2)$ **i** $(9x+2)(x-1)$

EXERCISE 3F

1 **a** $(x+1)(x+2)$ **b** $(x+2)(x+3)$ **c** $(x+2)(x-3)$
d $(x-2)(x+5)$ **e** $(x-3)(x+7)$ **f** $(x+4)^2$
g $(x-7)^2$ **h** $(x-4)(x+7)$ **i** $(x-3)(x-8)$
j $(x+4)(x+11)$ **k** $(x+7)(x-8)$ **l** $(x-9)^2$
m $(x+4)(x-8)$ **n** $(x-5)(x+9)$
o $(x+8)(x-12)$ **p** $(x-8)(x+12)$

2 **a** $2(x+1)(x+4)$ **b** $3(x-1)(x-6)$
c $2(x+3)(x+4)$ **d** $5(x+2)(x-8)$
e $4(x+1)(x-3)$ **f** $3(x-3)(x-11)$
g $2(x+9)(x-10)$ **h** $3(x+2)(x-4)$
i $2(x+4)(x+5)$ **j** $x(x+1)(x-8)$
k $4(x-3)^2$ **l** $3(x-3)(x+9)$
m $2(x-10)(x-12)$ **n** $x(x+4)(x-7)$
o $x^2(x+1)^2$

3 **a** $-(x-6)(x+9)$ **b** $-(x+2)(x+5)$
c $-(x+3)(x+7)$ **d** $-(x-1)(x-3)$
e $-(x-2)^2$ **f** $-(x-1)(x+3)$

4 **a** $-(x+6)(x-8)$ **b** $-(x-3)^2$
c $-3(x-3)(x-7)$ **d** $-2(x+7)(x-9)$
e $-2(x-5)^2$ **f** $-x(x+1)(x-2)$

5 As $b=m+n,\quad -b=-m-n$
Since $c=mn,\quad x^2-bx+c=x^2-mx-nx+mn$
$\qquad\qquad\qquad\qquad\qquad\quad =(x-m)(x-n)$

EXERCISE 3G

1 **a** **i** $3x^2+7x+2$
$=3x^2+6x+x+2$
$=3x(x+2)+1(x+2)$
$=(x+2)(3x+1)$

ii $3x^2+7x+2$
$=3x^2+x+6x+2$
$=x(3x+1)+2(3x+1)$
$=(3x+1)(x+2)$

b Yes, as $ab=ba$.

2 **a** $(2x+3)(x+1)$ **b** $(2x+9)(x+2)$
c $(7x+2)(x+1)$ **d** $(3x+1)(x+4)$
e $(3x+2)(x+2)$ **f** $(3x+7)(x+3)$
g $(4x+1)(2x+3)$ **h** $(7x+1)(3x+2)$
i $(3x+1)(2x+1)$ **j** $(6x+1)(x+3)$
k $(5x+1)(2x+3)$ **l** $(7x+1)(2x+5)$

3 **a** **i** $(2x+3)(2x-1)$ **ii** $(2x-1)(2x+3)$ **b** yes

4 **a** $(2x+1)(x-5)$ **b** $(3x-1)(x+2)$
c $(3x+1)(x-2)$ **d** $(2x-1)(x+2)$
e $(2x+5)(x-1)$ **f** $(5x-3)(x-1)$
g $(11x+2)(x-1)$ **h** $(2x+3)(x-3)$
i $(3x-2)(x-5)$ **j** $(5x+2)(x-3)$
k $(3x-2)(x+4)$ **l** $(2x-1)(x+9)$
m $(2x-3)(x+6)$ **n** $(5x+2)(3x-1)$
o $(21x+1)(x-3)$

5 **a** $-(3x+7)(x-2)$ **b** $-(5x-1)(x-2)$
c $-(4x-3)(x+3)$ **d** $-(3x-2)^2$
e $-(4x+1)(2x+3)$ **f** $-(6x+1)(2x-3)$

6 **a** $(3x+5)^2-(2x-3)^2$
$=9x^2+30x+25-[4x^2-12x+9]$
$=9x^2+30x+25-4x^2+12x-9$
$=5x^2+42x+16$

b $(5x+2)(x+8)$
$\{ac=80,\ b=42,$ split is $+40x+2x\}$

c $(3x+5)^2-(2x-3)^2$
$=(3x+5+2x-3)(3x+5-2x+3)$
$=(5x+2)(x+8)$

EXERCISE 3H

1 **a** $x(3x+2)$ **b** $(x+9)(x-9)$
c $2(p^2+4)$ **d** $3(b+5)(b-5)$
e $2(x+4)(x-4)$ **f** $n^2(n+2)(n-2)$
g $(x-9)(x+1)$ **h** $(d+7)(d-1)$
i $(x+9)(x-1)$ **j** $4t(1+2t)$
k $(2x+1)(2x+5)$ **l** $2(g+5)(g-11)$
m $(2a+3d)(2a-3d)$ **n** $5(a+1)(a-2)$
o $2(c-1)(c-3)$ **p** $(x+7)(2x+3)$
q $d^2(d+3)(d-1)$ **r** $x(x+2)^2$

2 **a** $7(x-5y)$ **b** $2(g+2)(g-2)$
c $-5x(x+2)$ **d** $m(m+3p)$
e $(a+3)(a+5)$ **f** $(m-3)^2$
g $5x(x+y-xy)$ **h** $(x+2)(y+2)$
i $(y+5)(y-9)$ **j** $(x+5)(2x+1)$
k $3(y+7)(y-7)$ **l** $(6x+1)(x-5)$
m $(2c+1)(2c-1)$ **n** $3(x+4)(x-3)$
o $2(x-3)(b+5)$ **p** $(4x+3)(3x+1)$
q $-2(x-1)(x-3)$ **r** $(4x+1)^2$
s $-2x(x-1)^2$ **t** $(a+b+3)(a+b-3)$
u $2(x-3)(6x-1)$

REVIEW SET 3A

1 Show that the total area equals the sum of the two smaller areas.

2 **a** x^2-x-30 **b** $6x^2+13x-5$ **c** $18x-x^2$

3 **a** $x(7x-4)$ **b** $x(x-1)(x+6)$ **c** $(x-8)(x+5)$

4 **a** $x^3+4x^2-7x-10$ **b** $2x^3+5x^2-8x-6$

5 **a** $(4+3m)(4-3m)$ **b** $x(x+9)(x-9)$
c $(x+12)(x+2)$

6 **a** t^2-49 **b** $4y^2-25$ **c** $4m^2-20mn+25n^2$

7 **a** $2(x+5)^2$ **b** $(b+d)(2-c)$

8 **a** $8k^3+36k^2+54k+27$ **b** $r^3-12r^2t+48rt^2-64t^3$

9 **a** $(x-2)(x+9)$ **b** $3(x+2)(x-5)$ **c** $-2(x+4)(x-8)$

10 **a** $(4x+3)(2x+1)$ **b** $(5x-3)(x-2)$
c $-(3x+1)(3x-2)$

11 **b** $3(x+2)(x+12)$
c $(2x+9+x-3)(2x+9-x+3)=3(x+2)(x+12)$

12 **a** **i** $a^2+2ab+b^2$ **ii** $a^3+3a^2b+3ab^2+b^3$
iii $a^4+4a^3b+6a^2b^2+4ab^3+b^4$
iv $a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

b **i** 8 **ii** 16 **iii** 32 **c** 2^n

d From **12 a**, we see that if $a=b=1$ in these four expansions, we obtain results of $4=2^2$, $8=2^3$, $16=2^4$, and $32=2^5$.
Letting $a=b=1$ in $(a+b)^n$ we get $(1+1)^n=2^n$.

REVIEW SET 3B

1 **a** $20x-25$ **b** $12x-4x^2$ **c** $3x^2-5x+12$

2 **a** $x^3+2x-20$ **b** $3b^2$

3 **a** $2(x+7)(x-7)$ **b** $(4x-3)(2x+5)$

4 Show $(10a+b)(10c+d) = 100ac + 10(ad+bc) + bd$.

5 **a** $(x-6)(x+9)$ **b** $3(x+4)^2$ **6** 16 terms

7 **a** $9x^4 - 30x^2 + 25$ **b** $8a^3 - 12a^2b + 6ab^2 - b^3$

8 **a** $(x+6)(x-11)$ **b** $2(x-3)(x+13)$

c $(2x+3)(2x-7)$

9 $x^4 + x^3 + 5x^2 + 5x + 12$

10 **a** $-(x+3)(x-4)$ **b** $-(3x+10)(2x-5)$

11 **a** We seek two numbers with product $6 \times 12 = 72$ and sum 17. These are 9 and 8.

$\therefore$ to factorise $6x^2 + 17x + 12$ we split the $17x$ into $9x + 8x$.

b $(2x+3)(3x+4)$ **c** $(3x+4)(2x+3)$ ✓

12 **a** **i** $23^2 = 529$, $27^2 = 729$ **ii** $18^2 = 324$, $32^2 = 1024$

iii $11^2 = 121$, $39^2 = 1521$ **iv** $14^2 = 196$, $36^2 = 1296$

b From **a**, if $a + b = 50$, it appears that the last two digits of a^2 and b^2 are the same. If this is always true then their difference will always be a multiple of 100.

c As $a + b = 50$, $b = 50 - a$

Show $a^2 - b^2 = 100(a-25)$ where $a - 25$ is an integer

$\therefore$ $a^2 - b^2$ is a multiple of 100.

$\therefore$ the last two digits of a^2 and b^2 are the same.

EXERCISE 4A

1 **a** 7 **b** 13 **c** 15 **d** 24 **e** $2\sqrt{2}$ **f** 4

g $4\sqrt{2}$ **h** $\frac{1}{2}$ **i** $\frac{1}{3}$ **j** $\frac{4}{11}$ **k** $\frac{3}{17}$ **l** $\frac{2}{23}$

2 **a** 2 **b** -5 **c** $\frac{1}{5}$

3 **a** 12 **b** 6 **c** 30 **d** 24 **e** -30

f -30 **g** 12 **h** 18 **i** $54\sqrt{2}$ **j** 48

k $192\sqrt{3}$ **l** 64 **m** $45\sqrt{5}$ **n** $-150\sqrt{3}$ **o** -224

4 **a** $\sqrt{10}$ **b** $\sqrt{21}$ **c** $\sqrt{33}$ **d** 7

e 6 **f** $-2\sqrt{10}$ **g** $6\sqrt{6}$ **h** $6\sqrt{15}$

i $\sqrt{30}$ **j** $4\sqrt{3}$ **k** -12 **l** $162\sqrt{6}$

5 **a** 2 **b** $\frac{1}{2}$ **c** 3 **d** $\frac{1}{3}$ **e** 2 **f** $\frac{1}{2}$

g 3 **h** $\sqrt{6}$ **i** $\frac{1}{\sqrt{10}}$ **j** 5 **k** 1 **l** 25

6 **a** $\frac{1}{5}$ **b** $\frac{4}{3}$ **c** $\frac{7}{4}$ **d** $\frac{9}{2}$

7 **a** $(\sqrt{a}\sqrt{b})^2 = \sqrt{a}\sqrt{b} \times \sqrt{a}\sqrt{b} = ab$

$(\sqrt{ab})^2 = \sqrt{ab} \times \sqrt{ab} = ab$

As $a \geqslant 0$, $b \geqslant 0$, $(\sqrt{a}\sqrt{b})^2 = (\sqrt{ab})^2$

b Using **a**, $\sqrt{b} \times \sqrt{\frac{a}{b}} = \sqrt{b \times \frac{a}{b}} = \sqrt{a}$

$\therefore \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

8 **a** $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$, whereas $\sqrt{9+16} = \sqrt{25} = 5$

$\therefore$ $\sqrt{9} + \sqrt{16} \neq \sqrt{9+16}$

$\sqrt{25} - \sqrt{16} = 5 - 4 = 1$, whereas $\sqrt{25-16} = \sqrt{9} = 3$

$\therefore$ $\sqrt{25} - \sqrt{16} \neq \sqrt{25-16}$

b No, as they are not true for all $a \geqslant 0$, $b \geqslant 0$.

EXERCISE 4B

1 **a** $2\sqrt{6}$ **b** $5\sqrt{2}$ **c** $3\sqrt{6}$ **d** $2\sqrt{10}$

e $2\sqrt{14}$ **f** $3\sqrt{7}$ **g** $2\sqrt{13}$ **h** $2\sqrt{11}$

i $2\sqrt{15}$ **j** $3\sqrt{10}$ **k** $4\sqrt{6}$ **l** $2\sqrt{17}$

m $5\sqrt{7}$ **n** $9\sqrt{2}$ **o** $8\sqrt{2}$ **p** $10\sqrt{7}$

2 **a** $\frac{\sqrt{5}}{3}$ **b** $\frac{3\sqrt{2}}{2}$ **c** $\frac{\sqrt{3}}{2}$ **d** $\frac{5\sqrt{3}}{6}$

3 **a** $2+\sqrt{2}$ **b** $3-\sqrt{3}$ **c** $1+\frac{1}{2}\sqrt{5}$ **d** $2-\sqrt{2}$

e $2+\sqrt{2}$ **f** $3+\frac{1}{2}\sqrt{3}$ **g** $\frac{7}{4}-\frac{1}{4}\sqrt{15}$ **h** $\frac{1}{2}-\sqrt{2}$

EXERCISE 4C

1 **a** $10\sqrt{2}$ **b** $3\sqrt{3}$ **c** $-\sqrt{5}$ **d** $\sqrt{10}$

e $5\sqrt{6}$ **f** $-6\sqrt{15}$

2 **a** $6\sqrt{2} - 2\sqrt{3}$ **b** $7\sqrt{7} + 3\sqrt{6}$ **c** $8\sqrt{10} + 3\sqrt{5}$

d $-2\sqrt{3} - 7\sqrt{13}$ **e** $6\sqrt{7} + 5\sqrt{11}$ **f** $\sqrt{14} - 2\sqrt{6}$

g $11 - 2\sqrt{17}$ **h** $-21 + 6\sqrt{15}$

3 **a** $\frac{7\sqrt{5}}{12}$ **b** $\frac{5\sqrt{6}}{14}$ **c** $\frac{23\sqrt{3}}{24}$ **d** $-\frac{14\sqrt{11}}{45}$

e $\frac{13\sqrt{10}}{12}$ **f** $-\frac{11\sqrt{2}}{28}$

4 **a**
$$\begin{aligned}&\sqrt{20}+\sqrt{5}\\&= 2\sqrt{5}+\sqrt{5}\\&= 3\sqrt{5}\\&= \sqrt{9}\times\sqrt{5}\\&= \sqrt{45}\end{aligned}$$

b
$$\begin{aligned}&\sqrt{147}-\sqrt{75}\\&= 7\sqrt{3}-5\sqrt{3}\\&= 2\sqrt{3}\\&= \sqrt{4}\times\sqrt{3}\\&= \sqrt{12}\end{aligned}$$

5 **a** **i** 9 units^2 **ii** 4 units^2 **b** 5 units^2

c $A = l^2 = 5$ units^2

Since $(\sqrt{5})^2 = 5$, the side length is $\sqrt{5}$ units.

d $4\sqrt{5}$ units

EXERCISE 4D

1 **a** $\sqrt{10}+2$ **b** $3\sqrt{2}-2$ **c** $3+\sqrt{3}$

d $\sqrt{3}-3$ **e** $7\sqrt{7}-7$ **f** $2\sqrt{5}-5$

g $22-\sqrt{11}$ **h** $\sqrt{6}-12$ **i** $3+\sqrt{6}-\sqrt{3}$

j $6-2\sqrt{15}$ **k** $6\sqrt{5}-10$ **l** $30+3\sqrt{10}$

2 **a** $2-3\sqrt{2}$ **b** $-2-\sqrt{6}$ **c** $2-4\sqrt{2}$

d $-3-\sqrt{3}$ **e** $-3-2\sqrt{3}$ **f** $-5-2\sqrt{5}$

g $-3-\sqrt{2}$ **h** $-5+4\sqrt{5}$ **i** $\sqrt{7}-3$

j $11-2\sqrt{11}$ **k** $\sqrt{7}-\sqrt{3}$ **l** $4-2\sqrt{2}$

m $9-15\sqrt{3}$ **n** $-14-14\sqrt{3}$ **o** $4-6\sqrt{2}$

3 **a** $4+3\sqrt{2}$ **b** $7+4\sqrt{3}$ **c** $1+\sqrt{3}$

d $10+\sqrt{2}$ **e** -2 **f** $3-3\sqrt{7}$

g $-1-\sqrt{5}$ **h** $1-\sqrt{6}$ **i** $14-7\sqrt{2}$

4 **a** $3+2\sqrt{2}$ **b** $7-4\sqrt{3}$ **c** $7+4\sqrt{3}$

d $6+2\sqrt{5}$ **e** $5-2\sqrt{6}$ **f** $27-10\sqrt{2}$

g $9+2\sqrt{14}$ **h** $22-8\sqrt{6}$ **i** $8-4\sqrt{3}$

j $13+4\sqrt{10}$ **k** $13-4\sqrt{10}$ **l** $44+24\sqrt{2}$

m $51-10\sqrt{2}$ **n** $17-12\sqrt{2}$ **o** $19+6\sqrt{2}$

5 **a** $90+34\sqrt{7}$ **b** $9\sqrt{3}-11\sqrt{2}$

6 **a** 13 **b** 23 **c** 1 **d** -9 **e** 14

f 19 **g** -2 **h** -28 **i** -174

7 **a** 1 **b** -4 **c** $x-y$ **d** 15 **e** 20 **f** 2

EXERCISE 4E

1 **a** $\frac{\sqrt{2}}{2}$ **b** $\sqrt{2}$ **c** $2\sqrt{2}$ **d** $\frac{\sqrt{6}}{2}$ **e** $\frac{\sqrt{14}}{6}$

f $\frac{\sqrt{3}}{3}$ **g** $\sqrt{3}$ **h** $\frac{4\sqrt{3}}{3}$ **i** $\frac{\sqrt{21}}{3}$ **j** $\frac{\sqrt{33}}{12}$

k $\frac{\sqrt{5}}{5}$ **l** $\frac{3\sqrt{5}}{5}$ **m** $3\sqrt{5}$ **n** $\frac{\sqrt{15}}{5}$ **o** $\frac{25\sqrt{5}}{2}$

p $\sqrt{5}$ **q** $\frac{\sqrt{3}}{6}$ **r** $\frac{2\sqrt{6}}{3}$ **s** $\frac{3\sqrt{5}}{2}$ **t** $\frac{\sqrt{2}}{4}$

2 **a** $\dfrac{3+\sqrt{5}}{4}$ **b** $\dfrac{2-\sqrt{3}}{1}$ **c** $\dfrac{4+\sqrt{11}}{5}$

d $\dfrac{5\sqrt{2}-2}{23}$ **e** $\dfrac{\sqrt{3}-1}{2}$ **f** $\dfrac{10+15\sqrt{2}}{-14}$

g $\dfrac{3\sqrt{5}-10}{11}$ **h** $\dfrac{-48+13\sqrt{7}}{59}$

3 **a** $4+2\sqrt{2}$ **b** $5-5\sqrt{2}$ **c** $-3+2\sqrt{2}$

d $-\frac{4}{7}+\frac{1}{7}\sqrt{2}$ **e** $1+\sqrt{2}$ **f** $3+2\sqrt{2}$

g $\frac{9}{7}+\frac{3}{7}\sqrt{2}$ **h** $\frac{10}{7}+\frac{6}{7}\sqrt{2}$

4 **a** -6 **b** $-14-2\sqrt{5}$ **c** $12+8\sqrt{2}-9\sqrt{3}+2\sqrt{6}$

5 **a** $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$

$=(\sqrt{a})^2-(\sqrt{b})^2$ {difference of two squares}

$=a-b$ which is an integer since $a, b \in \mathbb{Z}^+$

b **i** $\dfrac{4\sqrt{7}-4\sqrt{2}}{5}$ **ii** $\dfrac{10+2\sqrt{10}}{3}$ **iii** $\dfrac{-29-3\sqrt{39}}{10}$

6 $3+2\sqrt{2}$ **7** $\sqrt{6}=\dfrac{\sqrt{2}-\sqrt{3}}{p}$ $\left(\text{or } \sqrt{6}=\dfrac{12}{5-6p^2}\right)$

8 $x^2=8-2\sqrt{15}$ and so $2\sqrt{15}=8-x^2$
Now square both sides.

9 $u_1=1,\ u_2=1,\ u_3=2,\ u_4=3,\ u_5=5,\ u_6=8$

EXERCISE 4F

1 **a** $x=3,\ y=2$ **b** $x=15,\ y=-4$

c $x=-11,\ y=-3$ **d** $x=6,\ y=0$

e $x=0,\ y=-3$ **f** $x=y=0$

2 **a** $x=2,\ y=\frac{3}{2}$ **b** $x=\frac{3}{7},\ y=-\frac{1}{7}$

c $x=-\frac{3}{7},\ y=-\frac{1}{7}$ **d** $x=-\frac{8}{7},\ y=-\frac{12}{7}$

3 **a** $a=3,\ b=1$ **b** $a=4,\ b=5$

c $a=5,\ b=2$ or $a=-5,\ b=-2$

d $a=3,\ b=-4$ or $a=-3,\ b=4$

4 Let $\sqrt{11-6\sqrt{2}}=a+b\sqrt{2};\ \ a, b \in \mathbb{Q}$

Show $a+b\sqrt{2}=3-\sqrt{2}$ or $-3+\sqrt{2}$

But $11-6\sqrt{2}>0$

$\therefore\ \sqrt{11-6\sqrt{2}}=3-\sqrt{2}$

5 **a** $\sqrt{11+4\sqrt{6}}=2\sqrt{2}+\sqrt{3}$

b No. (Suppose it can and see what happens.)

REVIEW SET 4A

1 **a** 18 **b** -24 **c** $\sqrt{2}$ **d** -27 **e** $1500\sqrt{3}$ **f** $\frac{9}{16}$

2 **a** $4\sqrt{3}$ **b** $12\sqrt{6}$ **3** **a** $2\sqrt{5}-\sqrt{3}$ **b** $\frac{29\sqrt{6}}{60}$

4 **a** $8\sqrt{3}-6$ **b** $16-6\sqrt{7}$ **c** 1 **d** $\sqrt{5}-4$

e $8+5\sqrt{2}$ **f** $79+12\sqrt{14}$

5 **a** $4\sqrt{2}$ **b** $5\sqrt{3}$ **c** $\dfrac{4\sqrt{3}-\sqrt{6}}{14}$ **d** $\dfrac{15+5\sqrt{3}}{12}$

6 $\frac{1}{7}\sqrt{7}$ **7** $155+77\sqrt{2}$ **8** **a** $7+4\sqrt{3}$ **b** 13

9 $\dfrac{-24+5\sqrt{35}}{23}$ **10** $x=-2,\ y=1$ or $x=-\frac{3}{5},\ y=\frac{10}{3}$

REVIEW SET 4B

1 **a** $6\sqrt{15}$ **b** 20 **c** $-2\sqrt{2}$ **d** $2-2\sqrt{2}$ **e** 9 **f** 15

2 $5\sqrt{3}$ **3** **a** $\frac{3}{2}+\sqrt{6}$ **b** $2-\frac{3}{2}\sqrt{2}$

4 **a** 22 **b** $4\sqrt{5}-9$ **c** $6-4\sqrt{3}$ **d** $7\sqrt{2}-9$

5 **a** $7\sqrt{2}$ **b** $\dfrac{-\sqrt{6}-\sqrt{2}}{2}$ **c** $\dfrac{6-\sqrt{2}}{17}$ **d** $\dfrac{-25-10\sqrt{3}}{13}$

6 $49+20\sqrt{6}$

7 **a** $-\frac{1}{11}+\frac{1}{11}\sqrt{5}$ **b** $\frac{1}{2}-\frac{5}{2}\sqrt{5}$ **c** $\frac{3}{2}-\frac{1}{2}\sqrt{5}$

8 **a** $-1+5\sqrt{2}$ **b** $\dfrac{16+5\sqrt{3}}{11}$ **9** $\frac{3}{17}\sqrt{34}+\frac{1}{17}\sqrt{17}$

10 $p=2,\ q=-3$ or $p=-\frac{63}{5},\ q=\frac{10}{21}$

EXERCISE 5A

1 **a** $\sqrt{65}$ cm **b** 10 cm **c** $\sqrt{233}$ km **d** $5\sqrt{2}$ cm

e $2\sqrt{61}$ m **f** $\sqrt{481}$ mm

2 **a** $\sqrt{85}$ cm **b** 8 m **c** $6\sqrt{2}$ cm **d** $3\sqrt{13}$ cm

e $\sqrt{26}$ m **f** 180 cm or 1.80 m

3 **a** ≈ 9.4 cm **b** ≈ 7.1 m **c** ≈ 8.8 cm **d** ≈ 5.9 cm

e ≈ 5.2 m **f** ≈ 16.9 cm

4 **a** $x=\sqrt{11}$ **b** $x=\sqrt{2}$ **c** $x=\sqrt{5}$ **d** $x=\sqrt{19}$

e $x=\frac{1}{\sqrt{2}}$ **f** $x=\frac{1}{2}$

5 **a** $x=3\sqrt{3}$ **b** $x=2\sqrt{13}$ **c** $x=2$

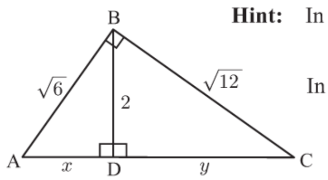
6 **a** $x=2\sqrt{2},\ y=\sqrt{17}$ **b** $x=3\sqrt{5},\ y=\sqrt{29}$

c $x=\sqrt{5},\ y=\sqrt{6}$

7 **a** $x=\sqrt{2}$ **b** $x=14$ **c** $x=12$

8 AC $=\sqrt{39}$ m ≈ 6.24 m

9



Hint: In $\triangle$ABD, $x^2=6-2^2$

$\therefore\ x=\sqrt{2},\ \ x>0$

In $\triangle$BCD,

$y^2+2^2=12$

$\therefore\ y=\sqrt{8},\ \ y>0$

In $\triangle$ABC, AC$^2=(\sqrt{6})^2+(\sqrt{12})^2=18$ $\therefore$ AC $=\sqrt{18}$

10 **a** $\sqrt{17}$ cm **b** $\sqrt{29}$ m **c** $\sqrt{41}$ m

EXERCISE 5B

1 **a** No {as $4^2+5^2=41$ and $7^2=49$}

b Yes {as $9^2+12^2=225$ and $15^2=225$}

c No {as $5^2+8^2=89$ and $9^2=81$}

d No {as $3^2+(\sqrt{7})^2=16$ and $(\sqrt{12})^2=12$}

e Yes {as $(\sqrt{27})^2+(\sqrt{48})^2=75$ and $(\sqrt{75})^2=75$}

f Yes {as $8^2+15^2=289$ and $17^2=289$}

2 **a** As $(\sqrt{5})^2=1^2+2^2$, it is right angled at A.

b As $8^2+12^2=208\neq(\sqrt{218})^2$, it is not right angled.

c As $7^2=49=(\sqrt{24})^2+5^2$, it is right angled at C.

d As $4^2+(3\sqrt{2})^2\neq(\sqrt{19})^2$, it is not right angled.

e $5^2+(\sqrt{47})^2=72=(6\sqrt{2})^2$

$\therefore$ it is right angled at A.

f $(\sqrt{13})^2+(5\sqrt{2})^2=13+50=63$ and $(3\sqrt{7})^2=63$

$\therefore$ it is right angled at B.

3 $x=\sqrt{101}\approx 10.0$

Hint: Show right-most triangle to be right angled first.

EXERCISE 5C

1 **a** yes **b** no **c** yes

d no {numbers must be integers} **e** yes

f no {numbers must be positive}

2 **a** $k = 20$ **b** $k = 10$ **c** $k = 48$ **d** $k = 15$
e $k = 21$ **f** $k = 11$

3 **a** As $\{a, b, c\}$ is a Pythagorean triple, $a^2 + b^2 = c^2$
Hint: $(ka)^2 + (kb)^2 = k^2a^2 + k^2b^2 = k^2(a^2 + b^2)$
b **i** $\{6, 8, 10\}$, $\{9, 12, 15\}$, $\{12, 16, 20\}$, $\{15, 20, 25\}$
ii $\{10, 24, 26\}$, $\{15, 36, 39\}$, $\{20, 48, 52\}$, $\{25, 60, 65\}$

4 **a** Show $(be - ad)^2 + (bd + ae)^2 = (cf)^2$
b $\{36, 77, 85\}$

5 **a** **i** $\{3, 4, 5\}$, $3^2 + 4^2 = 5^2$
ii $\{5, 12, 13\}$, $5^2 + 12^2 = 13^2$
iii $\{7, 24, 25\}$, $7^2 + 24^2 = 25^2$
iv $\{9, 40, 41\}$, $9^2 + 40^2 = 41^2$

EXERCISE 5D

1 $\sqrt{73}$ cm ≈ 8.54 cm

2 $3\sqrt{10}$ cm by $\sqrt{10}$ cm, or, ≈ 9.49 cm by 3.16 cm

3 **a** $24\sqrt{5}$ cm ≈ 53.7 cm **b** 160 cm^2

4 $2\sqrt{11}$ cm ≈ 6.63 cm **5** $5\sqrt{2}$ cm ≈ 7.07 cm

6 $4\sqrt{41}$ cm ≈ 25.6 cm

7 $8^2 + 11.55^2 = 197.4025$ and $14.05^2 = 197.4025$
$\therefore$ is right angled.

8 7.75 m **9** $\sqrt{89}$ km ≈ 9.43 km

10 $2\sqrt{445}$ km ≈ 42.2 km **11** ≈ 11.2 km/h and ≈ 22.4 km/h

12 **a** ≈ 115.4 m **b** 100 m **c** ≈ 15.4 m

13 D is 8 km from B.

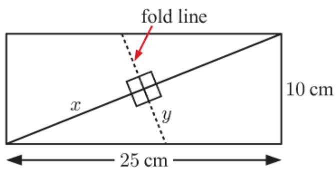
14 $6\sqrt{3}$ cm ≈ 10.4 cm

15 ≈ 22.2 cm^2

16 8 cm

17 6.025 m

18



$x \approx 13.463$, crease ≈ 10.8 cm

EXERCISE 5E

1 BC $= 6$ cm **2** ≈ 6.26 cm **3** ≈ 3.71 cm

4 $3\sqrt{2}$ cm ≈ 4.24 cm **5** $2\sqrt{39}$ cm ≈ 12.5 cm

6 $6\sqrt{2}$ cm ≈ 8.49 cm **7** $4\sqrt{7}$ cm ≈ 10.6 cm

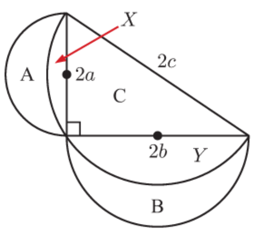
8 $(\sqrt{2} - 1)$ m ≈ 41.4 cm **9** ≈ 717 km

10 AB $= 4\sqrt{6}$ cm ≈ 9.80 cm **11** $\sqrt{101}$ m ≈ 10.050 m

12 AB $= 2\sqrt{7}$ cm ≈ 5.29 cm **13** 5 cm

14 **a** **i** 6 cm **ii** 4 cm **b** $\frac{13}{18}$

15



Hint:
$A + X = \frac{1}{2}\pi a^2$
$B + Y = \frac{1}{2}\pi b^2$
$C + X + Y = \frac{1}{2}\pi c^2$

EXERCISE 5F

1 15 cm **2** 10 cm **3** $\sqrt{10}$ cm ≈ 3.16 cm

4 $3\sqrt{3}$ cm ≈ 5.20 cm **5** ≈ 6.80 m **6** ≈ 4.12 cm

7 No, the maximum length would be about 8.06 m.

8 height $= \frac{100}{\sqrt{2}} \approx 71$ m

9 $5\sqrt{10}$ cm $\times$ $5\sqrt{10}$ cm ≈ 15.8 cm $\times$ 15.8 cm

10 ≈ 42.9 m **11** ≈ 29.87 m

12 $L = \sqrt{50^2 + (20\pi)^2}$ m ≈ 80.30 m

REVIEW SET 5A

1 **a** $\sqrt{29}$ cm ≈ 5.39 cm **b** $\sqrt{33}$ cm ≈ 5.74 cm
c $\sqrt{69}$ cm ≈ 8.31 cm

2 No, $5^2 + 11^2 = 146$ but $13^2 = 169$.

3 Yes, as $4^2 + 1^2 = (\sqrt{17})^2$. The right angle is at A.

4 **a** $x = \sqrt{113}$, $y = 2\sqrt{14}$ **b** $x = 4\sqrt{5}$, $y = 6$

5 **Hint:** Find the two lengths that make up the base of the triangle, then use Pythagoras in the largest triangle.

6 $30\sqrt{2}$ m ≈ 42.4 m

7 **a** $k = 36$
b **i** **Hint:** Show $(4k)^2 + (k^2 - 4)^2 = (k^2 + 4)^2$
ii $\{20, 21, 29\}$

8 ≈ 41.4 cm **9** $5\sqrt{2}$ cm ≈ 7.07 cm

10 **a** $x = 2\sqrt{5}$ **b** $x = 2\sqrt{14}$

11 **a** $\sqrt{61}$ m ≈ 7.81 m **b** $\sqrt{70}$ m ≈ 8.37 m

12 **b** **i** $a^2 + b^2$ **ii** c^2
c Area of large square $-$ areas of 4 triangles must be equal in both figures. $\therefore$ $a^2 + b^2 = c^2$

13 $h = \sqrt{\frac{2}{3}} \approx 0.816$ m

REVIEW SET 5B

1 **a** $x = 3\sqrt{2}$ **b** $x = \sqrt{2}$

2 $2^2 + 5^2 = 29$ and $AC^2 = 29$ $\therefore$ right angled at B.

3 $k = 35$ **4** 6 cm

5 **a** AB $= \sqrt{85}$ m ≈ 9.22 m **b** AB $= \sqrt{33}$ cm ≈ 5.74 cm

6 ≈ 1.431 m **7** $6\sqrt{5}$ m ≈ 13.42 m

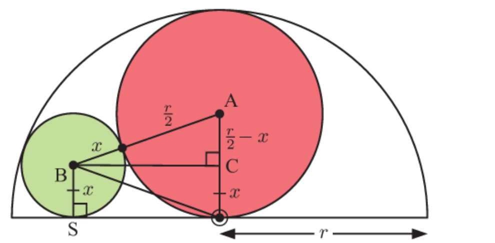
8 Max. distance $= \sqrt{122}$ m ≈ 11.05 m which is > 10.5 m.
$\therefore$ the beam will fit.

9 $\sqrt{99}$ cm ≈ 9.95 cm **10** AB ≈ 6.55 m

11 **a** AB $= \frac{s}{\sqrt{2}}$ **b** **i** $\frac{s^2}{4}$ **ii** $\frac{s^2}{\sqrt{2}}$
c $A = 4\left(\frac{s^2}{4}\right) + 4\left(\frac{s^2}{\sqrt{2}}\right) + s^2$, etc.

12 **a** **i** PQ $= \sqrt{a^2 + b^2}$ units **ii** $\frac{1}{2}$PQ $\times$ RN $= \frac{1}{2}ab$, etc.
b ≈ 81.6 m

13 **a** radius $= \frac{r}{2}$
b **i**



Hint: Find expressions for BC^2 and OS^2.

ii $\frac{r}{4}$

EXERCISE 6A

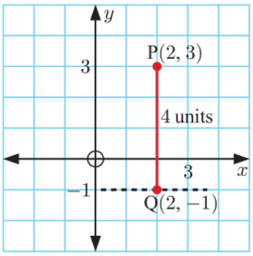
1 **a** $2\sqrt{2}$ units **b** 7 units **c** $2\sqrt{5}$ units **d** $3\sqrt{5}$ units
e 7 units **f** $\sqrt{5}$ units **g** $\sqrt{10}$ units **h** $\frac{\sqrt{5}}{2}$ units

2 **a** $10\sqrt{2} \approx 14.1$ km **b** $20\sqrt{5} \approx 44.7$ km
c $10\sqrt{26} \approx 51.0$ km

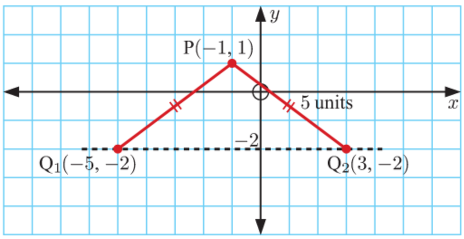
3 **a** isosceles with $AB = AC = \sqrt{85}$ units **b** scalene
c isosceles (and right angled at B) with $AB = BC$
d isosceles with $BC = AC = \sqrt{7}$ units
e equilateral, all sides $2\sqrt{3}$ units
f isosceles $(AC = BC)$ if $b \neq 2 \pm a\sqrt{3}$
equilateral if $b = 2 \pm a\sqrt{3}$

4 **a** right angled at B **b** not right angled
c right angled at A **d** not right angled

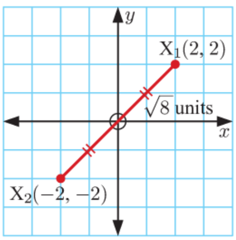
5 **a** $a = 2$



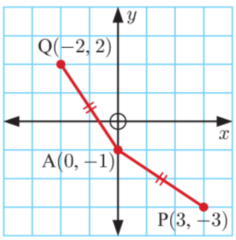
b $a = 3$ or -5



c $a = \pm 2$

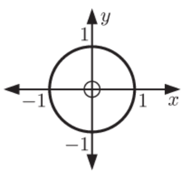


d $a = -1$



6 **a** **i** $x^2 + y^2 = 9$ **ii** $(x-1)^2 + (y-3)^2 = 4$

b



This set represents a circle with centre (0, 0) and radius 1 unit.

7 Let R be $(a, 0)$. Find PQ, PR, and RQ.
Solve $PQ = PR$, $PQ = RQ$, $PR = RQ$.
$(10, 0)$, $(-8, 0)$, $(-3, 0)$, $(-7, 0)$, and $(-\frac{101}{12}, 0)$

EXERCISE 6B

1 **a** $(5, 3)$ **b** $(1, -1)$ **c** $(\frac{3}{2}, 3)$ **d** $(0, 4)$
e $(2, -\frac{3}{2})$ **f** $(1, \frac{5}{2})$

2 **a** $B(0, -6)$ **b** $B(5, -2)$ **c** $B(0, 6)$ **d** $B(0, 7)$
e $B(-7, 3)$ **f** $B(-3, 0)$

3 $C(1, -3)$ **4** $P(7, -3)$ **5** $X(\frac{3}{2}, \frac{3}{2})$, $S(-2, 0)$

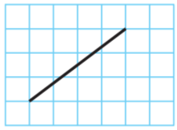
6 $\frac{\sqrt{89}}{2}$ units ≈ 4.72 units **7** $a = \frac{7}{3}$, $b = \frac{11}{2}$

8 **Hint:** Let the vertices be $A(a, b)$, $B(c, d)$, $C(e, f)$.
Use the midpoint formula to find six simple equations and solve them simultaneously.
Points are $(3, -1)$, $(7, 9)$, $(9, 1)$.

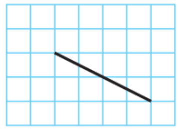
EXERCISE 6C.1

1 **a** $\frac{1}{3}$ **b** 0 **c** -3 **d** $\frac{2}{3}$ **e** $-\frac{3}{4}$
f undefined **g** -4 **h** $-\frac{2}{5}$

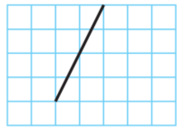
2 **a**



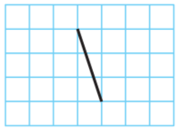
b



c



d



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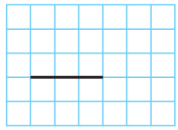

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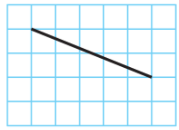

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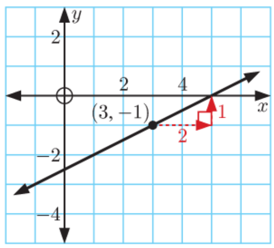

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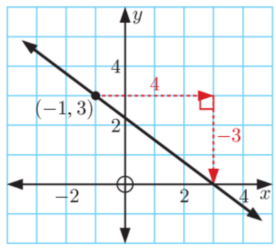

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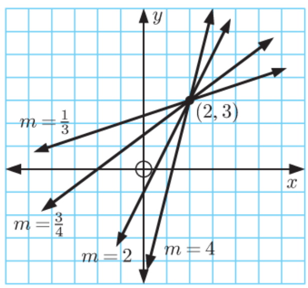

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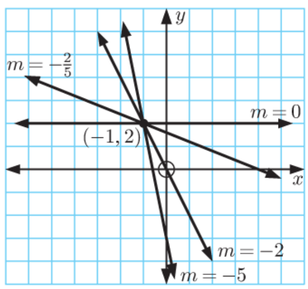
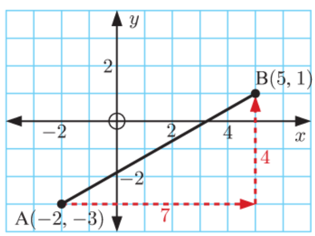


EXERCISE 6C.2

1 $\frac{4}{7}$



2 **a** $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** 4 **d** 0 **e** undefined
f $\frac{2}{7}$ **g** $-\frac{2}{7}$ **h** 1 **i** $-\frac{4}{3}$

3 **a** $t = 19$ **b** $t = \frac{29}{2}$ **c** $t = \frac{13}{3}$ **d** $t = \frac{2}{3}$
e $t = 9$ **f** $t = 5$

4 $Q(0, 7)$, $R(\frac{28}{3}, 0)$ **5** **c** gradient of $[AC] = 2$

EXERCISE 6D.1

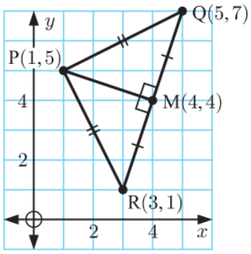
1 **a** -2 **b** $-\frac{5}{2}$ **c** $-\frac{1}{3}$ **d** $-\frac{1}{7}$ **e** $\frac{5}{2}$ **f** $\frac{2}{7}$
g $\frac{3}{4}$ **h** 1

2 The line pairs in **c**, **d**, **f**, and **h** are perpendicular.

3 **a** [AB]: $\frac{3}{5}$, [BC]: -2, [CD]: $\frac{9}{2}$, [DE]: $-\frac{1}{2}$, [EF]: -2, [FA]: 2
b **i** $[BC] \parallel [EF]$ **ii** $[DE] \perp [FA]$

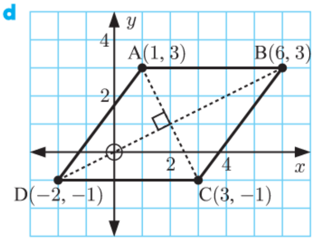
4 $k = -2$ **5** **a** $t = 4$ **b** $t = 4$ **c** $t = 14$ **d** $t = \frac{22}{7}$

6 **a** $PQ = PR = 2\sqrt{5}$ units **b** $M(4, 4)$
c gradient of $[PM] = -\frac{1}{3}$, gradient of $[QR] = 3$, and their product is -1
$\therefore$ $[PM] \perp [QR]$
d



7 $M(0, 3)$ and $N(3, 3)$
a gradient of $[MN] = 0$, gradient of $[AC] = 0$
$\therefore$ $[MN] \parallel [AC]$
b $MN = 3$ units and $AC = 6$ units $\therefore$ $MN = \frac{1}{2}(AC)$

8 **a** All sides have length 5 units. $\therefore$ ABCD is a rhombus.
b $(2, 1)$ and $(2, 1)$
c gradient of $[AC] = -2$, gradient of $[BD] = \frac{1}{2}$ and their product $= -1$
$\therefore$ $[AC] \perp [BD]$
d



9 **a** **i** $P(0, 5)$ **ii** $Q(\frac{9}{2}, 2)$ **iii** $R(\frac{1}{2}, -\frac{5}{2})$ **iv** $S(-4, \frac{1}{2})$
b **i** $-\frac{2}{3}$ **ii** $\frac{9}{8}$ **iii** $-\frac{2}{3}$ **iv** $\frac{9}{8}$
c PQRS is a parallelogram.

10 **a** $s = 6$ **b** **i** $\frac{1}{2}$ **ii** -2
c gradient of $[PS] \times$ gradient of $[SQ] = -1$
$\therefore$ $P\widehat{S}Q = 90°$

EXERCISE 6D.2

1 **a** gradient of $[AB] = \frac{4}{3}$, gradient of $[BC] = \frac{5}{4}$
$\therefore$ not collinear
b gradient of $[PQ] =$ gradient of $[QR] = \frac{6}{5}$
$\therefore$ P, Q, R are collinear
c gradient of $[RS] = -\frac{3}{11}$, gradient of $[ST] = -\frac{3}{2}$
$\therefore$ not collinear
d gradient of $[AB] =$ gradient of $[BC] = 3$
$\therefore$ A, B, C are collinear

2 **a** $c = 3$ **b** $c = -5$

3 **a** $M(1, -4)$
b gradient of $[AM] =$ gradient of $[MC] = 1$, $\therefore$ collinear
c gradient of $[AC] = 1$, gradient of $[BD] = -1$,
$\therefore$ perpendicular

EXERCISE 6E.1

1 **a** $y = 2x + 1$ **b** $y = -x + 1$ **c** $y = \frac{2}{3}x + 3$
d $y = -\frac{4}{5}x + \frac{6}{5}$ **e** $y = -\frac{3}{4}x - \frac{1}{2}$

2 **a** $4x - y = 7$ **b** $3x + 5y = -1$ **c** $x - 3y = -11$
d $3x + 4y = 24$ **e** $2x - 7y = 25$

3 **a** $y = x - 4$ **b** $y = -x + 4$ **c** $y = 6x + 16$
d $y = \frac{1}{2}x + 8$ **e** $y = -\frac{1}{3}x - \frac{7}{3}$ **f** $y = -\frac{3}{5}x - \frac{14}{5}$

4 **a** $3x - y = -1$ **b** $2x + y = 3$ **c** $x + 4y = 1$
d $5x - 3y = 10$ **e** $y = -3$ **f** $x = 2$

5 **a** $4x + 5y = 13$ **b** $4(12) + 5(-7) = 48 - 35 = 13$ ✓

6 **a** $2x - 5y = 10$ **b** $y = -2x - 2$ **c** $y = 2x + 6$
d $4x + 3y = 20$ **e** $x - 2y = -8$

7 **a** $-\frac{3}{4}$ **b** $4x - 3y = 10$ **8** $7x - 5y = -42$

9 $y = \frac{3}{2}x - 3$ **10** **a** $y = \frac{2}{3}x - \frac{5}{3}$ **b** $-\frac{5}{3}$

11 $x + 3y = 13$

12 **a** **i** $y = -\frac{1}{2}x + 2$ **ii** $y = 2x - 3$ **iii** $y = -\frac{1}{2}x + \frac{19}{2}$
b **Hint:** Solve $2x - 3 = -\frac{1}{2}x + \frac{19}{2}$.

13 **Hint:** P is $(a + b, c)$, Q is (b, c)
a gradient of $[OP] = \dfrac{c}{a+b}$ etc.
b gradient of $[AQ] = \dfrac{c}{b-2a}$ etc.
c Solve the equations in **a** and **b** simultaneously.

EXERCISE 6E.2

1 **a** $x - 2y = 2$ **b** $2x - 3y = -19$ **c** $3x - 4y = 15$
d $3x - y = 11$ **e** $x + 3y = 13$ **f** $3x + 4y = -6$
g $2x + y = 4$ **h** $3x + y = 4$

2 **a** $-\frac{2}{3}$ **b** $\frac{3}{7}$ **c** $\frac{6}{11}$ **d** $-\frac{5}{6}$ **e** $-\frac{1}{2}$ **f** 3

3 **a** Parallel lines have the same gradient of $-\frac{3}{5}$.
$\therefore$ equations have the form $3x + 5y =$ a constant.
b $3x + 5y = 2$ has gradient $-\frac{3}{5}$
$\therefore$ perpendicular lines have gradient $\frac{5}{3}$.
$\therefore$ equations have the form $5x - 3y =$ a constant.

4 **a** $3x + 4y = 10$ **b** $2x - 5y = 3$ **c** $3x + y = -12$
d $x - 3y = 0$

5 **a** $\frac{2}{3}$ and $-\frac{6}{k}$ **b** $k = -9$ **c** $k = 4$

6 **a** $2\sqrt{34}$ km ≈ 11.7 km **b** $(6, -\frac{1}{2})$ **c** no
d **i** $11x + 8y = 31$
ii No, as $11(2) + 8(1) = 30 \neq 31$.

7 **a** $x - 3y = -16$ **b** $3x - 2y = 13$ **c** $2x - y = -3$
d $x = 5$

8 **a** **i** $x - 7y = -12$ **ii** $x + y = 8$
b $(\frac{11}{2}) - 7(\frac{5}{2}) = -\frac{24}{2} = -12$ ✓ $(\frac{11}{2}) + (\frac{5}{2}) = 8$ ✓
c $PR = QR = \frac{5\sqrt{2}}{2}$ units

EXERCISE 6F

1 a $x-y=4$ b $2x-y=-6$ c $12x-10y=-35$ d $y=1$

2 a $2x+3y=-1$ b $2(-5)+3(3)=-1$ ✓ c $\text{AC}=\text{BC}=\sqrt{65}$ units

3 $2x-3y=-5$

4 a $x+2y=5$, $3x+y=10$, $x-3y=0$ b $(3,\ 1)$

5 **Hint:** Start by finding the gradient and midpoint of [AB].

6 The perpendicular bisectors of [PQ] and [QR] are $x-3y=-6$ and $2x-y=3$. They meet at $(3,\ 3)$.

7 a P(5, 3), Q(4, 0), R(2, 2)

b i $x-y=2$ ii $3x+y=12$ iii $x+3y=8$

c $\text{X}(3\frac{1}{2},\ 1\frac{1}{2})$ d Yes, $(\frac{7}{2})+3(\frac{3}{2})=8$ ✓

e The perpendicular bisectors meet at a point.

f X is the centre of the circle which could be drawn through A, B, and C.

EXERCISE 6G

1 a $2\sqrt{10}$ units b $3\sqrt{5}$ units c $3\sqrt{5}$ units d $2\sqrt{17}$ units e $2\sqrt{10}$ units f 5 units

2 a $\sqrt{10}$ units b 4 units

3 a N(4.44, 0.92) b 3.8 km

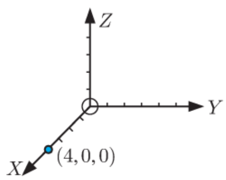
4 a $A=\dfrac{ab}{2}$, $A=\dfrac{cd}{2}=\dfrac{d\sqrt{a^2+b^2}}{2}$

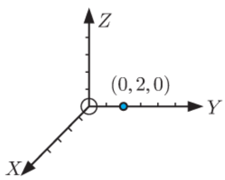
b i When $y=k$, $Ax+Bk+C=0$

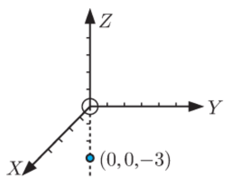
$$\therefore\ x=\frac{-Bk-C}{A}$$

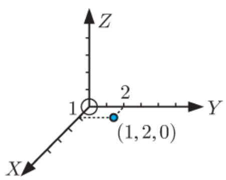
$\therefore$ P is $\left(\dfrac{-Bk-C}{A},\ k\right)$ etc.

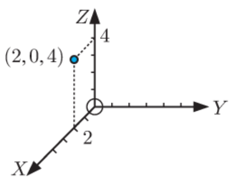
EXERCISE 6H

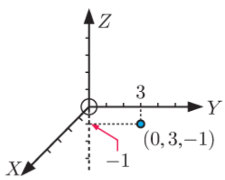
1 a


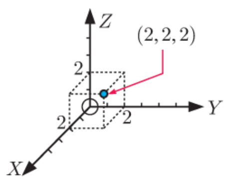
b


c


d


e


f


g


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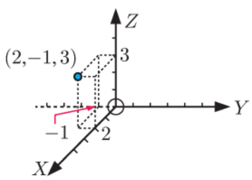
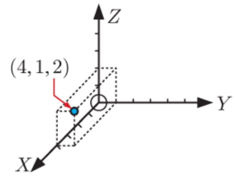


i


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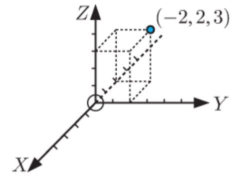
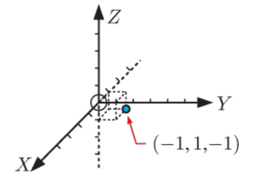
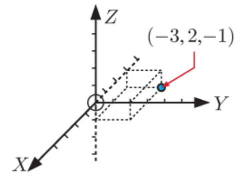


k


l


2 a i $2\sqrt{14}$ units ii M(1, 1, −1)

b i 6 units ii M(1, −2, 2)

c i $2\sqrt{3}$ units ii M(2, 2, 2)

d i $\sqrt{35}$ units ii $\text{M}(\frac{3}{2},\ \frac{1}{2},\ \frac{7}{2})$

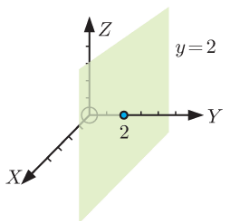
3 a Isosceles with $\text{AC}=\text{BC}=\sqrt{14}$ units.

b $\triangle$ABC is right angled at A.

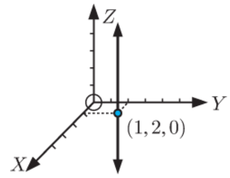
4 $k=1\pm\sqrt{19}$

5 a $x^2+y^2+z^2=4$ and must be the equation of a sphere, centre (0, 0, 0), radius 2 units.

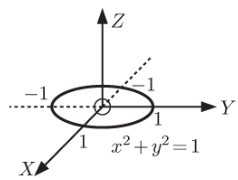
b $(x-1)^2+(y-2)^2+(z-3)^2=16$ which is the equation of a sphere, centre (1, 2, 3), radius 4 units.

6 a


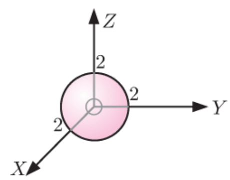
A plane parallel to the XOZ plane passing through (0, 2, 0).

b


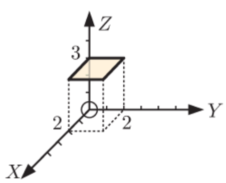
A line parallel to the Z-axis passing through (1, 2, 0).

c


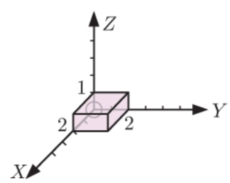
A circle in the XOY plane, centre (0, 0, 0), radius 1 unit.

d


A sphere, centre (0, 0, 0), radius 2 units.

e


A 2 by 2 square plane 3 units above the XOY plane (as shown).

f


All points on and within a $2\times2\times1$ rectangular prism (as shown).

REVIEW SET 6A

1 $(-3,\ 3)$ 2 $\sqrt{58}$ units 3 $-\frac{3}{2}$ 4 $m=6$ or -2

5 $t=-11$

6 Gradient of [AB] and [BC] $= 2$.
$\therefore$ [AB] $\parallel$ [BC] with B common.

7 **a** $y = -2x + 7$ **b** $2x + 3y = 7$ **c** $3x - 2y = 15$

8 $2x + 3y = 10$ **9** $a = 4$

10 **a** $AB = BC = 5$ units, $m_{AB} = \frac{3}{4}$, $m_{BC} = -\frac{4}{3}$
$\therefore$ right angle at B.
b $X(\frac{1}{2}, \frac{1}{2})$
c gradient of [BX] $= 7$, gradient of [AC] $= -\frac{1}{7}$
and $7 \times -\frac{1}{7} = -1$, $\therefore$ [BX] $\perp$ [AC].

11 **a** $x - 5y = -19$ **b** P(3, 7) **c** $5x + y = 22$
d **i** $(\frac{7}{2}, \frac{9}{2})$ **ii** $5(\frac{7}{2}) + \frac{9}{2} = \frac{44}{2} = 22$ ✓

12 **a** $y = 4x - 11$ **b** $6x - 16y = -11$ **13** $\sqrt{13}$ units

14 **a** $2\sqrt{14}$ units **b** M(0, 0, 0) **15** $k = 4$ or -6

REVIEW SET 6B

1 **a** $ST = \sqrt{73}$ units **b** $(3, -\frac{1}{2})$ **2** $x + 2y = 1$

3 **a** 2 **b** No, as $2x + y = 3$ has gradient -2.

4 $a = -1$ or -3 **5** $b = -3$

6 $x + 2y = 8$ **7** $5x + 6y = 29$ **8** $y = 3x - 5$

9 **a** **i** [AB]: $\frac{1}{5}$, [BC]: -2, [CD]: $\frac{1}{5}$, [AD]: -2
ii ABCD is a parallelogram.
b **i** $(\frac{1}{2}, \frac{1}{2})$ for both diagonals.
ii The diagonals of a parallelogram bisect each other.
c **i** [AC]: $-\frac{3}{7}$, [BD]: $\frac{5}{3}$
ii product $= -\frac{5}{7}$ $\therefore$ not a rhombus

10 **a** $N(\frac{66}{13}, \frac{18}{13})$ **b** $\frac{7}{\sqrt{13}}$ units

11 **a** 5 units **b** $4x - 3y = -18$ **c** B(5, -4)
d $4x - 3y = 32$

12 **a** R is (b, c), [OB] has gradient $\frac{c}{b}$, etc.
c **Hint:** Show that the x-coordinate of T is a.

13 distance $= \frac{12}{\sqrt{5}}$ units **14** $\sqrt{30}$ units

EXERCISE 7A

1 **a** $\triangle ABC \cong \triangle FED$ {AAcorS}
b $\triangle PQR \cong \triangle ZYX$ {SAS}
c $\triangle ABC \cong \triangle EDF$ {AAcorS}
d $\triangle ABC \cong \triangle LKM$ {SSS}
e $\triangle XYZ \cong \triangle FED$ {RHS} **f** not congruent
g not enough information **h** not enough information
i $\triangle ABC \cong \triangle PQR$ {SSS} **j** $\triangle ABC \cong \triangle FED$ {AAcorS}

2 **a** **B** and **D** {SAS} **b** **A** and **D** {RHS}
c **B** and **C** {AAcorS} **d** **A** and **D** {SSS}

EXERCISE 7B

1 $\triangle ABC \cong \triangle EDC$ {AAcorS}

2 **a** $\triangle ABD \cong \triangle CBD$ {SSS} **b** **i** $47°$ **ii** $51°$

3 **a** Join [AC].
Hint: Show $\triangle ABC \cong \triangle CDA$ {AAcorS}
b Now join [DB] and let the diagonals meet at M.
Hint: Show $\triangle AMD \cong \triangle CMB$ {AAcorS}

4 **a** **Hint:** Show $\triangle XYZ \cong \triangle ZWX$ {SSS}
b $W\widehat{Z}X = Y\widehat{X}Z$ so WZ $\parallel$ XY
and $W\widehat{X}Z = Y\widehat{Z}X$ so WX $\parallel$ ZY

5 **Hint:** Join [OA], [OB], and [OP], and show $\triangle OAP \cong \triangle OBP$. {RHS}

6 **Hint:** Join [AC], [BC], and [CX], and show $\triangle ACX \cong \triangle BCX$. {RHS}

7 **Hint:** Show $\triangle APB \cong \triangle AQC$ {AAcorS}

8 **Hint:** Join [AX], [BX], and [CX].
Show $\triangle AQX \cong \triangle BQX$ {SAS}
and $\triangle APX \cong \triangle CPX$ {SAS}

9 **Hint:** Join [WZ] and [ZY].
Show $\triangle WAZ \cong \triangle YDZ$ {SAS}

EXERCISE 7C

1 **e** **Hint:** Let $N\widehat{J}M$ be $\alpha°$; find all other angles in terms of α.
f **Hint:** Let $A\widehat{C}B = \alpha$, then $A\widehat{B}C = \alpha$ and $B\widehat{D}C = \alpha$.

2 **a** $x = 1.2$ **b** $x = \frac{10}{3}$ **c** $x = \frac{20}{7}$ **d** $x = 7.5$
e $x = 10.8$ **f** $x = \frac{8}{3}$ **g** $x = \frac{4}{3}$ **h** $x = 4.8$
i $x = \frac{35}{3}$

3 7 m **4** ≈ 1.62 m **5** 10.625 km

6 ≈ 651 m **7** 8 cm

8 She is correct as the viewing region is always 20 m long no matter where the camera is located on [AB].

9 1.52 m **10** ≈ 35.7 m **11** ≈ 5.56 cm

EXERCISE 7D

1 **a** $x \approx 14.2$ **b** $x \approx 30.7$ **c** $x \approx 41.5$
d $x \approx 6.26$ **e** $x = 8$ **f** $x \approx 10.7$

2 **a** $x = 96$ **b** $x = 432$ **c** $x \approx 0.150$
d $x = 7.5$ **e** $x = 13.2$ **f** $x \approx 6.94$

3 **a** $12.544\ cm^2$ **b** $6.144\ cm^2$

4 **a** $x = 1.5$ **b** $17.6\ m^2$ **5** 30 cm

6 **a** **i** It is multiplied by 8. **ii** It is increased by 72.8%.
b **i** It is divided by 8. **ii** It is increased by 237.5%.
c **i** $\approx 48.8\ cm^3$ **ii** 1310.72 grams

7 Length [RS] is the length of [PM] enlarged with scale factor $k = 2$.
$\therefore$ the area of the similar triangle will be enlarged by a factor of $k^2 = 4$.

8 **a** similar **b** not similar

9 **a** 10 cm **b** $30\ cm^2$ **c** 6 mm, 2.25 mm **d** 1.25 mL

10 **B** and **F**
Hint: Create a table for each pair of glasses.

	k	k^3	*Calc. V*	*Actual V*	
A, B	$\frac{10}{8.5}$	≈ 1.628	≈ 204	160	✗

REVIEW SET 7A

1 **a** **B** and **C** {AAcorS} **b** **A** and **C** {AAcorS}

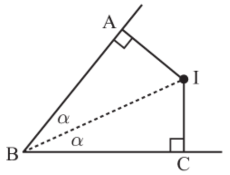
4 **a** $x = \frac{13}{8}$ **b** $x = \frac{23}{3}$ **c** $x = \frac{40}{3}$

5 **a** $38.4\ cm^2$ **b** $28.8\ cm^2$

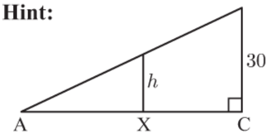
6 **Hint:** Show $\triangle$s APQ and ABC are similar, then $\triangle$s PBC and QCB congruent, etc.

7 ≈ 117 m

8 **Hint:** Explain why $\triangle$s ABI and CBI are congruent, etc.



9 **a** **Hint:**



Explain why $\dfrac{h}{30} = \dfrac{AX}{AC}$.

Similarly find $\dfrac{h}{50}$.

b 18.75 m

10 **a** **Hint:** Consider $\triangle$ADE, etc.

b **Hint:** Find the sizes of the interior angles of PQRST.

11 2 cm **12** **a** 5 : 4 **b** 25 : 16 **c** 125 : 64

REVIEW SET 7B

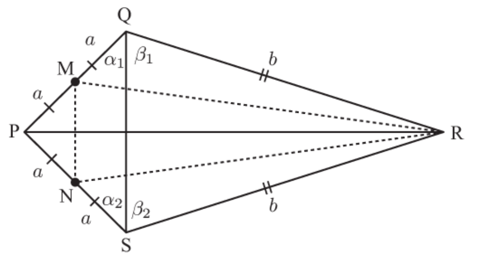
1 **a** yes {RHS} **b** no **c** no

2 **a** **Hint:** Use 'angle in semi-circle' theorem.

b BP = BQ, $\widehat{PAB} = \widehat{QAB}$, $\widehat{PBA} = \widehat{QBA}$
The triangles have equal area.

3 $x = 2.8$ **4** **a** $x = 3$ **b** $x = 4$ **5** AE $\approx$ 4.74 m

6 **Hint:** Explain equally marked sides and angles, then consider $\triangle$s MQR and NSR.



7 $x = 10$ **8** no **10** **a** ≈ 7.56 cm **b** ≈ 5.24 cm

EXERCISE 8A.1

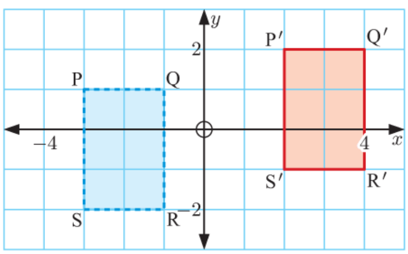
1 **a** (5, 3) **b** (4, 6) **2** **a** $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ -5 \end{pmatrix}$ **3** (0, 1)

4 **a** $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ **b** $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ **e** $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

f $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ **g** $\begin{pmatrix} -8 \\ -4 \end{pmatrix}$ **h** $\begin{pmatrix} -4 \\ -6 \end{pmatrix}$ **i** $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ **j** $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

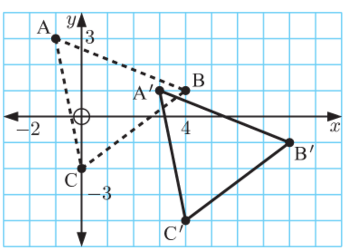
5 **a** P(−3, 1), Q(−1, 1), R(−1, −2), S(−3, −2)

b



c P′(2, 2), Q′(4, 2), R′(4, −1), S′(2, −1)

6 **a**, **b**



c A′(3, 1), B′(8, −1), C′(4, −4) **d** $2\sqrt{5}$ units

7 a translation of $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$

EXERCISE 8A.2

1 **a** $y = 2x + 7$ **b** $y = \frac{1}{3}x + 1$ **c** $y = -x + 7$

d $y = -\frac{1}{2}x - 6$ **e** $3x + 2y = 11$ **f** $x = 6$

g $y = 2x$ **h** $y = 0$

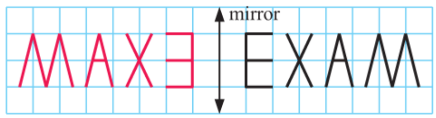
2 **a** $y = x^2 + 3$ **b** $y = -2(x - 3)^2 + 2$

c $y = \dfrac{x + 9}{x + 4}$ **d** $y = \dfrac{-2x - 2}{x - 3}$ **e** $y = 2^x - 3$

f $y = 9 \times 3^{-x}$ or $y = 3^{2-x}$

EXERCISE 8B

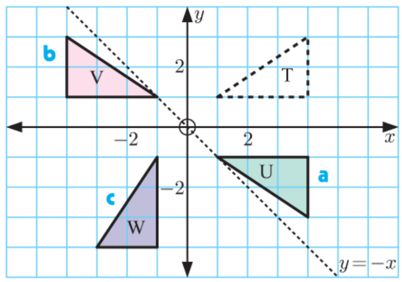
1



2 **a** (4, 1) **b** (−4, −1) **c** (−1, 4) **d** (1, −4)

3 **a** (−1, 3) **b** (1, −3) **c** (−3, −1) **d** (3, 1)

4



5 **a** $y = -2x - 3$ **b** $y = -x^2$ **c** $xy = -5$

d $x = 2^y$ **e** $3x + 2y = -4$ **f** $x^2 + y^2 = 4$

g $y = x^2$ **h** $2x + 3y = -4$ **i** $y = -3$ **j** $x = 2y^2$

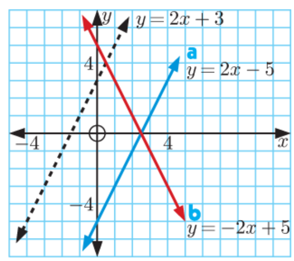
6 **a** (1, −1) **b** (5, −1) **c** (3, −9) **d** (1, 7)

e (5, 1) **f** (−8, 3)

7 **a** $y = 2x - 5$

b $y = -2x + 5$

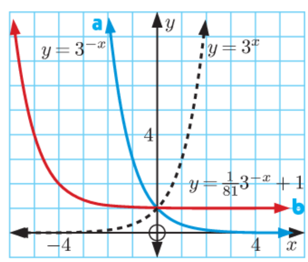
c



8 **a** $y = 3^{-x}$

b $y = \frac{1}{81}3^{-x} + 1$

c

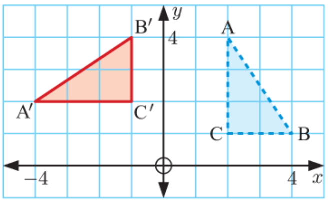


EXERCISE 8C

1 **a** (−3, −2) **b** (3, 2) **c** (2, −3)

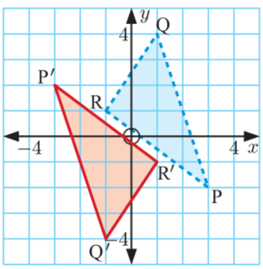
2 **a** (1, 4) **b** (−1, −4) **c** (−4, 1)

3 **a**



b A′(−4, 2), B′(−1, 4), C′(−1, 2)

4 **a**



b P′(−3, 2), Q′(−1, −4), R′(1, −1)

5 **a** $4x + 3y = -7$ **b** $x = 3$ **c** $x = -7$
d $y = -x^2$ **e** $3x - 2y = 12$

6 **a** $(-3, -2)$ **b** $(2, 5)$ **c** $(1, 3)$ **d** $(0, 1)$

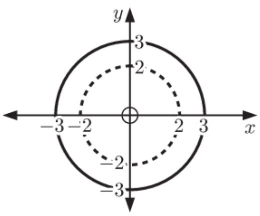
7 **a** $x - y = 13$ **b** $2x - y = 5$ **c** $y = x - 3$

EXERCISE 8D

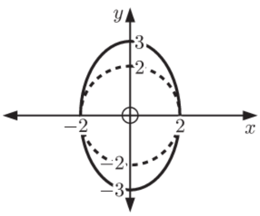
1 **a** $(6, 9)$ **b** $(-\frac{1}{3}, \frac{4}{3})$ **c** $(3, -4)$ **d** $(4, 10)$
e $(-1, 1)$ **f** $(\frac{9}{2}, -4)$

2 **a** $y = 2x + 6$ **b** $y = -2x^2$ **c** $y = \frac{1}{8}x^2$
d $xy = 4$ **e** $y = -3x + 6$ **f** $y = 2^{x+1}$

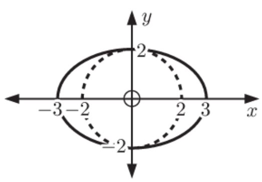
3 **a** circle, centre O, radius 3 is $x^2 + y^2 = 9$



b ellipse, centre O, x-intercepts ± 2, y-intercepts ± 3



c ellipse, centre O, x-intercepts ± 3, y-intercepts ± 2



4 **a** Vertical dilation of factor $k = \frac{3}{4}$ with invariant x-axis.
b Horizontal dilation of factor $k = 2$, with invariant y-axis.
c Horizontal dilation of factor $k = 4$, with invariant y-axis.
d Dilation with centre O and scale factor $k = \frac{1}{2}$.

REVIEW SET 8A

1 **a** $(1, 0)$ **b** $(-3, -2)$

2 **a** $3x - 2y = -5$ **b** $y = 3$
c $2x + y = 12$ **d** $y = 2(x - 2)^2 - 5$

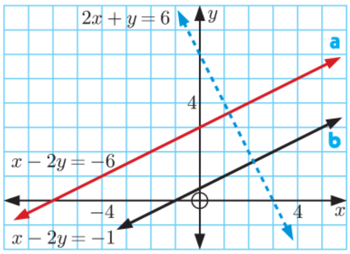
3 **a** $(-2, -5)$ **b** $(6, -3)$ **c** $(5, 1)$ **d** $(5, 3)$

4 **a** $y = -3x + 2$ **b** $y = -3x^2$ **c** $x = 3^y$ **d** $xy = 6$

5 **a** $(-3, 2)$ **b** $(2, -4)$

6 **a** $x - 2y = -6$ **b** $x - 2y = -1$

c



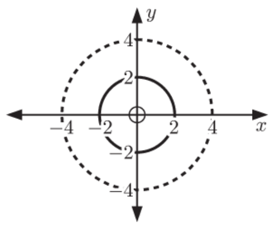
7 **a** $(4, -2)$ **b** $(3, 20)$ **c** $(-8, -7)$

8 Vertical dilation with scale factor $\frac{5}{3}$.

9 **a** $y = 3x + 6$ **b** $4x - 5y = 20$ **c** $y = -4x + 1$
d $y = -x^2 + 3x - 5$ **e** $(x - 5)^2 + (y + 2)^2 = 4$

10 **a** **i** $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ **ii** $\mathbf{M}_{y=x}$ **iii** $\mathbf{R}_{180}$
iv Horizontal dilation with scale factor $\frac{1}{3}$.
b Rotation of 180° about $(-\frac{1}{2}, -\frac{3}{2})$.

11 circle, centre O, radius 2 units, $x^2 + y^2 = 4$



REVIEW SET 8B

1 **a** $(3, 2)$ **b** $(-3, -2)$ **c** $(2, -3)$

2 **a** $(-5, -11)$ **b** $(-7, -3)$

3 **a** $(5, 2)$ **b** $(-5, -2)$ **c** $(-2, 5)$

4 **a** $(3, 2)$ **b** $(4, -5)$ **c** $(2, 5)$ **d** $(7, 2)$

5 **a** $5x - 2y = 27$ **b** $y = -(x + 2)^2 + 5$
c $y = -2 - \dfrac{4}{x - 1}$ **d** $(x + 3)^2 + (y + 4)^2 = 9$

6 **a** $4x - 3y = -8$ **b** $xy = -12$
c $3x - 2y = -9$ **d** $y = 2x^2$

7 **a** $(-1, -13)$ **b** $(-3, -2)$

8 **a** $(9, 15)$ **b** $(-4, 3)$ **c** $(-5, -\frac{3}{2})$

9 Horizontal dilation with scale factor 3.

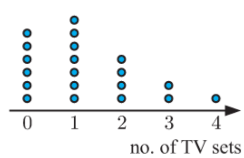
10 **a** $y = -6x + 3$ **b** $y = 2 - 15x$ **c** $x - 3y = 7$
d $x = 1 + 3y - 2y^2$ **e** $(x + 3)^2 + (y + 4)^2 = 8$

EXERCISE 9A

1 **a** 45 shoppers **b** 18 shoppers **c** ≈ 15.6%
d positively skewed

2 **a** 10 employees **b** ≈ 4.44%
c positively skewed **d** It is an outlier.

3 **a** **Number of TV sets in students' households**

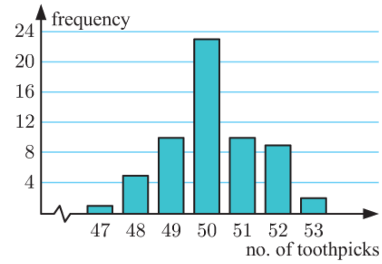


b positively skewed, no outliers
c 6 households
d 15%

4 **a**

No. of toothpicks	Tally	Frequency
47	|	1
48	𝍸	5
49	𝍸 𝍸	10
50	𝍸 𝍸 𝍸 𝍸 |||	23
51	𝍸 𝍸	10
52	𝍸 ||||	9
53	||	2
	Total	60

b **Number of toothpicks in boxes**

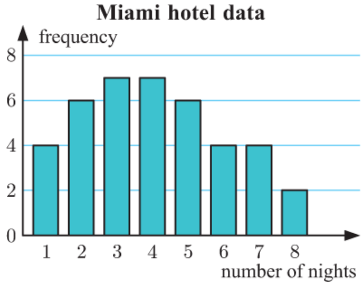


c approximately symmetrical **d** $\approx 38.3\%$

5 **a**

No. of nights	Tally	Frequency
1	||||	4
2	𝍸 |	6
3	𝍸 ||	7
4	𝍸 ||	7
5	𝍸 |	6
6	||||	4
7	||||	4
8	||	2
	Total	40

b **Miami hotel data**



c no **d** slightly positively skewed **e** the Miami hotel

6 **a**

Test score	Tally	Frequency
20 - 29	|	1
30 - 39	||	2
40 - 49	|||	3
50 - 59	𝍸 ||||	9
60 - 69	𝍸 𝍸 |||	13
70 - 79	𝍸 |||	8
80 - 89	𝍸 𝍸	10
90 - 100	||||	4
	Total	50

b 28%
c 12%

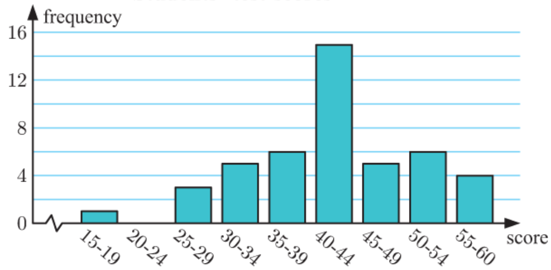
d More students had a test score in the interval 60 - 69 than in any other interval.

e negatively skewed

7 **a**

Test score	Tally	Frequency
15 - 19	|	1
20 - 24		0
25 - 29	|||	3
30 - 34	𝍸	5
35 - 39	𝍸 |	6
40 - 44	𝍸 𝍸 𝍸	15
45 - 49	𝍸	5
50 - 54	𝍸 |	6
55 - 60	||||	4
	Total	45

b **Students' test scores**

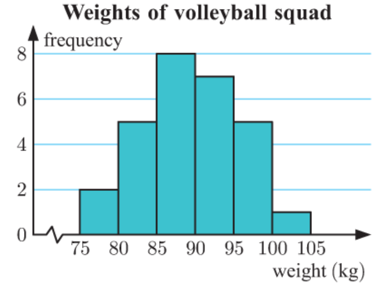


c negatively skewed with no outliers **d** 22.2%

EXERCISE 9B

1 **a** *Weight* can take any value in the given range, and is measured, not counted.

b **Weights of volleyball squad**

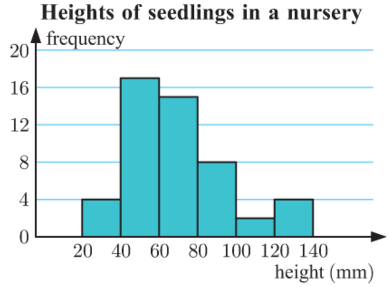


c $85 \leqslant w < 90$ kg. More people in the volleyball squad have weights between 85 and 90 kg than in any other interval.

d approximately symmetrical

2 **a** 6 seedlings **b** 30%

c **Heights of seedlings in a nursery**



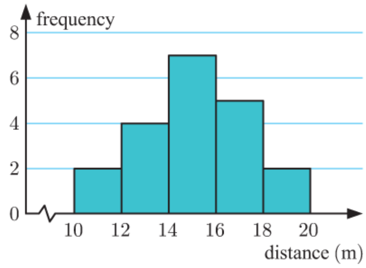
d $40 \leqslant h < 60$ mm **e** positively skewed

f **i** ≈ 754 seedlings **ii** ≈ 686 seedlings

3 **a**

Distance d (m)	Frequency
$10 \leqslant d < 12$	2
$12 \leqslant d < 14$	4
$14 \leqslant d < 16$	7
$16 \leqslant d < 18$	5
$18 \leqslant d < 20$	2

b **Shotput distances thrown**



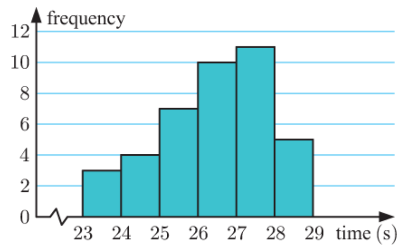
c $14 \leqslant d < 16$ m **d** approximately symmetrical

4 **a**

Time t (s)	*Frequency*
$23 \leqslant t < 24$	3
$24 \leqslant t < 25$	4
$25 \leqslant t < 26$	7
$26 \leqslant t < 27$	10
$27 \leqslant t < 28$	11
$28 \leqslant t < 29$	5

b 17.5%

c **200 m times run by athletics students**



d $27 \leqslant t < 28$ s **e** negatively skewed

EXERCISE 9C.1

1 **a** **i** 25 **ii** 24 **iii** 30
b **i** ≈ 13.3 **ii** 11.5 **iii** 8
c **i** ≈ 10.3 **ii** 10 **iii** 11.2
d **i** ≈ 429 **ii** 428 **iii** 415, 427

2 **a** *Data set A*: $\overline{x} \approx 7.73$ *Data set B*: $\overline{x} \approx 8.45$
b *Data set A*: 7 *Data set B*: 7
c The data sets are the same except for the last value, and the last value of A is less than the last value of B, so the mean of A is less than the mean of B.
d The middle value of both data sets is the same, so the median is the same.

3 **a** mean: \$582 000, median: \$420 000, mode: \$290 000
b The mode is the second lowest value, so does not take the higher values into account.
c No, since the data is unevenly distributed, the median is not in the centre.

4 **a** mean: 3.11 mm, median: 0, mode: 0
b **i** The data is very positively skewed so the median is not in the centre.
ii The mode is the lowest value so does not take the higher values into account.
Both median and mode indicate no rain for February if daily values are unknown.
c 15 and 27

5 **a** 44 points **b** 44 points **c** ≈ 40.6 points
d **i** increase **ii** 40.75 points

6 **a** 1 head **b** 1 head **c** 1.4 heads

7 **a**

Donation (\$)	*Frequency*
1	7
2	9
5	2
10	4
20	8
Total	30

b 30 donations
c **i** $\approx \$7.83$ **ii** \$2 **iii** \$2
d the mode

8 **a** **i** 4.25 ducklings **ii** 5 ducklings **iii** 5 ducklings
b Yes, it is negatively skewed.
c The mean is lower than the mode and median.

9 **a** **i** New York: 3.475 nights, Miami: 4.075 nights
ii New York: 2 nights, Miami: 3 nights and 4 nights
iii New York: 3 nights, Miami: 4 nights
b the Miami hotel

10 26 goals **11** 7.875 **12** 48 trees

13 **a** $x = 12$ **b** $x = 8$ **14** 15 **15** 6

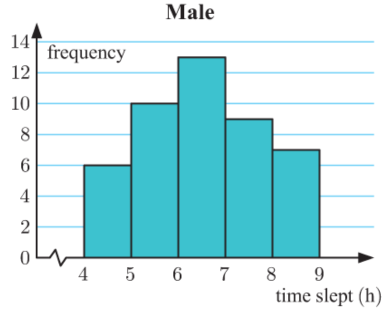
EXERCISE 9C.2

1 $\approx 32.1°$C

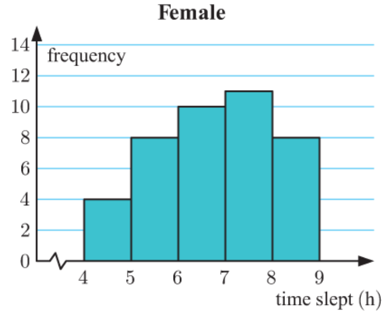
2 **a** $165 \leqslant s < 170$ km h^{-1}
b **i** 76 serves **ii** not possible **iii** 73.5%
c ≈ 167 km h^{-1}

3 **a** 23 times **b** 16 times **c** ≈ 24.1 runs

4 **a** **Male**



Female



b Male: approximately symmetrical,
Female: negatively skewed
c Male: ≈ 6.52 hours, Female: ≈ 6.77 hours
d The female students. However the means are only estimates, and the difference in means is small, so the answer may not be reliable.

5 **a** $\overline{p} \approx 42.28$ litres
b **i**

Petrol bought p (L)	*Frequency*
$20 \leqslant p < 30$	8
$30 \leqslant p < 40$	13
$40 \leqslant p < 50$	17
$50 \leqslant p < 60$	12

ii $\overline{p} \approx 41.6$ litres

c **i**

Petrol bought p (L)	*Frequency*
$20 \leqslant p < 25$	2
$25 \leqslant p < 30$	6
$30 \leqslant p < 35$	6
$35 \leqslant p < 40$	7
$40 \leqslant p < 45$	9
$45 \leqslant p < 50$	8
$50 \leqslant p < 55$	3
$55 \leqslant p < 60$	9

ii $\overline{p} \approx 42.1$ litres

d The estimate found in **c ii** is closer to the actual mean found in **a**. However, both estimates are reasonable approximations.

e Using smaller intervals should produce a more accurate estimate of the mean. This is because as the intervals get smaller, the values in those intervals get closer to the interval midpoint used in estimating the mean.

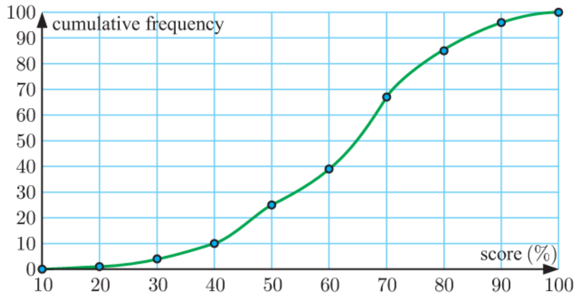
EXERCISE 9D

1 **a** 50 trout **b** **i** ≈ 5 trout **ii** ≈ 15 trout
c ≈ 26.8 cm

2 **a**

Score x (%)	*Frequency*	*Cumulative frequency*
$10 \leqslant x < 20$	1	1
$20 \leqslant x < 30$	3	4
$30 \leqslant x < 40$	6	10
$40 \leqslant x < 50$	15	25
$50 \leqslant x < 60$	14	39
$60 \leqslant x < 70$	28	67
$70 \leqslant x < 80$	18	85
$80 \leqslant x < 90$	11	96
$90 \leqslant x < 100$	4	100

b **Cumulative frequency graph of examination scores**

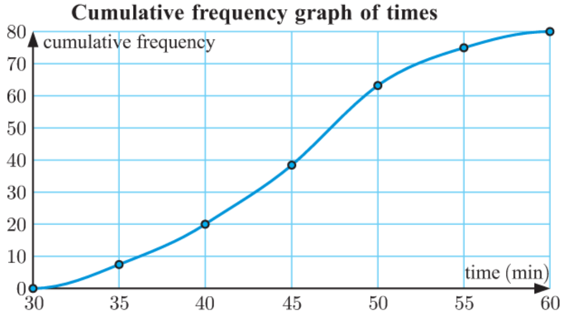


c **i** $\approx 64\%$ **ii** ≈ 52 students **iii** $\approx 74\%$ or more

3 **a**

Time t (min)	*Frequency*	*Cumulative frequency*
$30 \leqslant t < 35$	7	7
$35 \leqslant t < 40$	13	20
$40 \leqslant t < 45$	18	38
$45 \leqslant t < 50$	25	63
$50 \leqslant t < 55$	12	75
$55 \leqslant t < 60$	5	80

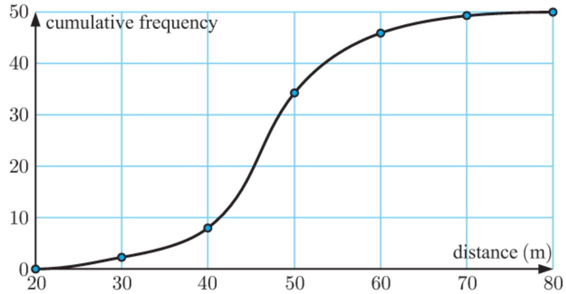
b **Cumulative frequency graph of times**



c **i** ≈ 45.4 min **ii** ≈ 14 runners **iii** ≈ 43 minutes

4 **a** ≈ 47.2 m

b **Baseball distances thrown**



c **i** ≈ 46 m **ii** ≈ 4 students **iii** ≈ 53 m

EXERCISE 9E

1 **a** **i** 7 **ii** 9 **iii** $Q_1 = 7$, $Q_3 = 10$ **iv** 3
b **i** 14 **ii** 18.5 **iii** $Q_1 = 16$, $Q_3 = 20$ **iv** 4
c **i** 7.7 **ii** 26.9 **iii** $Q_1 = 25.5$, $Q_3 = 28.1$ **iv** 2.6

2 range $= 12$ seeds, IQR $= 7$ seeds

3 **a** ≈ 6.33 tows **b** 7 tows **c** 6 tows **d** 2.5 tows

4 **a** **i** Kylie: 11, Chris: 12.5 **ii** Kylie: 17, Chris: 9
iii Kylie: 8, Chris: 4.5
b Chris **c** Kylie

5 **a** **i** Year 6: 4 visits, Year 10: 2 visits
ii Year 6: 4 visits, Year 10: 8 visits
iii Year 6: 2 visits, Year 10: 3 visits
b **i** the Year 6 class **ii** the Year 10 class

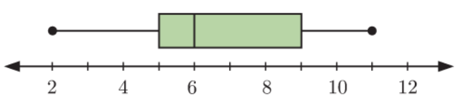
6 **a** ≈ 9.8 min **b** ≈ 6.3 min **c** ≈ 13 min **d** ≈ 6.7 min

EXERCISE 9F.1

1 **a** **i** 31 **ii** 54 **iii** 16 **iv** 40 **v** 26
b **i** 38 **ii** 14

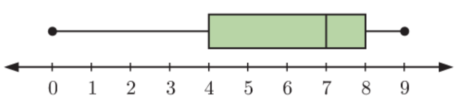
2 **a** **i** ...... 89 points **ii** ...... 25 points **iii** ...... 62 points
iv ...... 73 points **v** ...... between 45 and 73 points
b 64 **c** 28

3 **a** **i** min $= 2$, $Q_1 = 5$, median $= 6$, $Q_3 = 9$, max $= 11$
ii



iii range $= 9$ **iv** IQR $= 4$

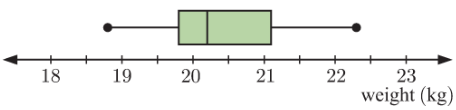
b **i** min $= 0$, $Q_1 = 4$, median $= 7$, $Q_3 = 8$, max $= 9$
ii



iii range $= 9$ **iv** IQR $= 4$

4 **a** median $= 20.2$ kg, $Q_1 = 19.8$ kg, $Q_3 = 21.1$ kg, max. weight $= 22.3$ kg, min. weight $= 18.8$ kg

b



c **i** IQR $= 1.3$ kg **ii** range $= 3.5$ kg

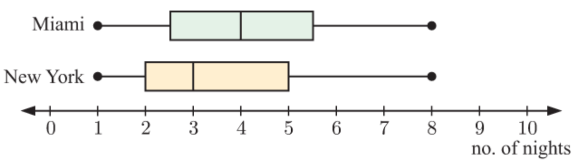
d **i** ...... 20.2 kg **ii** 31.8% of the bags ......
iii ...... 1.3 kg **iv** ...... 19.8 kg or less
e positively skewed

EXERCISE 9F.2

1 **a** Year 10: min = 4 hours, $Q_1 = 6.5$ hours, med = 9 hours, $Q_3 = 11$ hours, max = 15 hours
Year 12: min = 8 hours, $Q_1 = 10$ hours, med = 12 hours, $Q_3 = 16$ hours, max = 17 hours
b **i** Year 10: 11 hours, Year 12: 9 hours
ii Year 10: 4.5 hours, Year 12: 6 hours

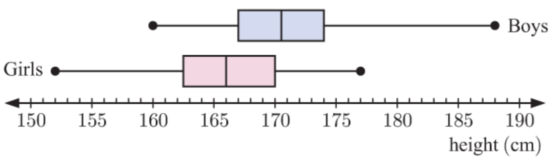
2 **a** **i** Indonesia: 88 cm, Australia: 93 cm
ii Indonesia: 43 cm, Australia: 41 cm
iii Indonesia: 45 cm, Australia: 52 cm
iv Indonesia: 15 cm, Australia: 20 cm
b **i** 75% **ii** 50% **c** **i** Indonesia **ii** Australia

3 **a** New York: min = 1 night, $Q_1 = 2$ nights, med = 3 nights, $Q_3 = 5$ nights, max = 8 nights
Miami: min = 1 night, $Q_1 = 2.5$ nights, med = 4 nights, $Q_3 = 5.5$ nights, max = 8 nights
b



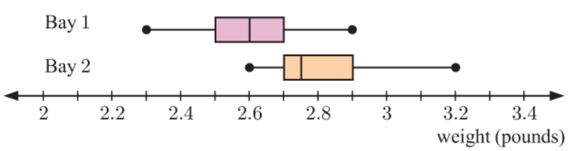
c New York: positively skewed,
Miami: slightly positively skewed
d the Miami hotel

4 **a** Boys: min = 160 cm, $Q_1 = 167$ cm, med = 170.5 cm, $Q_3 = 174$ cm, max = 188 cm
Girls: min = 152 cm, $Q_1 = 162.5$ cm, med = 166 cm, $Q_3 = 170$ cm, max = 177 cm
b



c The distributions show that in general, the boys are taller than the girls and are more varied in their heights.

5 **a** Bay 1: min = 2.3 pounds, $Q_1 = 2.5$ pounds, med = 2.6 pounds, $Q_3 = 2.7$ pounds, max = 2.9 pounds
Bay 2: min = 2.6 pounds, $Q_1 = 2.7$ pounds, med = 2.75 pounds, $Q_3 = 2.9$ pounds, max = 3.2 pounds
b



c Clearly Bay 2 catches weigh more than those from Bay 1. Median weight 2.75 pounds compared with 2.6 pounds. The range (0.6 pounds) and IQR (0.2 pounds) are identical for each bay indicating consistency of spread.

EXERCISE 9G.1

1 **a** ≈ 1.87 **b** ≈ 5.44

2 **a** ≈ 9.22 **b** ≈ 2.45
c The outlier increases the standard deviation.

3 **a** Sample A **b** A: 7, B: 5
c **i** A: 8, B: 4 **ii** A: 2, B: 1
iii A: 1.90, B: 0.894
d s takes all values into account, whereas the range and IQR each use only 2 values.

4 **a** Each brother was given the same number of eggs, and ate them over the same number of days, $\therefore$ the mean is the same.
b Kento: IQR = 2 eggs, Dongming: IQR = 9 eggs
c Kento: $s \approx 1.41$ eggs, Dongming: $s \approx 4.36$ eggs
d Dongming

5 **a** They both have mean = 3, range = 5 **b** Mickey's
c Mickey: $s \approx 1.95$ hits, Julio: $s \approx 1.26$ hits
d The standard deviation is better.

6 **a** Finnley: $\overline{x} = 23$ people, Florence: $\overline{x} \approx 17.4$ people
b Finnley: $s \approx 4.29$ people, Florence: $s \approx 9.24$ people
c Finnley 332, Florence 244 $\therefore$ Finnley
d Florence, as s is greater.

EXERCISE 9G.2

1 1.5 **2** $\overline{x} = 28.8$ chocolates, $s = 1.64$ chocolates

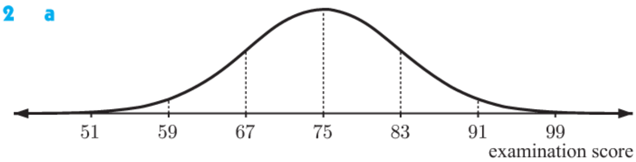
3 mean length = 38.3 cm, $s = 2.66$ cm

4 mean wage = €412.11, $s =$ €16.35

EXERCISE 9H

1 **a** $\approx 68\%$ **b** $\approx 95\%$ **c** $\approx 99.7\%$

2 **a**



b **i** ≈ 79 students **ii** ≈ 11 or 12 students
iii ≈ 409 students

3 **a**

b **i** ≈ 286 bottles **ii** ≈ 252 bottles **c** $\approx 15.9\%$

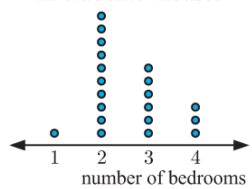
4 **a** $\approx 2.28\%$ **b** $\approx 84.1\%$ **c** $\approx 97.6\%$ **d** $\approx 84.1\%$

5 once

REVIEW SET 9A

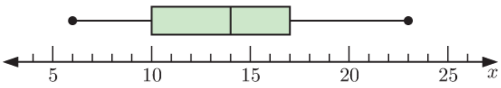
1 positively skewed

2 **a** discrete
b no
c **Number of bedrooms in students' houses**

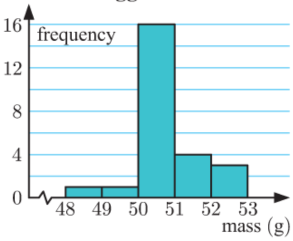


3 **a** **i** 14 **ii** 13.8 **iii** 14 **iv** 17
v $Q_1 = 10$, $Q_3 = 17$ **vi** 7

b



4 **a** **Masses of eggs in a carton**



b $50 \leqslant m < 51$ g. This is the most common weight interval for the eggs.

c approximately symmetric

d ≈ 50.8 g

5 **a** **i** 48 **ii** 98 **iii** 15 **iv** 66 **v** 42

b **i** 83 **ii** 24

6 $x = 13$

7 **a** Comparing the median swim times for girls and boys shows that, in general, the boys swim 2.5 seconds faster than the girls.

b The range of the girls' swim times is 10 seconds compared to the range of 7 seconds for the boys.

c The fastest 25% of the boys swim as fast as or faster than 100% of the girls.

d 100% of the boys swim faster than 60 seconds whereas 75% of the girls swim faster than 60 seconds.

8 **a** 60 competitors **b** 25% **c** ≈ 207 points

9 **a** Davis: $\overline{x} = \$115.28$, $s \approx \$9.60$
Douglas: $\overline{x} = \$102.37$, $s \approx \$20.42$

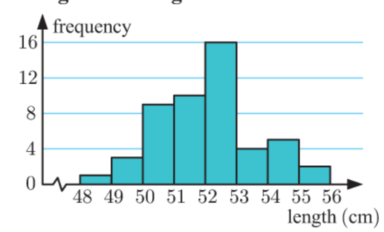
b the Davis family **c** the Douglas family

10 **a** $\overline{x} = 0.88$ kg, $s \approx 0.0980$ kg

b $\overline{x} = 1.76$ kg, $s \approx 0.196$ kg

c Both the mean and the standard deviation doubled when the individual scores were doubled.

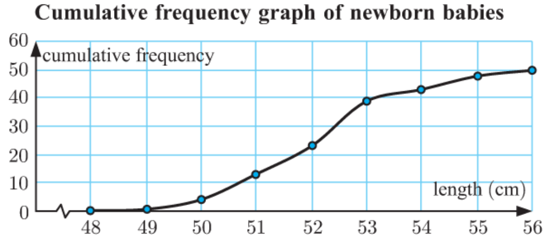
11 **a** **Histogram of lengths of newborn babies**



b 27 babies

c 70%

d **Cumulative frequency graph of newborn babies**



e **i** ≈ 52.1 cm **ii** ≈ 18 babies

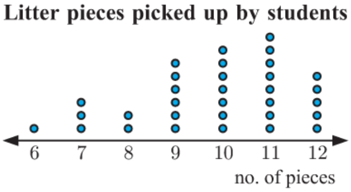
12 336 or 337 apples

REVIEW SET 9B

1 **a** 32 students

b **i** ≈ 9.84 pieces
ii 11 pieces
iii 10 pieces

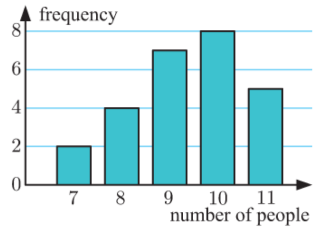
c **Litter pieces picked up by students**



d negatively skewed

2 **a** $\overline{x} \approx 14.6$ **b** 14.5 **c** 14, 15 **3** $\overline{x} \approx 13.6$

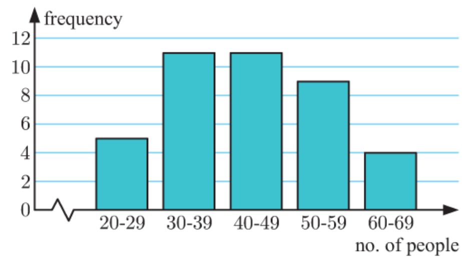
4 **a** **Number of people at judo class** **b** negatively skewed



5 **a**

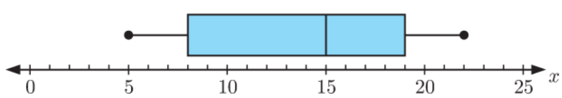
Number of people	*Tally*	*Frequency*
20 - 29	𝍸	5
30 - 39	𝍸 𝍸 𝍷	11
40 - 49	𝍸 𝍸 𝍷	11
50 - 59	𝍸 𝍷𝍷𝍷𝍷	9
60 - 69	𝍷𝍷𝍷𝍷	4
	Total	40

b **Number of people using the swimming pool**



c 32.5%

6



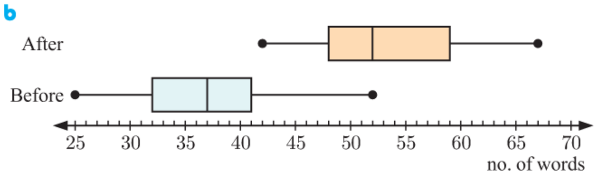
7 18 males **8** 19.5 **9** **a** 32 houses **b** $\approx \$383\,000$

10 **a** ≈ 7.00 **b** ≈ 0.920

11 **a** Before: min = 25 words, Q_1 = 32 words, med = 37 words, Q_3 = 41 words, max = 52 words

After: min = 42 words, Q_1 = 48 words, med = 52 words, Q_3 = 59 words, max = 67 words

b



c Yes. Each value in the 5-number summary for the After data is greater than the corresponding value for the Before data.

12 **a** ≈ 79 batteries **b** ≈ 11 batteries **c** ≈ 239 batteries

EXERCISE 10A

1 **a** 3 **b** 2 **c** $\frac{1}{2}$ **d** -6 **e** $\frac{9}{2}$ **f** 1

g $\frac{9}{2}$ **h** 12 **i** 2 **j** 6

2 **a** -2 **b** $-\frac{1}{2}$ **c** $\frac{1}{3}$ **d** 8 **e** 4 **f** -1

g $\frac{1}{2}$ **h** -3 **i** $-\frac{2}{3}$ **j** -4

EXERCISE 10B.1

1 **a** $\frac{a}{2}$ **b** $2m$ **c** 6 **d** 3 **e** $2a$ **f** x^2
g $2x$ **h** 2 **i** $\frac{1}{2a}$ **j** $2m$ **k** 4 **l** $\frac{2}{t}$
m $2d$ **n** $\frac{b}{2}$ **o** $\frac{2b}{3a}$

2 **a** t **b** cannot be simplified **c** y
d cannot be simplified **e** $\frac{a}{b}$ **f** cannot be simplified
g $\frac{a}{2}$ **h** cannot be simplified **i** $\frac{7c}{4d}$

3 **a** 4 **b** $2n$ **c** a **d** 1
e a^2 **f** $\frac{3n}{2}$ **g** $\frac{1}{2b^3}$ **h** $\frac{k^4}{2a^3}$

4 **a** $2(x+5)$ **b** $\frac{n+5}{6}$ **c** $\frac{b+2}{2}$ **d** $\frac{3(k-2)}{4}$
e $\frac{5}{t-1}$ **f** $\frac{2}{5(k+4)}$ **g** $\frac{1}{3(x-3)}$ **h** $\frac{5(p+4)}{3}$

5 **a** $\frac{x+2}{9}$ **b** $\frac{12}{a+1}$ **c** $\frac{x+y}{3}$ **d** $x+y$
e $\frac{2}{x+2}$ **f** $\frac{a+5}{3}$ **g** $\frac{y}{3}$ **h** $\frac{y-3}{3}$
i $\frac{x+1}{3}$ **j** $\frac{1}{2(b-4)}$ **k** $\frac{2(p+q)}{3}$ **l** $\frac{8}{5(r-2)}$
m $\frac{x}{x-1}$ **n** $\frac{(x+2)(x+1)}{4}$ **o** $\frac{(x+2)(x-1)^2}{4x}$

EXERCISE 10B.2

1 **a** $x+2$ **b** $x-2$ **c** $\frac{x+2}{2}$ **d** $\frac{x-5}{2}$
e $\frac{y+3}{3}$ **f** $\frac{3x-15}{2}$ **g** $a+b$ **h** $\frac{a+b}{c+d}$

2 **a** $\frac{2x+3}{3}$ **b** cannot be simplified
c cannot be simplified **d** $2a-1$ **e** $\frac{3a+1}{2}$
f cannot be simplified **g** $\frac{b+3}{2}$ **h** $\frac{4b-6}{3}$

3 **a** $\frac{3}{4}$ **b** $\frac{5}{3}$ **c** x **d** $\frac{4}{5}$ **e** $\frac{1}{y}$ **f** $\frac{x}{y}$ **g** $4x$
h $3x$ **i** $\frac{5x}{7}$ **j** $\frac{3b}{4}$ **k** $\frac{2x}{3}$ **l** $\frac{3(a+b)}{2a^2(2a+b)}$

4 **a** -2 **b** $-\frac{3}{2}$ **c** -1 **d** $-\frac{1}{2}$ **e** -3
f $-\frac{2}{x}$ **g** $-\frac{ab}{2}$ **h** $-2x$

5 **a** $x+1$ **b** $x-1$ **c** $-(x+1)$ **d** $\frac{1}{x-2}$
e $a-b$ **f** $-(a+b)$ **g** $\frac{2}{x-1}$ **h** $\frac{3+x}{x}$
i $\frac{3(x+y)}{2y}$ **j** $-\frac{2(b+a)}{a}$ **k** $\frac{y}{4x+y}$ **l** $-\frac{4x}{x+4}$

6 **a** $x+1$ **b** $\frac{1}{x-5}$ **c** $\frac{2x}{x-5}$ **d** $\frac{x-2}{x+2}$
e $\frac{x+3}{x-1}$ **f** $\frac{x+1}{1-x}$ **g** $\frac{x-5}{x+3}$ **h** $\frac{x+2}{x+3}$
i $\frac{x+2}{2x-1}$ **j** $\frac{2x+1}{x-1}$ **k** $\frac{4x-3}{2x+1}$ **l** $\frac{3x+4}{x+2}$

EXERCISE 10C

1 **a** $\frac{xy}{10}$ **b** $\frac{3}{2}$ **c** $\frac{a^2}{2}$ **d** $\frac{1}{6}$ **e** $\frac{1}{5}$ **f** $\frac{c^2}{10}$
g $\frac{ac}{bd}$ **h** 1 **i** $\frac{1}{2m}$ **j** 2 **k** $\frac{a}{b}$ **l** 4
m $\frac{3}{m}$ **n** $\frac{a^2}{b^2}$ **o** $\frac{4}{x^2}$ **p** $\frac{1}{c}$

2 **a** $\frac{3}{2}$ **b** $\frac{3}{a}$ **c** $\frac{3x}{16}$ **d** $\frac{3}{4}$ **e** 2 **f** $\frac{c}{25}$
g $\frac{1}{5}$ **h** $\frac{m^2}{2}$ **i** 2 **j** $\frac{n}{m}$ **k** $\frac{3}{4g}$ **l** $\frac{g}{3}$
m $\frac{8}{x^3}$ **n** $\frac{x^2}{3}$ **o** $\frac{1}{a}$ **p** $\frac{3a}{5}$

3 **a** $\frac{5x}{2(x-2)}$ **b** $\frac{4}{3t}$ **c** $\frac{20}{a}$ **d** $\frac{4k}{3}$
e $-\frac{x}{2}$ **f** $\frac{3m}{2(5m-1)}$

EXERCISE 10D

1 **a** $\frac{5a}{6}$ **b** $\frac{b}{10}$ **c** $\frac{7c}{4}$ **d** $\frac{5d-6}{10}$
e $\frac{2x+15}{24}$ **f** $-\frac{5x}{14}$ **g** $\frac{4a+3b}{12}$ **h** $-\frac{2t}{9}$
i $\frac{5m}{21}$ **j** $\frac{d}{2}$ **k** $\frac{11p}{35}$ **l** $\frac{22t}{45}$
m $\frac{19k}{72}$ **n** m **o** $\frac{5a}{12}$ **p** $\frac{x}{12}$
q $\frac{11z}{20}$ **r** $\frac{41q}{21}$

2 **a** $\frac{7b+3a}{ab}$ **b** $\frac{3c+2a}{ac}$ **c** $\frac{4d+5a}{ad}$ **d** $\frac{2an-am}{mn}$
e $\frac{2a+b}{2x}$ **f** $\frac{5}{2a}$ **g** $\frac{4y-1}{xy}$ **h** $\frac{31}{5x}$
i $\frac{35}{12z}$ **j** $\frac{ad+bc}{bd}$ **k** $\frac{6+a^2}{2a}$ **l** $\frac{3x+2y}{3y}$
m $\frac{40-2p}{5p}$ **n** $\frac{7x}{18y}$ **o** $-\frac{19}{40t}$ **p** $\frac{5x+6}{2x^2}$

3 **a** $\frac{x+2}{2}$ **b** $\frac{y-3}{3}$ **c** $\frac{3a}{2}$ **d** $\frac{b-12}{4}$
e $\frac{x-8}{2}$ **f** $\frac{6+a}{3}$ **g** $\frac{4x}{5}$ **h** $\frac{2x+1}{x}$
i $\frac{5x-2}{x}$ **j** $\frac{a^2+2}{a}$ **k** $\frac{3+b^2}{b}$ **l** $\frac{1-2x^3}{x^2}$

4 **a** $\frac{5x+2}{6}$ **b** $\frac{-x-1}{4}$ **c** $\frac{11x+9}{12}$ **d** $\frac{5x+1}{6}$
e $\frac{-2x-1}{12}$ **f** $\frac{10x+3}{6}$ **g** $\frac{7x+3}{12}$ **h** $\frac{5x+2}{4}$
i $\frac{23x-6}{30}$ **j** $\frac{5b-a}{6}$ **k** $\frac{14x-1}{20}$ **l** $\frac{23-5x}{14}$
m $\frac{11x-12}{30}$ **n** $\frac{3x-4}{20}$ **o** $\frac{3x-2}{8}$ **p** $\frac{3x-2}{10}$
q $\frac{32-7x}{15}$ **r** $\frac{-17x-1}{12}$

5 **a** $\frac{7x+3}{x(x+1)}$ **b** $\frac{2x-6}{x(x+2)}$ **c** $\frac{x-7}{(x+1)(x-1)}$
d $\frac{3x+7}{x+2}$ **e** $\frac{5x-4}{x(x-4)}$ **f** $\frac{-4x-10}{x+3}$
g $\frac{2x^2+x+1}{(x+1)(x-1)}$ **h** $\frac{11x-10}{x(x-2)}$

i $\dfrac{-2x-6}{(x+1)(x+2)}$ **j** $\dfrac{2x^2+5x-4}{(x-1)(2x+1)}$

k $\dfrac{x^2+2x+17}{(x-1)(x+3)}$ **l** $\dfrac{-8x}{(x+5)(x-3)}$

m $\dfrac{3x^2+6x+2}{x(x+1)(x+2)}$ **n** $\dfrac{27x-24}{x(x+3)(x-4)}$

o $\dfrac{2x^3-x^2+1}{x(x-1)(x+1)}$

6 **a** $\$\left(\dfrac{210x+150}{x(x+1)}\right)$ **b** **i** \$49.50 **ii** \$40

7 **a** $\dfrac{2+x}{x(x+1)}$ **b** $\dfrac{2(x^2+2x+2)}{(x+2)(x-3)}$

c $\dfrac{x^2-2x+3}{(x-2)(x+3)}$ **d** $\dfrac{x+14}{x+7}$

8 **a** $\dfrac{-2}{x-2}$ **b** $\dfrac{2}{x+4}$ **c** $\dfrac{x-2}{x+2}$

d $\dfrac{x-6}{2-x}$ **e** $\dfrac{-(x+2)}{4x^2}$ **f** $\dfrac{12-x}{16x^2}$

9 **a** $\dfrac{2x}{x+1}-\dfrac{4}{(x-1)(x+1)}=\dfrac{2(x-2)}{x-1}$ is zero when $x=2$, and undefined when $x=\pm1$.

b $\dfrac{6}{(x+2)(x+5)}+\dfrac{x}{x+2}=\dfrac{x+3}{x+5}$ is zero when $x=-3$, and undefined when $x=-5$ or -2.

EXERCISE 10E

1 **a** $x=15$ **b** $x=\frac{9}{2}$ **c** $x=\frac{35}{2}$ **d** $x=\frac{3}{8}$

e $x=-\frac{35}{12}$ **f** $x=\frac{15}{2}$ **g** $x=-\frac{25}{21}$ **h** $x=-\frac{3}{7}$

2 **a** $x=2$ **b** $x=-9$ **c** $x=-\frac{4}{3}$ **d** $x=-27$

e $x=-\frac{7}{2}$ **f** $x=-1$

REVIEW SET 10A

1 **a** -2 **b** 4 **c** $\frac{1}{3}$ **d** $-\frac{13}{11}$

2 **a** $\dfrac{2t}{3}$ **b** $\frac{8}{3}$ **c** $\dfrac{x}{3}$ **d** $\dfrac{2}{x+2}$

3 **a** $\dfrac{2}{x-3}$ **b** $\dfrac{x+2}{x}$ **c** $\dfrac{3x}{3x+1}$

4 **a** -2 **b** $-\dfrac{5}{x}$ **c** $-\dfrac{x+4}{2}$

5 **a** $\dfrac{a}{3}$ **b** $\dfrac{3a}{b^2}$ **c** $\dfrac{3a+b^2}{3b}$ **d** $\dfrac{3a-b^2}{3b}$

6 **a** $\dfrac{21}{x}$ **b** $\dfrac{t}{24}$ **7** **a** $\dfrac{3}{2n}$ **b** $\dfrac{7x}{3(x+5)}$

8 **a** $\dfrac{11x}{12}$ **b** $\dfrac{14+x}{7}$ **c** $\dfrac{x-4}{4}$ **d** $\dfrac{5x}{12}$

9 **a** $\dfrac{7x-3}{12}$ **b** $\dfrac{3x+2}{6}$ **c** $\dfrac{3x+3}{10}$

10 **a** $\dfrac{3x}{(x+1)(x-2)}$ **b** $\dfrac{x+9}{(x-1)(x+1)}$ **c** $\dfrac{x^2+x+1}{x^2(x+1)}$

11 $x=6$

12 **a** **i** $\dfrac{a^2-9}{a}$ **ii** $\dfrac{3-a}{3}$ **b** $-\dfrac{3(a+3)}{a}$

c **i** -12 **ii** undefined **iii** $-\frac{24}{5}$

REVIEW SET 10B

1 **a** -6 **b** 0 **c** 2

2 **a** $\dfrac{3}{2x}$ **b** $a+2b$ **c** $\dfrac{x+2}{x}$

3 **a** $\frac{1}{3}$ **b** $2(x-2)$ **c** $\dfrac{x-3}{4}$

4 **a** $\dfrac{2m}{n^2}$ **b** $\dfrac{m}{2}$ **c** $\dfrac{m^3}{n}$

5 **a** $\dfrac{11}{2x}$ **b** $\dfrac{6b-ay}{by}$ **c** $\dfrac{35}{12x}$

6 **a** $\dfrac{5x}{14}$ **b** $\dfrac{4x+9}{3x^2}$

7 **a** $\dfrac{10+x}{2}$ **b** $\dfrac{3x-y}{x}$ **c** $\dfrac{6+3x+2y}{6}$

8 **a** $\dfrac{3y}{2(y+2)}$ **b** $-\dfrac{3}{4x}$

9 **a** $\dfrac{3x-2}{8}$ **b** $\dfrac{9x+27}{10}$ **c** $\dfrac{9-7x}{18}$

10 **a** $\dfrac{7-x}{(x-1)(x+2)}$ **b** $\dfrac{x^2-2x+2}{x^2(x-1)}$

c $\dfrac{2x^3+3x^2-10x-12}{x(x+3)(x+2)}$

11 **a** **i** 12 **ii** 50 **b** xy

c $(x+y)\div\left(\dfrac{1}{x}+\dfrac{1}{y}\right)=(x+y)\div\left(\dfrac{y+x}{xy}\right)$ **d** 420

$=\overset{1}{\cancel{(x+y)}}\times\dfrac{xy}{\underset{1}{\cancel{x+y}}}$

$=xy$

12 $x=\frac{1}{2}$

EXERCISE 11A

1 **a** $x=\pm10$ **b** $x=\pm5$ **c** $x=\pm2$

d $x=\pm3$ **e** no real solution **f** $x=0$

g $x=\pm3$ **h** no real solution **i** $x=\pm\sqrt{2}$

2 **a** $x=4$ or -2 **b** $x=0$ or -8 **c** no real solution

d $x=4\pm\sqrt{5}$ **e** no real solution **f** $x=-2$

g $x=2\frac{1}{2}$ **h** $x=0$ or $-\frac{4}{3}$ **i** $x=\frac{8}{3}$ or $-\frac{10}{3}$

j $x=-\frac{1}{2}\pm2\sqrt{3}$ **k** $x=\dfrac{3\pm\sqrt{7}}{2}$ **l** $x=\dfrac{\pm\sqrt{6}-3}{2}$

3 **a** $x=\pm2$ **b** $x=\pm\sqrt{10}$ **c** $x=\pm4$

4 **a** $x=1$ **b** $x=\frac{3}{5}$ or 1

EXERCISE 11B

1 **a** $x=0$ or 7 **b** $x=0$ or 5 **c** $x=0$ or 8

d $x=0$ or 4 **e** $x=0$ or -2 **f** $x=0$ or $-\frac{5}{2}$

g $x=0$ or $\frac{3}{4}$ **h** $x=0$ or $\frac{5}{4}$ **i** $x=0$ or 3

2 **a** $x=-1$ or -2 **b** $x=1$ or 2 **c** $x=5$

d $x=-2$ or -3 **e** $x=2$ or 3 **f** $x=-1$ or -6

g $x=-2$ or -7 **h** $x=-5$ or -6 **i** $x=-5$ or 3

j $x=-6$ or 2 **k** $x=3$ or 8 **l** $x=7$

3 **a** $x=-3$ or -6 **b** $x=-4$ or -7 **c** $x=-4$ or 2

d $x=-4$ or 3 **e** $x=3$ or 2 **f** $x=2$

g $x=3$ or -2 **h** $x=-12$ or 5 **i** $x=10$ or -7

j $x=\frac{1}{2}$ or 2 **k** $x=-3$ or $\frac{1}{3}$ **l** $x=-4$ or $-\frac{5}{3}$

m $x=\frac{1}{2}$ or -3 **n** $x=\frac{1}{2}$ or 5 **o** $x=-1$ or $-\frac{5}{2}$

4 **a** $x=-\frac{1}{3}$ or -4 **b** $x=-\frac{2}{5}$ or 3 **c** $x=\frac{1}{2}$ or -9
d $x=-1$ or $\frac{5}{2}$ **e** $x=\frac{4}{3}$ or -2 **f** $x=\frac{3}{2}$ or -6
g $x=-\frac{1}{6}$ or 3 **h** $x=\frac{3}{2}$ or -4 **i** $x=\frac{3}{2}$ or $\frac{1}{3}$
j $x=-\frac{8}{3}$ or 2 **k** $x=\frac{4}{7}$ or $-\frac{1}{2}$ **l** $x=\frac{1}{4}$ or $-\frac{4}{3}$

5 **a** $x=-4$ or -3 **b** $x=-3$ or 1 **c** $x=\pm 3$
d $x=-1$ or $\frac{2}{3}$ **e** $x=-\frac{1}{2}$ **f** $x=\frac{5}{2}$ or 4
g $x=11$ or -3 **h** $x=-\frac{3}{4}$ or $\frac{5}{2}$

6 **a** $x=-2$ or 1 **b** $x=-6$ or 2 **c** $x=2$ or -1
d $x=4$ or -1 **e** $x=1$ or $-\frac{1}{3}$ **f** $x=\frac{1}{2}$ or -1

7 **a** $x=4$ or -1 **b** $x=15$ or 2 **c** $x=-9$ or -2
d $x=3$ or $-\frac{7}{2}$ **e** $x=4$ or $\frac{4}{3}$ **f** $x=1$ or $\frac{6}{5}$
g $x=0$ or $\pm\sqrt{11}$ **h** $x=0$ or -3

8 **a** $x=\pm 1$ or ± 2 **b** $x=\pm\sqrt{3}$ or ± 2 **c** $x=\pm\sqrt{5}$

9 **a** $x=0,\ 13,$ or -3 **b** $x=0$ or 4
c $x=0,\ \pm 2,$ or $\pm\sqrt{11}$

10 **a** $x=2$ **b** $x=3$

EXERCISE 11C

1 **a** **i** 1^2 **ii** $(x+1)^2=6$
b **i** 1^2 **ii** $(x-1)^2=-6$
c **i** 3^2 **ii** $(x+3)^2=11$
d **i** 3^2 **ii** $(x-3)^2=6$
e **i** 5^2 **ii** $(x+5)^2=26$
f **i** 4^2 **ii** $(x-4)^2=21$
g **i** 6^2 **ii** $(x+6)^2=49$
h **i** $(\frac{5}{2})^2$ **ii** $(x+\frac{5}{2})^2=\frac{17}{4}$
i **i** $(\frac{7}{2})^2$ **ii** $(x-\frac{7}{2})^2=\frac{65}{4}$

2 **a** $x=2\pm\sqrt{3}$ **b** $x=1\pm\sqrt{3}$ **c** $x=2\pm\sqrt{7}$
d $x=-1\pm\sqrt{2}$ **e** $x=-2\pm\sqrt{3}$ **f** $x=-3\pm\sqrt{6}$
g $x=-1$ or -2 **h** $x=-4\pm\sqrt{2}$ **i** $x=\dfrac{3\pm\sqrt{13}}{2}$

3 **a** no real solution **b** $x=3$ or 2 **c** no real solution
d $x=\dfrac{-1\pm\sqrt{5}}{2}$ **e** $x=\dfrac{-5\pm\sqrt{33}}{2}$ **f** no real solution

4 **a** $x=\dfrac{-2\pm\sqrt{6}}{2}$ **b** $x=\dfrac{6\pm\sqrt{15}}{3}$ **c** $x=\dfrac{5\pm\sqrt{10}}{5}$
d $x=\dfrac{-6\pm\sqrt{46}}{2}$ **e** no real solution **f** $x=\dfrac{-3\pm\sqrt{21}}{6}$

5 **a** $x=\dfrac{-b\pm\sqrt{b^2-4c}}{2}$
b **i** $b^2-4c>0$ **ii** $b^2-4c=0$ **iii** $b^2-4c<0$

EXERCISE 11D

1 **a** **i, ii** $x=-2$ or -4 **b** **i, ii** $x=5$
c **i, ii** $x=-\frac{2}{3}$ or 3

2 **a** $x=\dfrac{-1\pm\sqrt{21}}{2}$ **b** $x=\dfrac{5\pm\sqrt{5}}{2}$ **c** $x=2\pm\sqrt{5}$
d $x=\dfrac{-5\pm\sqrt{37}}{6}$ **e** $x=\dfrac{1\pm\sqrt{57}}{4}$ **f** $x=\dfrac{4\pm\sqrt{11}}{5}$
g $x=\dfrac{3\pm\sqrt{5}}{2}$ **h** $x=\dfrac{1\pm\sqrt{7}}{2}$ **i** $x=\dfrac{1\pm\sqrt{2}}{3}$
j $x=\dfrac{5\pm\sqrt{53}}{14}$ **k** $x=\dfrac{-1\pm\sqrt{7}}{3}$ **l** $x=\dfrac{2\pm\sqrt{3}}{5}$

3 **a** $x=\dfrac{-1\pm\sqrt{29}}{2}$ **b** $x=\dfrac{-1\pm\sqrt{5}}{2}$ **c** no real solution
d $x=1\pm 2\sqrt{2}$ **e** $x=\dfrac{7\pm\sqrt{217}}{6}$ **f** $x=\dfrac{3\pm\sqrt{13}}{2}$

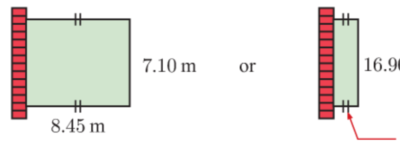
EXERCISE 11E

1 -11 or 10 **2** -3 or 8 **3** $x=3+\sqrt{5}$ and $3-\sqrt{5}$

4 The numbers are -2 and 5, or 2 and -5.

5 8 cm **6** 10 m **7** 18 m by 12 m

8



9 17.9 cm **10** BC $=$ 16 cm or 5 cm

11 **a** $x=2$ **b** $x=5$ **c** $x=6$ **d** $x=\sqrt{31}-1$
e $x=\dfrac{3+\sqrt{5}}{2}$ as $x>\frac{1}{2}$ **f** $x=3+\sqrt{34}$

12 BE $=$ 6 cm **13** CD $=(1+\sqrt{41})$ m **14** $n=6$

15 $\frac{2}{5}$ or $\frac{-9}{-6}$ **16** 40 oranges **17** $\frac{4}{3}$ or $\frac{3}{4}$

18 The numbers are $\dfrac{4+\sqrt{14}}{2}$ and $\dfrac{4-\sqrt{14}}{2}$.

19 Cut out squares with sides 2 cm.

20 **a** $x+2r=10 \quad \therefore\ r=5-\dfrac{x}{2}$
b **Hint:** Lawn area $=4\times$ total of flower bed areas
$\therefore\ 2(\pi\times 5^2)=4\times\pi r^2$
$\therefore\ 50\pi=4\pi\left(5-\dfrac{x}{2}\right)^2$ etc.
c 2.93 m

21 $x=5$

22 **a** **Hint:** Draw a diagram and show that area of pavement $=2\,[x(12+2x)+6x]$.
b $4x^2+36x=\frac{7}{8}(6\times 12)=63$ **c** 1.5 m

23 3.2 cm

EXERCISE 11F

1 **a** $i\sqrt{2}$ **b** $5i$ **c** $i\sqrt{11}$ **d** $9i$

2 **a** $x=\pm i\sqrt{3}$ **b** $x=\pm 2i$ **c** $x=\pm i\sqrt{14}$
d $x=\pm i\sqrt{7}$ **e** $x=\pm 4i$ **f** $x=\pm i\sqrt{6}$

3 **a** two real solutions **b** one real solution
c imaginary solutions

4 **a** $x=\dfrac{-1\pm i\sqrt{7}}{2}$ **b** $x=\dfrac{3\pm i\sqrt{15}}{2}$ **c** $x=-2\pm 3i$
d $x=\dfrac{1\pm i\sqrt{39}}{4}$ **e** $x=1\pm 4i$ **f** $x=\dfrac{3\pm i\sqrt{51}}{6}$

EXERCISE 11G

1 **a** sum of the roots is $\frac{7}{3}$, product of the roots is $\frac{2}{3}$
b roots are 2 and $\frac{1}{3}$, with sum $\frac{7}{3}$, product $\frac{2}{3}$

2 **a** sum of the roots is -2, product of the roots is -4
b roots are $-1\pm\sqrt{5}$, with sum -2 and product -4

3 **a** sum of the roots is 3, product of the roots is $\frac{1}{2}$
b sum of the roots is $-\frac{4}{5}$, product of the roots is -2

c sum of the roots is $\frac{1}{4}$, product of the roots is $\frac{3}{2}$

4 $-\frac{2}{3}$

5 **a** sum of the roots is -4, product of the roots is 5
b roots are $-2 \pm i$, with sum -4, product 5

REVIEW SET 11A

1 **a** $x = \pm\sqrt{11}$ **b** $x = \pm 4$

2 **a** $x = 9$ or -1 **b** $x = 0$ or -2

3 **a** $x = 7$ or -3 **b** $x = \pm\frac{5}{2}$ **c** $x = \frac{2}{3}$ or $-\frac{1}{2}$

4 **a** $x = 6$ or -4 **b** $x = -3$ **c** $x = \frac{1}{2}$ or 4

5 $x = -3 \pm \sqrt{5}$

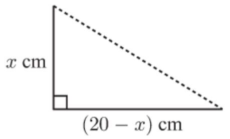
6 **a** $x = -12 \pm \sqrt{155}$ **b** $x = 9$ or -2 **c** $x = -\frac{2}{5}$ or $\frac{3}{2}$

7 $\frac{2}{3}$ or $\frac{3}{2}$

8 **a** $x = 2$ or $-\frac{1}{2}$ **b** $x = \dfrac{-2 \pm \sqrt{19}}{3}$ **c** $x = \dfrac{13 \pm \sqrt{105}}{8}$

9 $x = 7$

10



a **i** $(10 + 2\sqrt{10})$ cm, $(10 - 2\sqrt{10})$ cm, and hypotenuse $2\sqrt{70}$ cm
ii $(20 + 2\sqrt{70})$ cm

b **Hint:** We require $\frac{1}{2}x(20 - x) = 51$.

11 $x = 1$ or 5 **12** **a** $x = \pm i$ **b** $x = \dfrac{-5 \pm i\sqrt{11}}{2}$

13 sum of the roots is 4, product of the roots is $\frac{1}{3}$

REVIEW SET 11B

1 **a** $x = \pm\sqrt{6}$ **b** $x = \pm 1$

2 **a** $x = -3 \pm \sqrt{19}$ **b** $x = \dfrac{1 \pm \sqrt{17}}{3}$

3 **a** $x = 11$ or -3 **b** $x = \frac{1}{2}$ or $-\frac{3}{4}$ **4** $x = 1 \pm \sqrt{101}$

5 **a** $x = 9$ or -5 **b** $x = \frac{2}{3}$ or -5

6 **a** $x = \pm\sqrt{35}$ **b** $x = -3$ or 1

7 8 cm, 15 cm, and 17 cm

8 **a** $x = \dfrac{-1 \pm \sqrt{3}}{2}$ **b** $x = \dfrac{2 \pm \sqrt{2}}{2}$

9 12 people, \$40 each **10** 10 cm

11 **a** $$x + \frac{1}{x} = 5$$
$\therefore\ x^2 - 5x + 1 = 0$ has solutions $x = \dfrac{5 \pm \sqrt{21}}{2}$

b **Hint:** $\dfrac{5 + \sqrt{21}}{2} + \dfrac{2}{5 + \sqrt{21}}\left(\dfrac{5 - \sqrt{21}}{5 - \sqrt{21}}\right)$

12 **a** $x = \pm i\sqrt{5}$ **b** $x = -3 \pm 3i$ **13** $-\frac{3}{2}$

EXERCISE 12A.1

1 **a** $\tan 64° = \dfrac{c}{x}$ **b** $\sin 70° = \dfrac{a}{x}$ **c** $\cos 43° = \dfrac{x}{e}$
d $\sin 35° = \dfrac{x}{b}$ **e** $\cos 61° = \dfrac{g}{x}$ **f** $\tan 30° = \dfrac{h}{x}$

2 **a** $x \approx 12.99$ **b** $x \approx 15.52$ **c** $x \approx 9.84$
d $x \approx 6.73$ **e** $x \approx 3.31$ **f** $x \approx 20.01$
g $x \approx 16.86$ **h** $x \approx 4.45$ **i** $x \approx 1.65$
j $x \approx 26.75$ **k** $x \approx 7.53$ **l** $x \approx 9.61$

3 **a** $\theta = 22°$, $a \approx 8.09$, $b \approx 3.03$
b $\theta = 59°$, $a \approx 13.0$, $b \approx 7.83$
c $\theta = 65°$, $a \approx 5.42$, $b \approx 2.29$

4 ≈ 28.8 cm

EXERCISE 12A.2

1 **a** $\theta \approx 48.2°$ **b** $\theta \approx 45.6°$ **c** $\theta \approx 56.3°$
d $\theta \approx 37.4°$ **e** $\theta \approx 42.2°$ **f** $\theta = 45°$
g $\theta \approx 40.2°$ **h** $\theta \approx 35.3°$ **i** $\theta \approx 35.9°$

2 **a** $\phi \approx 45.6°$, $\theta \approx 44.4°$, $x \approx 7.14$
b $\alpha \approx 53.9°$, $\beta \approx 36.1°$, $x \approx 5.94$
c $a \approx 50.3°$, $b \approx 39.7°$, $x \approx 8.65$

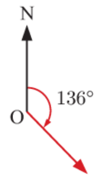
3 **a** **i** $\sin\theta = \frac{3}{8}$ **ii** $\cos\theta = \frac{\sqrt{55}}{8}$ **iii** $\tan\theta = \frac{3}{\sqrt{55}}$
b In each case, $\theta \approx 22.0°$.

EXERCISE 12B

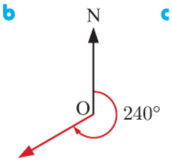
1 ≈ 110 m **2** $\approx 32.9°$ **3** ≈ 238 m **4** ≈ 765 m
5 ≈ 280 m **6** $6.89°$ **7** ≈ 23.5 m **8** $\approx 21.8°$
9 ≈ 15.8 cm **10** $\approx 106°$ **11** No, ≈ 0.721 cm.
12 $\approx 41.4°$ **13** $\approx 53.2°$ **14** **a** ≈ 248 m **b** ≈ 128 m
15 $\approx 14.3°$ **16** ≈ 729 m
17 **a** ≈ 1.66 units **b** ≈ 1.66 units

EXERCISE 12C

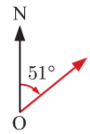
1 **a**



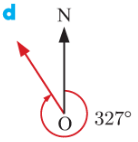
b



c



d



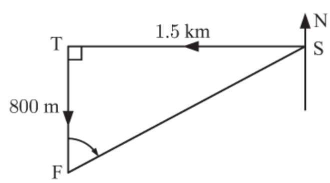
2 **a** 040° **b** 235° **c** 297° **d** 132° **e** 225°
f 337°

3 **a** 041° **b** 142° **c** 322° **d** 099° **e** 221°
f 279°

4 **a** 055° **b** 235° **c** 095° **d** 275° **e** 130°
f 310°

5 $\approx 057.3°$ **6** $\approx 308°$

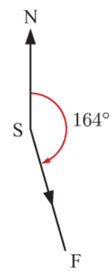
7 **a**



b 1.7 km
c 061.9°T

8 ≈ 7.81 km, $\approx 130°$ **9** ≈ 46.0 km

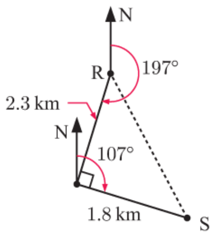
10 **a**



b 35 km
c **i** ≈ 9.65 km
ii ≈ 33.6 km

11 ≈ 2.44 km **12** ≈ 221 km

13 **a**



b 2.92 km
c 159°

14 ≈ 3.61 km, $\approx 057.7°$

15 **a** ≈ 33.5 km, $\approx 025.6°$ **b** $\approx 206°$

EXERCISE 12D.1

1 **a** ≈ 21.2 cm **b** $\approx 35.3°$

2 **a** HX ≈ 9.43 cm **b** $\approx 32.5°$ **c** HY ≈ 10.8 cm **d** $\approx 29.1°$

3 **a** DF ≈ 8.94 cm **b** $\approx 18.5°$

4 ≈ 69.2 cm **5** **a** $45°$ **6** $\theta \approx 61.9°$

EXERCISE 12D.2

1 **a** **i** [GF] **ii** [HG] **iii** [HF] **iv** [GM]
b **i** [MA] **ii** [MN]
c **i** [CD] **ii** [DE] **iii** [DF] **iv** [DX]

2 **a** **i** $\mathrm{B\widehat{G}F}$ **ii** $\mathrm{B\widehat{H}F}$ **iii** $\mathrm{D\widehat{F}H}$ **iv** $\mathrm{A\widehat{X}E}$
b **i** $\mathrm{P\widehat{Z}S}$ **ii** $\mathrm{Q\widehat{Y}R}$ **iii** $\mathrm{P\widehat{W}S}$ **iv** $\mathrm{Q\widehat{W}R}$
c **i** $\mathrm{A\widehat{S}X}$ **ii** $\mathrm{A\widehat{Y}X}$

3 **a** **i** $45°$ **ii** $\approx 35.3°$ **iii** $\approx 63.4°$ **iv** $\approx 41.8°$
b **i** $\approx 18.4°$ **ii** $\approx 15.5°$ **iii** $\approx 17.5°$
c **i** $\approx 26.6°$ **ii** $\approx 22.6°$ **iii** $\approx 26.6°$
d **i** $\approx 61.9°$ **ii** $\approx 69.3°$

EXERCISE 12E

1

θ	$\cos\theta$	$\sin\theta$	$\cos(180° - \theta)$	$\sin(180° - \theta)$
18°	0.9511	0.3090	−0.9511	0.3090
27°	0.8910	0.4540	−0.8910	0.4540
53°	0.6018	0.7986	−0.6018	0.7986
62°	0.4695	0.8829	−0.4695	0.8829
80°	0.1736	0.9848	−0.1736	0.9848
125°	−0.5736	0.8192	0.5736	0.8192

2 P(0.276, 0.961), Q(−0.906, 0.423)

3 **a** 154° **b** 135° **c** 111° **d** 94°

4 **a** 82° **b** 53° **c** 24° **d** 12°

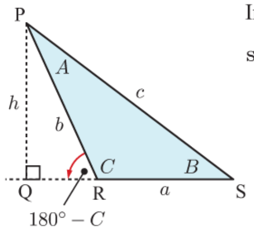
5 **a** 154° **b** 135° **c** 111° **d** 94°

6 **a** 82° **b** 53° **c** 24° **d** 12°

EXERCISE 12F

1 **a** $\approx 55.2\ \text{cm}^2$ **b** $\approx 347\ \text{km}^2$ **c** $\approx 1.15\ \text{m}^2$

2



In $\triangle$PQR,

$$\sin(180° - C) = \frac{h}{b}$$

$$\therefore\ h = b\sin(180° - C)$$

$$\therefore\ h = b\sin C$$

But, $\text{area}\ \triangle\text{PRS} = \frac{1}{2}ah$

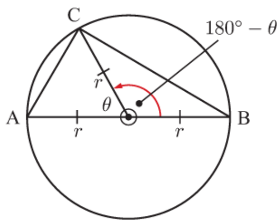
$$= \tfrac{1}{2}ab\sin C$$

3 **a** $\approx 13.6\ \text{cm}^2$ **b** $\approx 58.6\ \text{m}^2$ **c** $\approx 5.81\ \text{m}^2$

4 **a** $\approx 41.6\ \text{cm}^2$ **b** $\approx 36.7\ \text{m}^2$ **c** $\approx 7.70\ \text{cm}^2$

5 $50.0\ \text{cm}^2$ **6** $x \approx 21.9$ **7** ≈ 13.1 cm

8



Area $\triangle$OBC
$= \frac{1}{2}r^2\sin(180° - \theta)$
$= \frac{1}{2}r^2\sin\theta$
$=$ area $\triangle$OAC

9 **a** **i** $\text{Area} = \frac{1}{2}bc\sin A$ **ii** $\text{Area} = \frac{1}{2}ab\sin C$
b From **a**, $\frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C$
Divide both sides by $\frac{1}{2}abc$, etc.

EXERCISE 12G.1

1 **a** $x \approx 11.05$ **b** $x \approx 11.52$ **c** $x \approx 5.19$

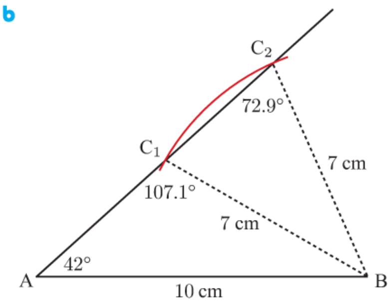
2 **a** $x \approx 9.43$ **b** $x \approx 11.9$ **c** $x \approx 6.37$

3 $\approx 10.2\ \text{cm}^2$

4 **a** $\theta = 59°$, $x \approx 96.7$, $y \approx 90.1$
b $\phi = 62°$, $x \approx 4.17$, $y \approx 5.62$
c $\theta = \phi = 28.5°$, $x \approx 5.30$

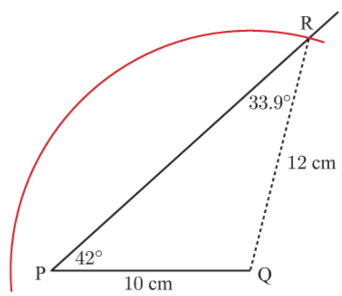
EXERCISE 12G.2

1 **a** $\mathrm{A\widehat{C}B} \approx 72.9°$ or $\approx 107°$ **b**



2 **a** Using the sine rule, $\mathrm{P\widehat{R}Q} \approx 33.9°$ or $146.1°$.
In the case of $146.1°$, $42° + 146.1°$ is already $> 180°$.
$\therefore$ $\mathrm{P\widehat{R}Q} \approx 33.9°$ only.

b



3 **a** $\theta \approx 31.4°$ **b** $\theta \approx 77.5°$ or $102°$
c $\theta \approx 43.6°$ or $136°$ **d** $\theta \approx 40.8°$

4 **a** $\widehat{\mathrm{A}} \approx 49.1°$ **b** $\widehat{\mathrm{B}} \approx 71.6°$ or $108°$ **c** $\widehat{\mathrm{C}} \approx 44.8°$

EXERCISE 12H

1 **a** ≈ 2.66 cm **b** ≈ 9.63 m **c** 10.6 m
d 27.5 cm **e** 4.15 km **f** 15.2 m

2 **a** $\theta \approx 36.3°$ **b** $\theta \approx 53.2°$ **c** $\theta \approx 115.6°$

3 **a** $\widehat{\mathrm{A}} \approx 51.8°$, $\widehat{\mathrm{B}} \approx 40.0°$, $\widehat{\mathrm{C}} \approx 88.3°$
b $\widehat{\mathrm{P}} \approx 34.0°$, $\widehat{\mathrm{Q}} \approx 96.6°$, $\widehat{\mathrm{R}} \approx 49.3°$

4 **a** $\cos\theta = \dfrac{m^2+c^2-a^2}{2cm}$

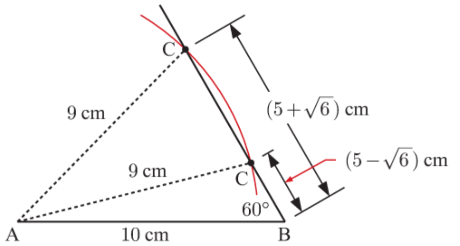
b $\cos(180° - \theta) = \dfrac{m^2+c^2-b^2}{2cm}$

c **Hint:** $\cos(180° - \theta) = -\cos\theta$

d **i** $x \approx 9.35$ **ii** $x \approx 4.24$

5 **b** $x = 5 \pm \sqrt{6}$

c



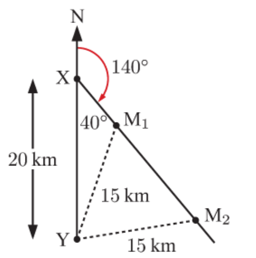
EXERCISE 12I

1 AC $\approx$ 14.3 km **2** AC $\approx$ 1300 m **3** $\theta \approx 13.4°$

4 **a** $\widehat{A} \approx 35.69°$ **b** $\approx$ 4 ha **5** $\approx$ 100 m

6 19.6 km in direction 106° **7** $\approx 214°$

8 **a**



b $XM_1 \approx 7.59$ km, $XM_2 \approx 23.0$ km

9 $\approx$ 63.4 m

10 AB $\approx$ 6.43 m, BC $\approx$ 9.85 m, AC $\approx$ 8.66 m

11 $\approx$ 3.58 m, $\approx$ 1.95 m, $\approx$ 4.47 m

REVIEW SET 12A

1 **a** $x \approx 14.0$ **b** $x \approx 35.2$

2 $\theta = 36°$, $x \approx 12.4$, $y \approx 21.0$ **3** $\approx$ 55.2 m

4 **a** 157° **b** 281° **5** $\approx 201°$

6 **a** $\approx 56.3°$ **b** $\approx 33.9°$

7 P(0.545, 0.839), Q(−0.978, 0.208)

8 **a** 29° **b** $\approx$ 117 m **c** $\approx$ 160 m

9 **a** $\approx 77\ \text{m}^2$ **b** $\approx$ 15.9 m

10 **a** $x \approx 223$ **b** $x \approx 99.4$

11 $\approx$ 337 m in the direction $\approx 138°$

12 **a** DC $\approx$ 10.2 m **b** BE $\approx$ 7.00 m **c** $\approx 82.0\ \text{m}^2$

13 XR $\approx$ 5.62 cm

REVIEW SET 12B

1 **a** $\theta \approx 38.7°$ **b** $\theta \approx 37.1°$

2 **a** $x \approx 3.18$ **b** $x \approx 9.40$

3 $\alpha \approx 36.4°$, $\theta \approx 53.6°$, $x \approx 25.7$ **4** $\approx 32.2°$

5 187 m **6** **a** 230° **b** 165° **c** 140°

7 **a** 45° **b** 60° **8** **a** 78.1 km **b** $\approx 051.2°$

9 **a** 142° **b** 128° **10** $\approx 69.3\ \text{m}^2$

11 AC $\approx$ 10.7 m **12** $\widehat{ACB} \approx 50.5°$ or 129.5°

13 **a** $\approx$ 185 m **b** $\approx 184°$

14 **a** $\cos B\widehat{Q}P = \dfrac{2^2+5^2-4^2}{2\times 2\times 5} = \frac{13}{20}$

b $\cos(B\widehat{Q}R) = \cos(180° - B\widehat{Q}P)$
$= -\cos B\widehat{Q}P$
$= -\frac{13}{20}$

c $BR^2 = 2^2+5^2-2\times 2\times 5(-\frac{13}{20})$
$= 2^2+5^2+13$
etc.

EXERCISE 13A

1 $\frac{168}{200} = 0.84$ **2** $\frac{11}{123} \approx 0.0894$ **3** $\frac{169}{227} \approx 0.744$

4 $\frac{172}{417} \approx 0.412$

5 **a** $\frac{137}{200} = 0.685$ **b** $\frac{145}{200} = 0.725$

c We could combine Sam and Karla's results and estimate the chance of an end occurring using the combined data.

d Yes

EXERCISE 13B

1 **a** 407 people **b** **i** $\frac{93}{407} \approx 0.229$ **ii** $\frac{207}{407} \approx 0.509$

2 **a** 57 ice creams **b** **i** $\frac{17}{57} \approx 0.298$ **ii** $\frac{36}{57} \approx 0.632$

3 **a**

Councillor	*Frequency*
Mr Tony Trimboli	216
Mrs Andrea Sims	72
Mrs Sara Chong	238
Mr John Henry	74
Total	600

b **i** $\frac{74}{600} \approx 0.123$ **ii** $\frac{310}{600} \approx 0.517$

4 **a**

	Like	Dislike	*Total*
Junior students	87	38	125
Senior students	129	56	185
Total	216	94	310

b **i** $\frac{87}{310} \approx 0.281$ **ii** $\frac{129}{310} \approx 0.416$ **iii** $\frac{94}{310} \approx 0.303$

c The total is 1. This is because the three probabilities in **b** cover all possible outcomes that could occur.

5 **a** 100 students

b **i** $\frac{29}{100} = 0.29$ **ii** $\frac{8}{100} = 0.08$ **iii** $\frac{26}{100} = 0.26$

iv $\frac{68}{100} = 0.68$

6 **a** $\frac{69}{147} \approx 0.469$ **b** $\frac{26}{147} \approx 0.177$ **c** $\frac{43}{147} \approx 0.293$

7 **a** $\frac{3}{82} \approx 0.0366$ **b** $\frac{52}{82} \approx 0.634$ **c** $\frac{32}{82} \approx 0.390$

d $\frac{19}{82} \approx 0.232$

8 **a** $\frac{4822}{5038} \approx 0.957$ **b** $\frac{448}{5038} \approx 0.0889$

c $\frac{31}{5038} \approx 0.006\,15$ **d** $\frac{1864}{5038} \approx 0.370$

EXERCISE 13C

1 **a** {A, B, C, D} **b** {BB, BG, GB, GG}

c {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}

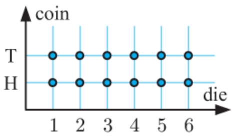
d {GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB}

e **i** {HH, HT, TH, TT}

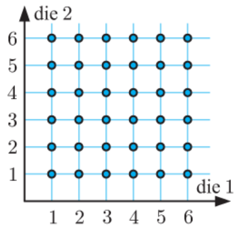
ii {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

iii {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

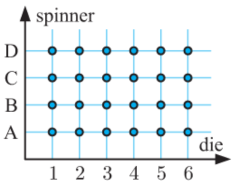
2 **a**



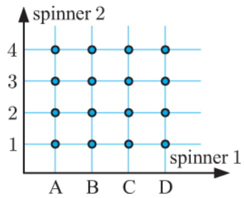
b



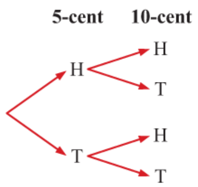
c



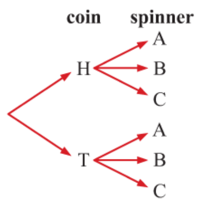
d



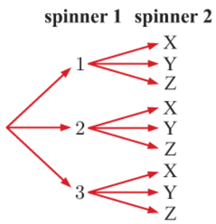
3 **a**



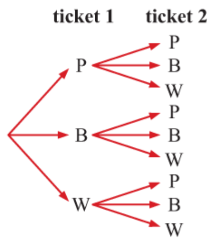
b



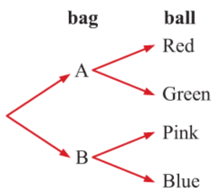
c



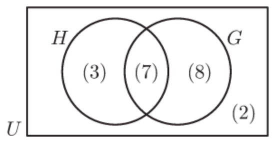
d



e



4



EXERCISE 13D

1 {ODG, OGD, DOG, DGO, GOD, GDO}

a $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{1}{3}$

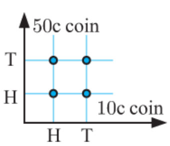
2 {BBB, GBB, BGB, BBG, BGG, GBG, GGB, GGG}

a $\frac{1}{8}$ **b** $\frac{1}{8}$ **c** $\frac{1}{8}$ **d** $\frac{3}{8}$ **e** $\frac{1}{2}$ **f** $\frac{7}{8}$

3 {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}

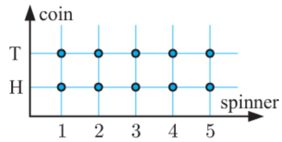
a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$

4 **a**



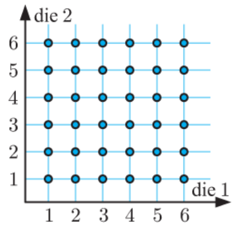
b **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$

5 **a**



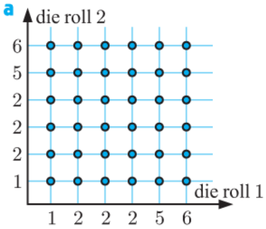
b **i** $\frac{1}{10}$ **ii** $\frac{3}{10}$ **iii** $\frac{2}{5}$ **iv** $\frac{3}{5}$

6 **a**



b **i** $\frac{1}{18}$ **ii** $\frac{1}{6}$ **iii** $\frac{11}{36}$ **iv** $\frac{5}{9}$ **v** $\frac{1}{4}$ **vi** $\frac{1}{6}$

7 **a**



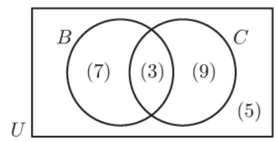
b **i** $\frac{1}{6}$ **ii** $\frac{2}{9}$ **iii** $\frac{1}{3}$ **iv** $\frac{11}{36}$ **v** $\frac{8}{9}$ **vi** $\frac{5}{9}$

8 **a** **i** $\frac{1}{15}$ **ii** $\frac{2}{15}$ **iii** $\frac{7}{15}$ **iv** $\frac{5}{15} = \frac{1}{3}$

b $P(\text{odd}) = \frac{6}{15}$, $P(\text{even}) = \frac{9}{15}$ $\therefore$ even

9 **a** 33 students **b** **i** $\frac{17}{33}$ **ii** $\frac{5}{33}$ **iii** $\frac{8}{11}$ **iv** $\frac{19}{33}$

10 **a**



b **i** $\frac{3}{8}$ **ii** $\frac{1}{8}$

11 **a** $\frac{11}{50}$ **b** $\frac{29}{50}$

EXERCISE 13E.1

1 **a** $\frac{1}{48}$ **b** $\frac{5}{16}$ **2** **a** $\frac{2}{9}$ **b** $\frac{4}{15}$

3 **a** $\frac{6}{625} = 0.0096 = 0.96\%$ **b** $\frac{506}{625} = 0.8096 \approx 81.0\%$

4 **a** $\frac{1}{49}$ **b** $\frac{10}{49}$ **c** $\frac{25}{49}$

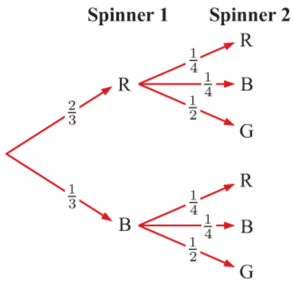
5 **a** ≈ 0.0545 **b** ≈ 0.0584 **c** ≈ 0.441 **d** ≈ 0.0840

6 **a** **i** ≈ 0.405 **ii** ≈ 0.595

b **i** ≈ 0.164 **ii** ≈ 0.354

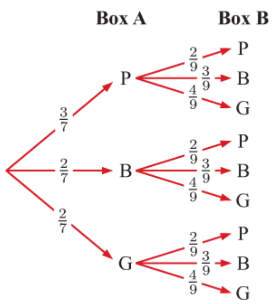
7 **a** ≈ 0.366 **b** ≈ 0.0231

8 **a**



b **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$ **iii** $\frac{5}{12}$

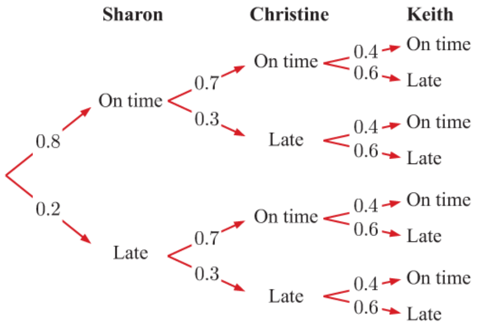
9 **a**



b **i** $\frac{2}{9}$ **ii** $\frac{20}{63}$ **iii** $\frac{29}{63}$

c 1; this is because every possible outcome is covered by these three events.

10 **a**

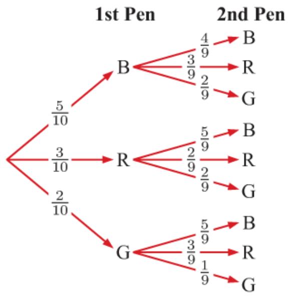


b 0.712

EXERCISE 13E.2

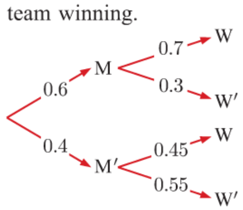
1 **a** $\frac{1}{45}$ **b** $\frac{1}{6}$ **2** **a** $\frac{25}{102}$ **b** $\frac{13}{204}$ **c** $\frac{1}{221}$

3 **a**



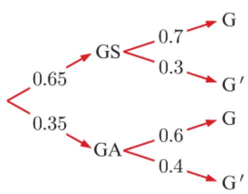
b **i** $\frac{14}{45}$ **ii** $\frac{16}{45}$

4 **a** M is the event of Matt playing in a game.
W is the event of his team winning.



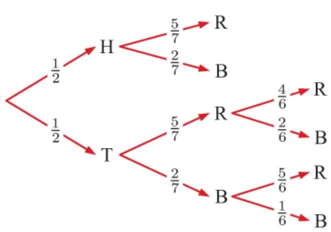
b 0.6

5



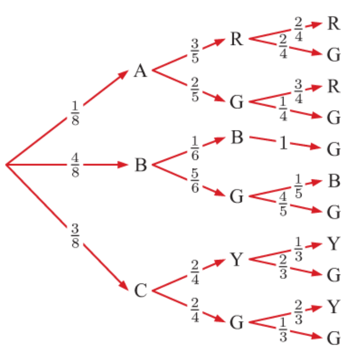
P(G) = 0.665

6



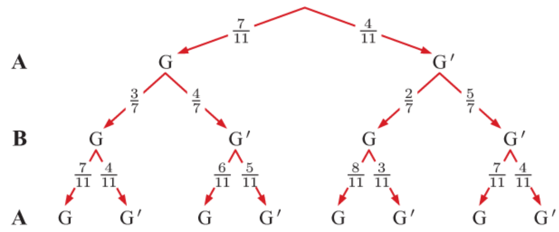
P(at least one red) $= \frac{5}{6}$

7



P(exactly one G) $= \frac{59}{120}$

8



P(gold coin in A) $= \frac{519}{847} \approx 0.613$

9 0.448 **10** **a** $\frac{13}{75}$ **b** $\frac{26}{75}$ **c** $\frac{12}{25}$

EXERCISE 13F

1 **a** $\frac{12}{25}$ **b** $\frac{12}{19}$ **2** **a** $\frac{24}{25}$ **b** $\frac{1}{25}$

3 **a** $\frac{13}{14}$ **b** $\frac{5}{23}$ **c** $\frac{9}{10}$

4 **a** $\frac{1}{20}$ **b** $\frac{12}{19}$ **c** $\frac{17}{78}$ **d** $\frac{8}{31}$

5 **a** $\frac{17}{37}$ **b** $\frac{11}{19}$ **c** $\frac{11}{17}$

6 **a** $\frac{11}{37}$ **b** $\frac{14}{47}$ **c** $\frac{14}{29}$ **7** **a** $\frac{3}{16}$ **b** $\frac{2}{5}$ **8** $\frac{3}{5}$

9 **a** $\frac{1}{4}$

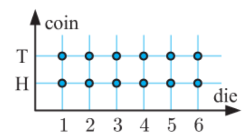
b **i** It increases the probability. In order for the sum to be 5, either the 3 or the 4 must be picked from bag A.

ii $\frac{1}{2}$

EXERCISE 13G

1 A and B, A and D, A and E, A and F, B and D, B and F, C and D.

2 **a**



b **i** No. It is possible to 'toss a head' *and* 'roll a 5'.

ii $P(A \text{ or } B) = \frac{7}{12}$, $P(A \text{ and } B) = \frac{1}{12}$

iii $P(A) + P(B) - P(A \text{ and } B) = \frac{1}{2} + \frac{1}{6} - \frac{1}{12}$

$= \frac{7}{12} = P(A \text{ or } B)$ ✓

3 **a** No, since $P(A \text{ and } B) \neq 0$ **b** 0.75

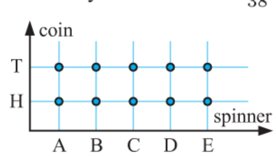
4 $P(Y) = 0.4$ **5** $P(C) + P(D) > 1$ **6** **a** 0 **b** 0.1

7 0.65 **8** $P(A) = 0.8$, $P(B) = 0.5$

REVIEW SET 13A

1 **a** 38 days **b** **i** $\frac{1}{38}$ **ii** $\frac{11}{38}$ **iii** $\frac{19}{38} = \frac{1}{2}$

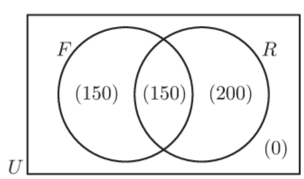
2



3 **a** $\frac{49}{81} \approx 0.60$ **b** $\frac{32}{81} \approx 0.40$

4 **a** $\frac{1}{20}$ **b** $\frac{3}{10}$ **c** $\frac{9}{20}$

5 **a**



b **i** $\frac{3}{10}$ **ii** $\frac{1}{2}$

6 **a** $\frac{2}{7}$ **b** $\frac{4}{7}$ **7** **a** $\frac{1}{9}$ **b** $\frac{2}{9}$

8

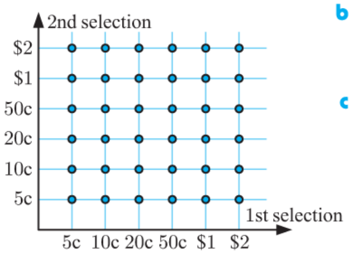
Injuries	0	1	2	3
Probability	0.336	0.452 ✓	0.188	0.024

$\therefore$ one injury is most likely.

9 0.66 **10** **a** 47 books **b** **i** $\frac{31}{47}$ **ii** $\frac{4}{47}$ **iii** $\frac{7}{16}$

11 $\frac{41}{90}$

12 **a**



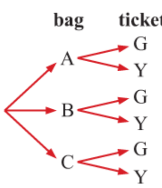
b **i** $\frac{4}{9}$ **ii** $\frac{11}{36}$ **iii** $\frac{23}{36}$ **c** $\frac{20}{23}$

13 **a** 0 **b** 0.14

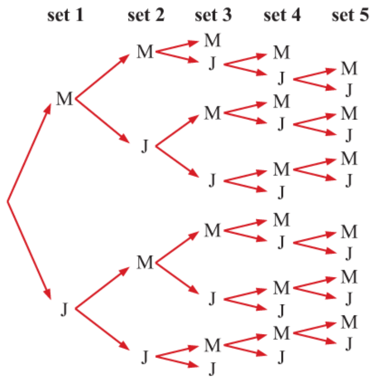
REVIEW SET 13B

1 **a** ≈ 0.364 **b** ≈ 0.551 **c** ≈ 0.814

2 **a**



b

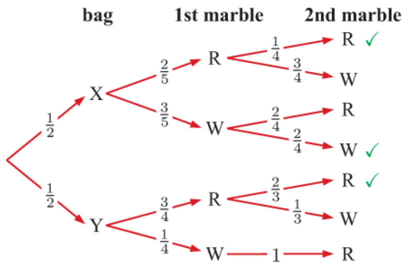


3 **a** $\frac{318}{450} \approx 0.707$ **b** $\frac{132}{450} \approx 0.293$ **c** $\frac{21}{450} \approx 0.0467$

4 $\frac{1}{2}$ **5** **a** $\frac{1}{9}$ **b** $\frac{4}{9}$ **c** $\frac{4}{9}$

6 **a** $\frac{37}{50} = 0.74$ **b** ≈ 0.0176

7 **a**



b $\frac{9}{20}$

8 $\frac{7}{12}$ **9** 0.697 **10** $\frac{2}{7}$

11 $\mathrm{P}(A) = \frac{3}{20}$, $\mathrm{P}(B) = \frac{7}{20}$, $\mathrm{P}(C) = \frac{2}{5}$

12 **a** $\frac{13}{55}$

b **i** Increases, as the number of tickets of the desired colour does not decrease with each draw.

ii $\frac{37}{121}$

13 **a** **i** 0.21 **ii** 0.28 **iii** 0.3

b P(Ihor wins) $= 0.238$ which is > 0.21

$\therefore$ the strategy increases his chance of winning.

EXERCISE 14A

1 **a** $A = 2000 + 150 \times 8$ **b** $A = 2000 + 150w$
c $A = 2000 + dw$ **d** $A = P + dw$

2 **a** $C = 40 + 60 \times 5$ **b** $C = 40 + 60t$
c $C = 40 + xt$ **d** $C = F + xt$

3 **a** $P = 3 \times 10 - 1(15 - 10)$
$\therefore P = 3 \times 10 - 5$

b $P = 3 \times c - 1(20 - c)$
$\therefore P = 4c - 20$

c $P = 3 \times c - 1(a - c)$
$\therefore P = 4c - a$

4 **a** $D = 4 \times 6 + 2(4 - 1)$ **b** $D = 5m + 3(5 - 1)$
c $D = 8m + b(8 - 1)$ **d** $D = mp + b(p - 1)$

5 **a** $G = 2 \times (3 - 1) + 3 \times (2 - 1)$
b $G = 3 \times (5 - 1) + 5 \times (3 - 1)$
c $G = 4 \times (4 - 1) + 4 \times (4 - 1)$
d $G = m(n - 1) + n(m - 1)$
$\therefore G = 2mn - m - n$

6 **a** $A = 2ab + \dfrac{\pi a^2}{2}$ **b** $A = ar + \dfrac{\pi r^2}{2}$

c $A = aw + \dfrac{\pi w^2}{4}$ **d** $A = ab - \dfrac{\pi a^2}{8}$

e $A = 2ar - \pi r^2$ **f** $A = \dfrac{\pi(b^2 - a^2)}{8} + \dfrac{a\sqrt{b^2 - a^2}}{2}$

7 **a** $V = Al$ **b** $V = \dfrac{\pi d^2 h}{4}$ **c** $V = \dfrac{abc}{2}$

8 **a** $A = 2ab + 2bc + 2ac$ **b** $A = 6a^2 + 8ab$
c $A = \pi r^2 + 2\pi rh + \pi rs$

9 **Hint:** Area of end section $= \pi R^2 - \pi r^2$
$= \pi(R + r)(R - r)$

EXERCISE 14B

1 **a** ≈ 26.4 cm **b** ≈ 17.8 cm **c** ≈ 127 m

2 **a** 19.6 m **b** ≈ 4.52 s

3 **a** ≈ 129 cm^2 **b** ≈ 7.14 m

4 **a** ≈ 4260 cm^3 **b** ≈ 1.06 cm **c** ≈ 4.99 mm

5 **a** ≈ 707 cm^2 **b** ≈ 39.9 cm

6 **a** ≈ 1.34 s **b** 81 cm

EXERCISE 14C

1 **a** $y = \dfrac{10 - 2x}{5}$ **b** $y = \dfrac{20 - 3x}{4}$ **c** $y = 2x - 8$

d $y = \dfrac{14 - 2x}{7}$ **e** $y = \dfrac{20 - 5x}{2}$ **f** $y = \dfrac{2x + 12}{3}$

2 **a** $x = r - p$ **b** $x = \dfrac{z}{y}$ **c** $x = \dfrac{d - a}{3}$

d $x = \dfrac{d - 2y}{5}$ **e** $x = \dfrac{p - by}{a}$ **f** $x = \dfrac{y - c}{m}$

g $x = \dfrac{s - 2}{t}$ **h** $x = \dfrac{m - p}{q}$ **i** $x = \dfrac{6 - a}{b}$

3 **a** $y = \dfrac{t - z}{5}$ **b** $y = \dfrac{c - p}{2}$ **c** $y = \dfrac{a - t}{3}$

d $y = \dfrac{n - 5}{k}$ **e** $y = \dfrac{a - n}{b}$ **f** $y = \dfrac{a - p}{n}$

4 **a** $z = \dfrac{b}{ac}$ **b** $z = \dfrac{a}{d}$ **c** $z = \dfrac{2d}{3}$

d $z = \pm\sqrt{2a}$ **e** $z = \pm\sqrt{bn}$ **f** $z = \pm\sqrt{m(a - b)}$

5 **a** $a=\dfrac{F}{m}$ **b** $r=\dfrac{C}{2\pi}$ **c** $d=\dfrac{V}{lh}$
d $K=\dfrac{b}{A}$ **e** $h=\dfrac{2A}{b}$ **f** $T=\dfrac{100I}{PR}$

6 $h=\dfrac{A-2\pi r^2}{2\pi r}$ or $h=\dfrac{A}{2\pi r}-r$

7 **a** $r=\sqrt{\dfrac{A}{\pi}}$ **b** $x=\sqrt[5]{aN}$ **c** $k=\sqrt[3]{\dfrac{M}{5}}$
d $x=\sqrt[3]{\dfrac{n}{D}}$ **e** $x=\pm\sqrt{\dfrac{y+7}{4}}$ **f** $Q=\pm\sqrt{P^2-R^2}$

8 **a** $a=d^2n^2$ **b** $l=25T^2$ **c** $a=\pm\sqrt{b^2+c^2}$
d $d=\dfrac{25a^2}{k^2}$ **e** $l=\dfrac{gT^2}{4\pi^2}$ **f** $b=\dfrac{16a}{A^2}$

9 **a** $x=\dfrac{c-a}{3-b}$ **b** $x=\dfrac{c}{a+b}$ **c** $x=\dfrac{a+2}{n-m}$
d $x=-\dfrac{a}{b+8}$ **e** $x=\dfrac{a-b}{1-c}$ **f** $x=\dfrac{e-d}{r+s}$

10 **a** $a=\dfrac{2-bP}{P}$ **b** $r=\dfrac{8-qT}{T}$ **c** $q=\dfrac{Ap-B}{A}$
d $x=\dfrac{3-Ay}{2A}$ **e** $y=\sqrt{\dfrac{4-Mx^2}{M}}$

11 **a** $x=\dfrac{y}{1-y}$ **b** $x=\dfrac{2y+3}{1-y}$ **c** $x=\dfrac{3y+1}{3-y}$
d $x=\dfrac{2y+1}{y+4}$ **e** $x=\dfrac{3y-7}{2y+3}$ **f** $x=\dfrac{3y-1}{y-1}$
g $x=\dfrac{-4y-3}{y+2}$ **h** $x=\dfrac{2y}{y+3}$ **i** $x=\dfrac{y+12}{5}$

12 $p=\sqrt{1-\dfrac{1}{N}}$ $\{\text{as } p>0\}$

EXERCISE 14D

1 **a** $\theta=\dfrac{360A}{\pi r^2}$
b **i** $\theta\approx 63.7°$ **ii** $\theta\approx 105°$ **iii** $\theta\approx 214°$

2 **a** $a=\dfrac{d^2}{2bK}$ **b** **i** $a\approx 1.29$ **ii** $a=16.2$

3 **a** $t=(H-1)^2$
b **i** 1 year **ii** 4 years **iii** $6\frac{1}{4}$ years

4 **a** $r=\sqrt[3]{\dfrac{3V}{4\pi}}$
b **i** ≈ 2.12 cm **ii** ≈ 5.76 cm **iii** ≈ 62.0 cm

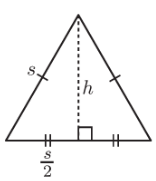
5 **a** $v=\sqrt{u^2+2as}$ **b** **i** ≈ 20.6 m/s **ii** ≈ 52.9 m/s

6 **a** $\approx 58.8\%$ **b** $w=\dfrac{Pl}{100-P}$ **c** 9 matches
d 7 consecutive matches

7 **a** $c=\dfrac{A-2ab}{2(a+b)}$ **b** **i** $c=3$ **ii** $c=7$ **iii** $c=7.5$

8 **a** $F\approx 2.01\times 10^{20}$ Newtons **b** $d=\sqrt{\dfrac{Gm_1m_2}{F}}$
c **i** $\approx 1.50\times 10^{11}$ m **ii** $\approx 1.43\times 10^{14}$ m

9 **a** **Hint:**



Use the diagram to show that the area of one triangle face

$=\dfrac{s}{2}\times\dfrac{\sqrt{3}s}{2}=\dfrac{\sqrt{3}s^2}{4}$

b $s^2=\dfrac{A}{1+\sqrt{3}}\left(\dfrac{1-\sqrt{3}}{1-\sqrt{3}}\right)$ etc.
c **i** ≈ 4.28 cm **ii** ≈ 7.41 cm **iii** ≈ 14.8 cm

10 **b** $p=\dfrac{3g}{g-1}$ **c** $0g$ $0p$, $2g$ $6p$, $4g$ $4p$

11 **a** **i** $C\approx 0.816$ **ii** $C\approx 0.742$ **b** $h=C^2H$
c TB: ≈ 2.65 m, GB: ≈ 3.68 m, M: ≈ 2.16 m, SB: ≈ 3.99 m

12 **a** $v=\dfrac{c}{m}\sqrt{m^2-m_0^2}$ **b** $v=\frac{2\sqrt{2}}{3}c$
c $v\approx 3.00\times 10^8$ m/s

EXERCISE 14E

1 **a** 6, 11, 16, 21, 26 **b** 51 **c** $5n+1$

2 **a** 7, 10, 13, 16, 19 **b** 34 **c** $3n+4$

3 **a** **i** 4 **ii** 9 **iii** 16 **iv** 25 **b** $S_n=n^2$

4 **a** **i** 3 **ii** 7 **iii** 15 **iv** 31
b As $3=2^2-1$, $7=2^3-1$, $15=2^4-1$, etc., $S_n=2^n-1$

5 **a** $S_1=\frac{1}{2}$, $S_2=\frac{2}{3}$, $S_3=\frac{3}{4}$, $S_4=\frac{4}{5}$, ...
b **i** $S_{10}=\frac{10}{11}$ **ii** $S_n=\dfrac{n}{n+1}$

6 **a** $S_1=\dfrac{1\times2\times3}{6}=1$ and $1^2=1$ ✓
$S_2=\dfrac{2\times3\times5}{6}=5$ and $1^2+2^2=5$ ✓
$S_3=\dfrac{3\times4\times7}{6}=14$ and $1^2+2^2+3^2=14$ ✓
$S_4=\dfrac{4\times5\times9}{6}=30$ and $1^2+2^2+3^2+4^2=30$ ✓
b $S_{100}=\dfrac{100\times101\times201}{6}=338\,350$

REVIEW SET 14A

1 **a** **i** $V=6\times8$ litres **ii** $V=8n$ litres **iii** $V=ln$ litres
b $V=25+ln$ litres

2 **a** 90 km/h **b** 3900 km **3** $A=\frac{\sqrt{3}}{2}a^2+3ab$

4 **a** $x=\dfrac{3p-n}{m}$ **b** $x=\dfrac{5y}{7}$

5 **a** $k=T^2+l^2$ **b** $k=-\sqrt{\dfrac{P+r}{2}}$

6 **a** 11 cm **b** $c=\sqrt{L^2-a^2-b^2}$ **c** 9 cm

7 $x=\dfrac{2y-3}{y-2}$

8 **a** 8 amperes **b** $r=\dfrac{E-IR}{I}$ **c** 0.15 ohms

9 **a** 4, 7, 10 **b** $M=3n+1$

10 **a** **i** 3 **ii** 6 **iii** 10 **b** **i** 9 **ii** 36 **iii** 100
c It seems that $C_n=S_n^2$.

11 **a** **i** ≈ 283.2 K **ii** ≈ 183.2 K **iii** ≈ 338.7 K
b $F=\frac{9}{5}(K-273.15)+32$
c **i** 104°F **ii** -459.67°F **iii** -99.67°F

REVIEW SET 14B

1 **a** $B=15+25\times5$ **b** $B=c+25\times p$ **c** $B=c+mp$

2 **a** $E=2\times(3-2)+2\times(5-2)=8$
b $E=2\times(4-2)+2\times(8-2)=16$
c $E=2(m-2)+2(n-2)$

3 **a** $M = 37$ **b** $r = 8$ **4** $V = 4x^2y$

5 **a** $a = \dfrac{B+f}{d}$ **b** $a = \dfrac{9Q^2}{t^2}$ **6** $h = \dfrac{5-G^2}{G^2}$

7 **a** $b = \dfrac{a}{a-1}$ **b** $b = \frac{3}{2}$; $3 \times \frac{3}{2} = 3 + \frac{3}{2} = 4\frac{1}{2}$ ✓

8 $x = \dfrac{2y+3}{4-3y}$ **9** 8, 13, 18, ... $\therefore$ $M = 5n + 3$

10 **a** **i** 6 **ii** 12 **iii** 20 **iv** 30
b $S_1 = 2 = 1 \times 2$
$S_2 = 6 = 2 \times 3$
$S_3 = 12 = 3 \times 4$
$S_4 = 20 = 4 \times 5$
$S_5 = 30 = 5 \times 6$ $\therefore$ $S_n = n(n+1)$

11 **a** $E = 1000$ joules **b** $v = \sqrt{\dfrac{2E}{m}}$ **c** 8 m/s

EXERCISE 15A

1 **a** Domain is $\{x \mid x > -4\}$. Range is $\{y \mid y > -2\}$.
b Domain is $\{x \mid -3 \leqslant x \leqslant 4\}$. Range is $\{y \mid -5 \leqslant y \leqslant 2\}$.
c Domain is $\{x \mid -3 < x < 4\}$. Range is $\{y \mid -5 < y < 6\}$.
d Domain is $\{x \mid x = 2\}$. Range is $\{y \mid y \in \mathbb{R}\}$.
e Domain is $\{x \mid -3 \leqslant x \leqslant 3\}$. Range is $\{y \mid -3 \leqslant y \leqslant 3\}$.
f Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \leqslant 0\}$.
g Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y = -5\}$.
h Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \geqslant 1\}$.
i Domain is $\{x \mid x \geqslant -5\}$. Range is $\{y \mid y \leqslant 7\}$.
j Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \leqslant 4\}$.
k Domain is $\{x \mid x \geqslant -5\}$. Range is $\{y \mid y \in \mathbb{R}\}$.
l Domain is $\{x \mid x \in \mathbb{R},\ x \neq 1\}$.
Range is $\{y \mid y \in \mathbb{R},\ y \neq 0\}$.

2 **a** Domain is $\{x \mid 0 \leqslant x \leqslant 2\}$. Range is $\{y \mid -3 \leqslant y \leqslant 2\}$.
b Domain is $\{x \mid -2 < x < 2\}$. Range is $\{y \mid -1 < y < 3\}$.
c Domain is $\{x \mid -4 \leqslant x \leqslant 4\}$. Range is $\{y \mid -2 \leqslant y \leqslant 2\}$.

EXERCISE 15B

1 **a**, **b**, and **e** are functions as no two ordered pairs have the same x-coordinate.

2 **a**, **b**, **d**, **e**, **g**, **h**, and **i** are functions.

3 No, a vertical line is not a function as it does not satisfy the vertical line test.

EXERCISE 15C.1

1 **a** 3 **b** 7 **c** 1 **d** -7 **e** 2

2 **a** -2 **b** -17 **c** 13 **d** $5x+3$ **e** $-5x-17$

3 **a** 2 **b** 11 **c** 46 **d** $2x^2+3x+2$
e $2x^2+x+1$

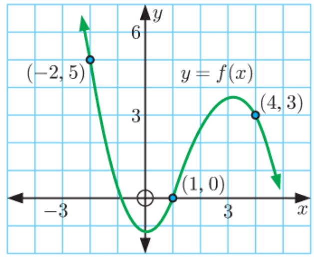
4 **a** 45 **b** -3 **c** 0 **d** $4x^2-20x+21$
e $16x^2+8x-3$

5 **a** **i** $-\frac{3}{2}$ **ii** $-\frac{1}{3}$ **iii** $-\frac{8}{3}$ **b** $x = -2$
c $2 - \dfrac{7}{x}$ **d** $x = -1$

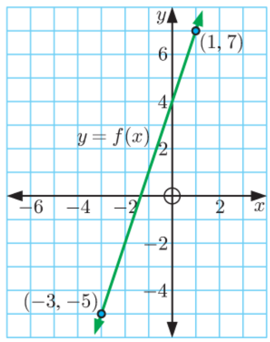
6 **a** $V(4) = 12\,000$. The value of the car after 4 years is \$12 000.
b $V(t) = 8000$ when $t = 5$. 5 years after purchase the value of the car is \$8000.
c \$28 000

7 **a** **i** $f(2) = 1$ **ii** $f(3) = -1$ **b** $x = -4$

8 **a** **i** $f(4) = 2$ **ii** $g(0) = -6$ **iii** $g(5) = -1$
b $x = 0$ and $x = 3$ **c** $x = 2$
d g has gradient 1 and y-intercept -6.

9


Other graphs are possible.

10 **a**


b $f(-3) = -5$, $f(1) = 7$
c $f(x) = 3x + 4$

EXERCISE 15C.2

1 **a** $\{x \mid x \in \mathbb{R}\}$ **b** $\{x \mid x \neq 0\}$
c $\{x \mid x \neq 3\}$ **d** $\{x \mid x \neq -2 \text{ and } x \neq 1\}$
e $\{x \mid x \neq 3 \text{ and } x \neq -3\}$ **f** $\{x \mid x \neq 1 \text{ and } x \neq 4\}$

2 **a** $\{x \mid x \geqslant 2\}$ **b** $\{x \mid x \leqslant 3\}$
c $\{x \mid 0 \leqslant x \leqslant 2\}$ **d** $\{x \mid x > 0\}$
e $\{x \mid x > 0\}$ **f** $\{x \mid x < 4 \text{ and } x \neq 0\}$

EXERCISE 15D

1 **a** $2-3x$ **b** $6-3x$ **c** $9x-16$ **d** x

2 **a** $\sqrt{4x-3}$ **b** $4\sqrt{x}-3$ **c** 5 **d** 5

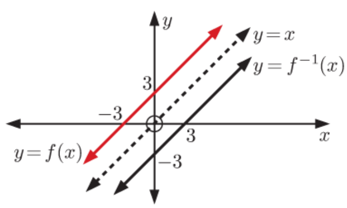
3 **a** $f(x) = \sqrt{x}$, $g(x) = x - 3$ **b** $f(x) = x^3$, $g(x) = x + 5$
c $f(x) = \dfrac{5}{x}$, $g(x) = x + 7$ **d** $f(x) = 3 - 4x$, $g(x) = \dfrac{1}{\sqrt{x}}$
e $f(x) = x^2$, $g(x) = 3^x$ **f** $f(x) = \dfrac{x+1}{x-1}$, $g(x) = x^2$
(**Note:** There may be other answers.)

4 **a** $f(g(x)) = 3x^2 + 6x + 1$ **b** $x = -3$ or 1

5 **a** $f(g(x)) = \dfrac{4}{x+2} + 1$ $\left(\text{or } \dfrac{x+6}{x+2}\right)$ **b** 5 **c** $x = 0$

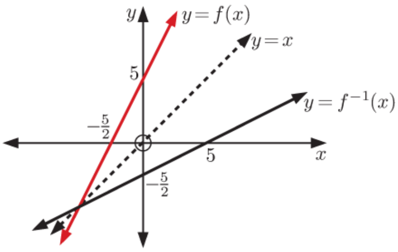
6 **a** **i** 13 **ii** 4 **iii** 6 **iv** 17
b $f(g(x)) = x$ and $g(f(x)) = x$ **c** **i** 3 **ii** -7

EXERCISE 15E

1 **a** **i** $f^{-1}(x) = x - 3$
ii


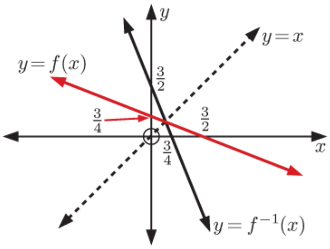
b **i** $f^{-1}(x) = \dfrac{x-5}{2}$

ii

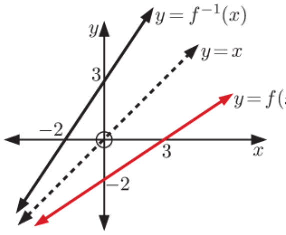


c **i** $f^{-1}(x) = -2x + \frac{3}{2}$

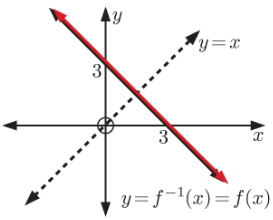
ii



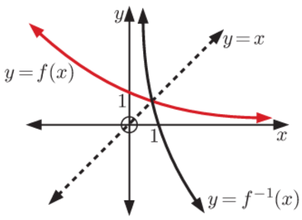
2 **a**



b



c



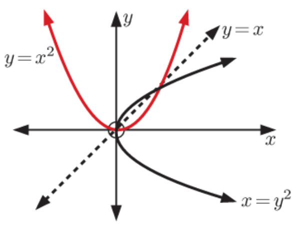
3 **a** $f^{-1}(x) = \dfrac{x-7}{2}$

4 **a** $f^{-1}(x) = \dfrac{1-3x}{x-2}$

5 **a** $g^{-1}(x) = \dfrac{5x}{x-1}$

c $g(6) = 6$, $g^{-1}(1)$ is undefined

6 **a**



b No, as the vertical line test fails.

c Yes, it is $y = \sqrt{x}$ (not $y = \pm\sqrt{x}$).

7 **b** **i** and **iii** have inverse functions.

8 **a** Is a function as it passes the vertical line test, but does not have an inverse as it fails the horizontal line test.

b It passes both the vertical line and horizontal line tests.

EXERCISE 15F

1 **a** 2 **b** 10 **c** 5 **d** 11 **e** 11 **f** 40 **g** 40 **h** -2

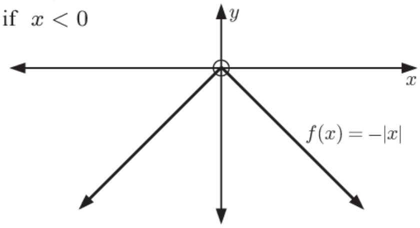
2 **a** 3 **b** 10 **c** 7 **d** 3 **e** 13 **f** 2 **g** $\frac{5}{2}$ **h** $\frac{4}{5}$

3 **a**

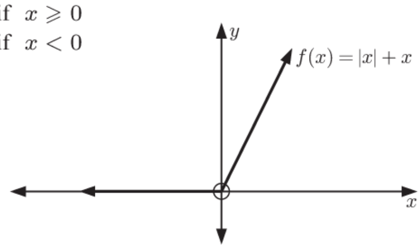
x	9	3	0	-3	-9
x^2	81	9	0	9	81
$\lvert x\rvert^2$	81	9	0	9	81

b $x^2 = |x|^2$

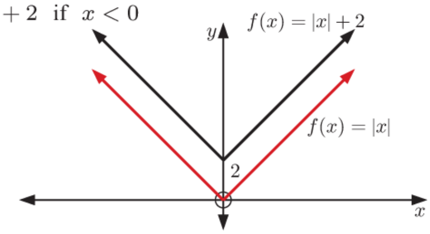
4 **a** $f(x) = \begin{cases} -x & \text{if } x \geqslant 0 \\ x & \text{if } x < 0 \end{cases}$



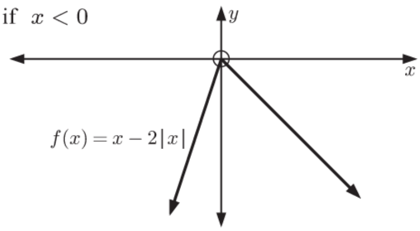
b $f(x) = \begin{cases} 2x & \text{if } x \geqslant 0 \\ 0 & \text{if } x < 0 \end{cases}$



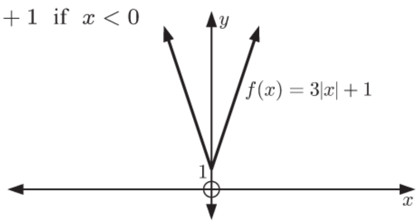
c $f(x) = \begin{cases} x+2 & \text{if } x \geqslant 0 \\ -x+2 & \text{if } x < 0 \end{cases}$



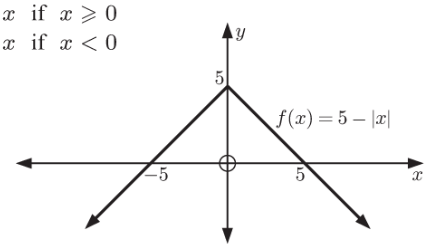
d $f(x) = \begin{cases} -x & \text{if } x \geqslant 0 \\ 3x & \text{if } x < 0 \end{cases}$



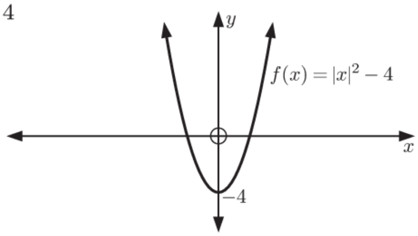
e $f(x) = \begin{cases} 3x+1 & \text{if } x \geqslant 0 \\ -3x+1 & \text{if } x < 0 \end{cases}$



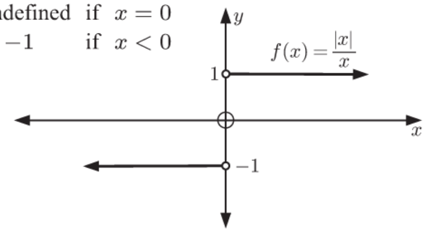
f $f(x) = \begin{cases} 5-x & \text{if } x \geqslant 0 \\ 5+x & \text{if } x < 0 \end{cases}$



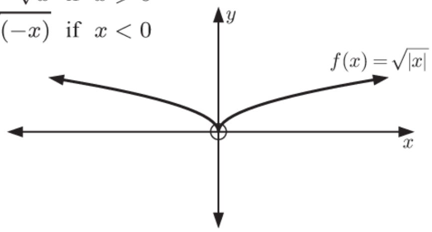
g $f(x) = x^2 - 4$



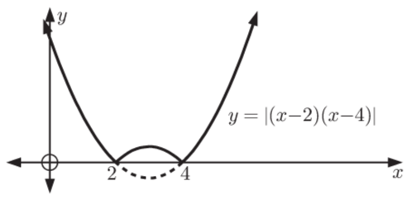
h $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ \text{undefined} & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$



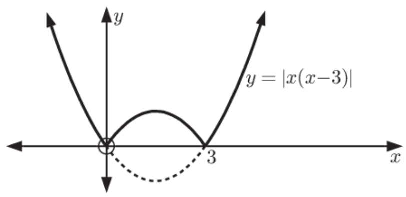
i $f(x) = \begin{cases} \sqrt{x} & \text{if } x \geqslant 0 \\ \sqrt{(-x)} & \text{if } x < 0 \end{cases}$



5 a i



ii



b The part of the graph $y = (x-2)(x-4)$ or $y = x(x-3)$ below the x-axis is reflected in the x-axis.

EXERCISE 15G

1 a $(2, 9)$ **b** $(\frac{5}{4}, \frac{43}{4})$ **c** $(5, \frac{1}{5})$ **d** $(1, 3)$

2 a at $(-3, 2)$ and $(2, 7)$ **b** at $(2, 1)$ and $(-1, -2)$
c at $(-\frac{1}{2}, -\frac{9}{4})$ and $(-3, 14)$ **d** at $(1, 1)$ and $(-\frac{1}{5}, -5)$

3 a at $(-1.62, -1.24)$ and $(0.62, 3.24)$ **b** at $(4.71, 0.64)$
c They do not intersect, $\therefore$ no solutions exist.
d at $(-0.75, -0.43)$

REVIEW SET 15A

1 a Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \geqslant -2\}$.
b Domain is $\{x \mid x \geqslant 0\}$. Range is $\{y \mid y \in \mathbb{R}\}$.

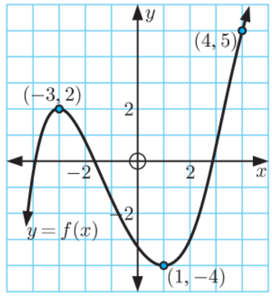
2 a 2 **b** -4 **c** $-x^2 + 9x - 18$

3 a function **b** not a function

4 a 8 **b** $9x^2 + 6x$ **c** $x = -5$ or 3

5 a function **b** not a function

6

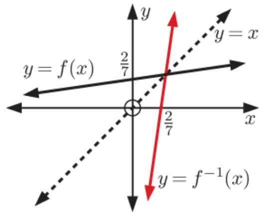


Note: There may be other answers.

7 a $0 \leqslant x \leqslant 15$ **b** $-5 \leqslant y \leqslant 15$
c When $x = 3$, $y = 10$, or $f(3) = 10$.
d When $y = 10$, $x = 3$, or $8 \leqslant x \leqslant 9$.

8 a $f(g(x)) = \sqrt{5x - 3}$ **b** $g(f(x)) = 5\sqrt{x} - 3$
c $g(g(x)) = 25x - 18$

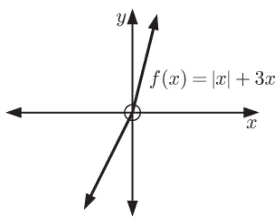
9 a $f^{-1}(x) = 7x - 2$ **c**

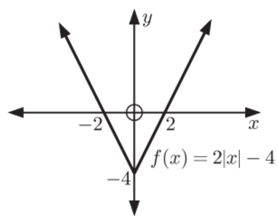


10 a $g^{-1}(x) = \dfrac{4}{x - 1}$ **c** $g(2) = 3$, $g^{-1}(3) = 2$

11 a 7 **b** -1 **c** 0

12 a $f(x) = \begin{cases} 4x & \text{if } x \geqslant 0 \\ 2x & \text{if } x < 0 \end{cases}$ **b** $f(x) = \begin{cases} 2x - 4 & \text{if } x \geqslant 0 \\ -2x - 4 & \text{if } x < 0 \end{cases}$





13 $(-3, -5)$ and $(1, 3)$

REVIEW SET 15B

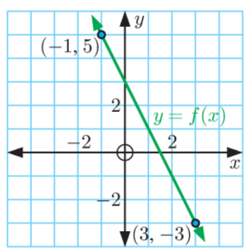
1 a Domain is $\{x \mid x > -6\}$. Range is $\{y \mid y > -2\}$.
b Domain is $\{x \mid x \leqslant 1\}$. Range is $\{y \mid y \in \mathbb{R}\}$.

2 a function **b** not a function

3 a -24 **b** $-5x - x^2$ **c** $-x^2 + 3x + 4$

4 a Domain is $\{x \mid x \neq -4\}$.
b Domain is $\{x \mid x \neq -5 \text{ and } x \neq 1\}$.
c Domain is $\{x \mid 0 \leqslant x \leqslant 6\}$.

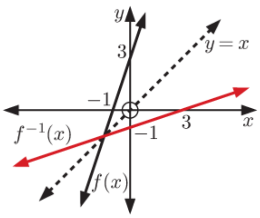
5 a



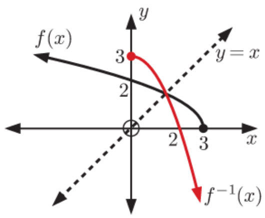
b $f(-1) = 5$, $f(3) = -3$
c $f(x) = -2x + 3$
d $f^{-1}(x) = \dfrac{3 - x}{2}$

6 **a** $f(g(x)) = 15 - 2x$ **b** $g(f(x)) = 6 - 2x$
c $f(g(-2)) = 19$

7 **a**



b

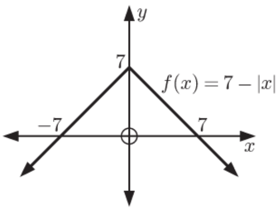


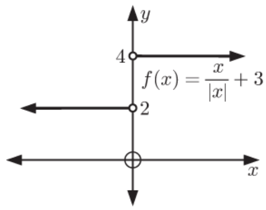
8 **a** 36 **b** 13 **c** $\frac{7}{4}$

9 **a** $f(g(x)) = \dfrac{7-x}{x-4}$ **b** 2 **c** $x = 10$

10 **a** $f^{-1}(x) = \dfrac{2x+1}{x-1}$ **c** 3

11 **a** $f(x) = \begin{cases} 7-x & \text{if } x \geqslant 0 \\ 7+x & \text{if } x < 0 \end{cases}$ **b** $f(x) = \begin{cases} 4 & \text{if } x > 0 \\ 2 & \text{if } x < 0 \\ \text{undef.} & \text{if } x = 0 \end{cases}$





12 $(-1, -3)$ and $(\frac{3}{2}, 2)$

EXERCISE 16A

1 **a** 4 **b** 16 **c** 13

2 **a** 3, 7, 11, 15, 19, ... **b** 40, 35, 30, 25, 20, ...
c 2, 3, 5, 7, 11, ...

3 **a** The sequence starts at 5 and increases by 3 each time.
b $u_1 = 5, \quad u_n = u_{n-1} + 3 \quad \text{for} \quad n \geqslant 2$

4 **a** The nth term of the sequence is n squared. **b** $u_n = n^2$

5 **a** 19 **b** 19 **c** 96 **d** 27

6 **a** $u_6 = 14$ **b** 136 **c** $u_8 = -14$

7 **a** 7, 10, 13, 16 **b** 25, 21, 17, 13 **c** 5, 15, 45, 135
d 100, 50, 25, 12.5 **e** 3, 5, 9, 17
f 4, 6, 4, 6 **g** 3, 4, 12, 48

EXERCISE 16B

1 **a** arithmetic **b** not arithmetic
c not arithmetic **d** arithmetic

2 **a** $\square = 16$ **b** $\square = 34$ **c** $\square = 15, \ \triangle = 3$
d $\square = 13, \ \triangle = -5$

3 **a** $u_1 = 41, \ d = 1$ **b** $u_1 = 1, \ d = 11$
c $u_1 = 98, \ d = -10$ **d** $u_1 = 91, \ d = -9$

4 **a** common difference is 7 **b** $u_n = 7n - 3$
c $u_{30} = 207$ **d** yes, the 49th term **e** no

5 **a** $d = -4$ **b** $u_n = 71 - 4n$ **c** $u_{60} = -169$
d no **e** no

6 **a** $u_1 = 4$ and $d = 11$ **b** $u_{37} = 400$ **c** $u_{24} = 257$

7 **a** $k = 22$ **b** $k = 2\frac{1}{2}$ **c** $k = \frac{13}{3}$

8 **a** $k = 6$ **b** 29, 37

9 **a** $u_n = 5n + 17$ **b** $u_n = 10 - 4n$
c $u_n = 3n - 16$ **d** $u_n = -\frac{17}{2} - \frac{3}{4}n$

10 **a** $u_n = 6n - 8$ **b** $u_{30} = 172$

EXERCISE 16C

1 **a** geometric **b** not geometric **c** geometric
d not geometric

2 **a** 5 **b** $\frac{1}{2}$ **c** -2 **d** $-\frac{1}{10}$

3 **a** $b = 12, \ c = 24$ **b** $b = \frac{1}{2}, \ c = \frac{1}{8}$ **c** $b = \frac{5}{3}, \ c = -\frac{5}{9}$

4 **a** $r = 3$ **b** $u_n = 3^{n-1}$ **c** $u_{10} = 19\,683$

5 **a** $r = -\frac{1}{2}$ **b** $u_n = 40 \times (-\frac{1}{2})^{n-1}$ **c** $u_{12} = -\frac{5}{256}$

6 **a** $r = -\frac{1}{4}$ **b** $-0.000\,976\,562\,5$

7 $u_n = 3 \times 2^{\frac{n-1}{2}}$ or $3\left(\sqrt{2}\right)^{n-1}$

8 **a** $k = \frac{2}{3}$ **b** $k = 9$ **c** $k = 2$ **d** $k = \pm 9$
e $k = -7$ **f** $k = -\frac{8}{7}$ or 2

9 **a** $u_n = \frac{16}{9} \times 3^{n-1}$ **b** $u_n = 128 \times (-\frac{1}{2})^{n-1}$
c $u_n = 2 \times 5^{n-1}$ or $u_n = (-2) \times (-5)^{n-1}$
d $u_n = 6 \times (\pm\frac{1}{\sqrt{2}})^{n-1}$

10 $\pm\frac{3}{32}$

EXERCISE 16D

1 **a** $u_3 = 8$ **b** $S_3 = 16$ **c** $u_5 = 19$ **d** $S_5 = 46$

2 **a** **i** 5, 7, 9, 11 **ii** $S_4 = 32$ **3** $u_6 = 12$
b **i** $-5, -2, 3, 10$ **ii** $S_4 = 6$
c **i** 7, 14, 28, 56 **ii** $S_4 = 105$
d **i** 3, 5, 11, 29 **ii** $S_4 = 48$

4 **a** $S_1 = \frac{1}{2}, \ S_2 = \frac{2}{3}, \ S_3 = \frac{3}{4}, \ S_4 = \frac{4}{5}$ **b** $\frac{100}{101}$

EXERCISE 16E

1 **a, b, c** 75 **2** $S_{12} = 450$ **3** $S_{20} = 220$

4 **a** 210 **b** 75 **c** 4075 **d** 6780
e -2280 **f** -2400 **g** 275 **h** 387.5

5 2500 **6** **a** 24 **b** 1476

7 **a** 775 **b** 1705 **c** 969 **d** 1040 **e** -345 **f** 306

8 **a** $S_n = \dfrac{n}{2}(2u_1 + (n-1)d)$ where $u_1 = 4, \ d = 4$
$= \dfrac{n}{2}(8 + 4(n-1))$ etc.
b 840

9 **a** €65 **b** €1350

10 **a** $\dfrac{n(n^2+1)}{2}$ **b** $\dfrac{3(3^2+1)}{2} = \dfrac{3 \times 10}{2} = 15$ ✓ **c** 260

EXERCISE 16F.1

1 **a, b** 315

2 **a** 3280 **b** 4 882 812 **c** $\frac{1533}{32}$ **d** $63 + 63\sqrt{2}$
e -1364 **f** $\frac{1640}{27}$ **g** ≈ 52.2 **h** ≈ 12.8

3 **a** $u_1 = 2, \ r = 3$ **b** 59 048

5 **a** **i** ≈ 158 mL **ii** ≈ 37.5 mL **iii** 8.91 mL
b The amount of water Doug drinks is a geometric sequence with $r = \frac{3}{4}$.
c **i** ≈ 1887.4 mL **ii** ≈ 1993.7 mL **iii** ≈ 1999.6 mL
d If Doug followed the formula, the amount would eventually become too small to measure, and too small to sustain him. (In theory Doug would never run out of water.)

EXERCISE 16F.2

1 **a** diverge **b** converge **c** diverge **d** converge

2 **a** **i** ≈ 23.44 **ii** ≈ 26.53 **iii** ≈ 26.99
b 27 **c** $S = \dfrac{9}{1 - \frac{2}{3}} = 27$

3 **a** 32 **b** $\frac{5}{4}$ **c** 27 **d** 128 **e** $\frac{432}{7}$ **f** $\frac{2}{3}$

4 **a** Start with $\frac{1}{3}$ unit^2 shaded blue. So $u_1 = \frac{1}{3}$.
Each step shades $\frac{1}{3}$ of the previous area, so $r = \frac{1}{3}$.
$\therefore \quad S = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots.$

b At each step, the same amount is coloured blue as is coloured red.

c If the process continues indefinitely, half the 1 unit^2 rectangle will be shaded blue and half will be shaded red. And from **a**, total blue area $= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots.$
$\therefore \quad \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots. = \frac{1}{2}$

d $S = \dfrac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$

5 **a** $(8-4) + (2-1) + (\frac{1}{2} - \frac{1}{4}) + \ldots.$
$= \quad 4 \quad + \quad 1 \quad + \quad \frac{1}{4} \quad + \ldots.$

b $\dfrac{8}{1 - -\frac{1}{2}} = \dfrac{16}{3} = \dfrac{4}{1 - \frac{1}{4}}$

6 $10 + 5\sqrt{2}$

REVIEW SET 16A

1 **a** 8, 13, 18, 23, 28 **b** 19, 12, 5, −2, −9
c 1, 8, 27, 64, 125

2 2, 8, 20, 44 **3** **a** $\square = 25$ **b** $\square = 21, \ \triangle = 9$

4 $k = 6$ **5** **a** $\frac{4}{5}$ **b** −3 **6** $u_n = -3 \times (-2)^{n-1}$

7 **a** 3, 8, 15, 24 **b** 50 **8** **a** 1180 **b** −1410

9 **a** 1 747 625 **b** ≈ 11.08 **10** **a** $\frac{125}{4}$ **b** 16

REVIEW SET 16B

1 **a** $u_8 = 29$ **b** $u_8 = 54$

2 **a** $d = 4$ **b** $u_n = 4n - 10$ **c** $u_{100} = 390$
d $u_{127} = 498$

3 **a** $r = -\frac{1}{2}$ **b** $u_{16} = -\frac{1}{512}$

4 **a** 5, 7, 5, 7, 5 **b** $u_{100} = 7$ **5** $S_{15} = 165$

6 $u_n = 39 - \frac{15}{2}(n-1)$

7 **a** 1070 **b** 270 **c** −119 **d** $\dfrac{58\,025}{32} \approx 1813.3$

8 **a** $k = -5$ or 10
b If $k = -5$, $r = -2$. If $k = 10$, $r = -\frac{1}{3}$.

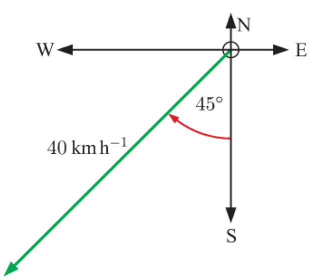
9 **a** $u_1 = 6$, $r = -\frac{1}{2}$ **b** $S_{10} = \frac{1023}{256}$

10 **a** **i** 42 km **ii** 58 km **iii** 78 km
b **i** 82 km **ii** 490 km **iii** 1180 km
c yes, using $u_n = 2n + 38$ and $S_n = n^2 + 39n$
d yes, $S_{30} = 2070$

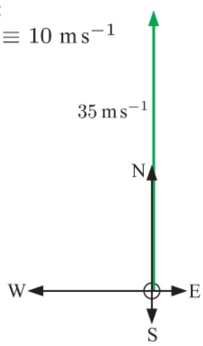
11 12 cm

EXERCISE 17A

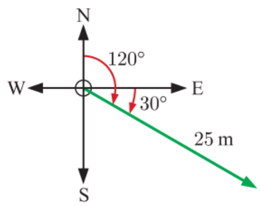
1 **a** *Scale*:
1 cm ≡ 10 km h^{-1}



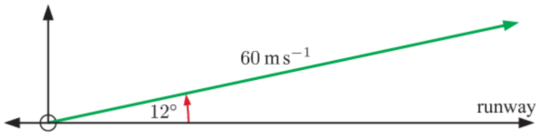
b *Scale*:
1 cm ≡ 10 m s^{-1}



c *Scale*: 1 cm ≡ 10 m

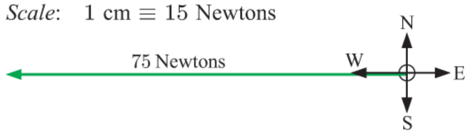


d

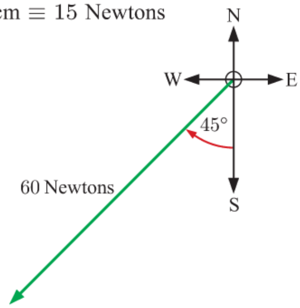


Scale: 1 cm ≡ 10 m s^{-1}

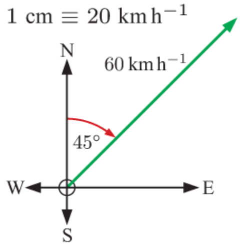
2 **a** *Scale*: 1 cm ≡ 15 Newtons



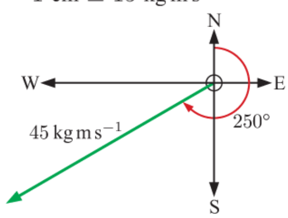
b *Scale*: 1 cm ≡ 15 Newtons



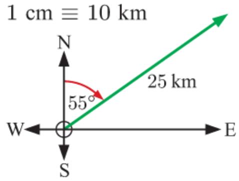
3 **a** *Scale*:
1 cm ≡ 20 km h^{-1}



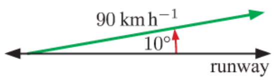
b *Scale*:
1 cm ≡ 15 kg m s^{-1}



c *Scale*:
1 cm ≡ 10 km



d *Scale*:
1 cm ≡ 30 km h^{-1}



EXERCISE 17B

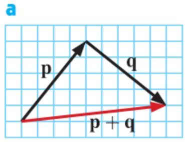
1 **a** **a**, **c**, and **e**; **b** and **d** **b** **a**, **b**, **c**, and **d**
c **a** and **b**; **c** and **d** **d** none are equal
e **a** and **c**; **b** and **d**

2 **a** false **b** true **c** false **d** true **e** true

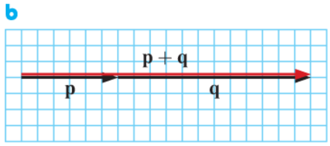
3 Statements **a**, **d**, and **f** are true.

EXERCISE 17C

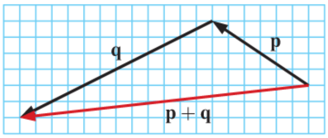
1 **a**



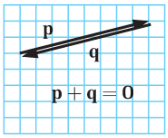
b



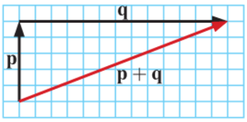
c



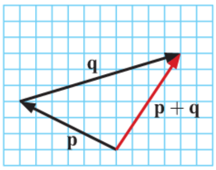
d



e

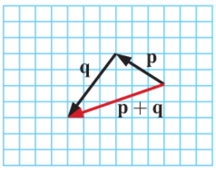


f

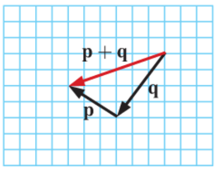


2 **a** $\overrightarrow{QS}$ **b** $\overrightarrow{PR}$ **c** $\overrightarrow{PQ}$ **d** $\overrightarrow{PS}$

3 **a** **i**



ii



b $\mathbf{p}+\mathbf{q}=\mathbf{q}+\mathbf{p}$ for any two vectors $\mathbf{p}$ and $\mathbf{q}$

4 **a** $600\ \text{km h}^{-1}$ due north **b** $400\ \text{km h}^{-1}$ due north
c $510\ \text{km h}^{-1}$ at a bearing of $011.3°$

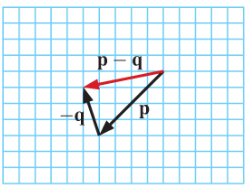
5 **a** $0\ \text{km h}^{-1}$ **b** $20\ \text{km h}^{-1}$ east **c** $14.1\ \text{km h}^{-1}$ north east

6 The aircraft is travelling at a speed of $404\ \text{km h}^{-1}$ at a bearing of $098.5°$.

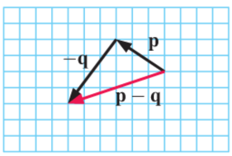
7 The ship is travelling at a speed of 23.3 knots at a bearing of $134°$.

EXERCISE 17D

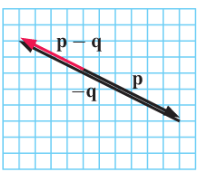
1 **a**



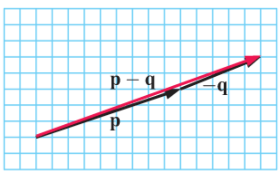
b



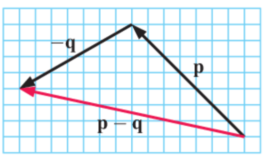
c



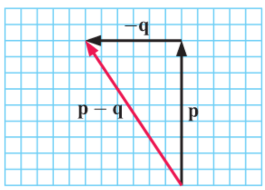
d



e



f

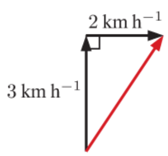


2 **a** $\overrightarrow{QS}$ **b** $\overrightarrow{PR}$ **c** $\mathbf{0}$ (zero vector) **d** $\overrightarrow{RQ}$ **e** $\overrightarrow{QS}$ **f** $\overrightarrow{RQ}$

3 The plane must fly $4.57°$ west of north at $502\ \text{km h}^{-1}$.

4 **a** The boat must head $25.5°$ west of north. **b** $28.3\ \text{km h}^{-1}$

5 **a**



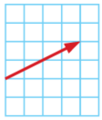
b The current will increase Ian's speed and push him to the right of the point he is swimming towards.

c Speed is $\approx 3.61\ \text{km h}^{-1}$, direction is $\approx 33.7°$ to the right.

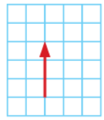
d Ian should face $\approx 41.8°$ left of where he is aiming.

EXERCISE 17E.1

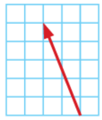
1 **a**



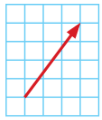
b



c



d

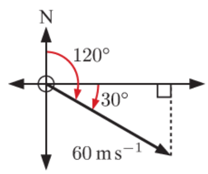


2 **a** $\begin{pmatrix}4\\2\end{pmatrix}$ **b** $\begin{pmatrix}0\\-3\end{pmatrix}$ **c** $\begin{pmatrix}-3\\-4\end{pmatrix}$ **d** $\begin{pmatrix}-6\\0\end{pmatrix}$ **e** $\begin{pmatrix}-6\\4\end{pmatrix}$
f $\begin{pmatrix}2\\-4\end{pmatrix}$

3 **a** $\begin{pmatrix}3\\4\end{pmatrix}$ **b** $\begin{pmatrix}-4\\-2\end{pmatrix}$ **c** $\begin{pmatrix}-2\\1\end{pmatrix}$ **d** $\begin{pmatrix}3\\-3\end{pmatrix}$ **e** $\begin{pmatrix}1\\5\end{pmatrix}$

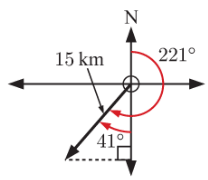
4 **a** $k=2$ **b** $k=-1$ **c** $k=3$

5 **a**



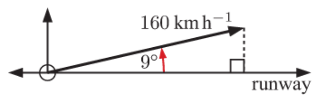
The vector is $\begin{pmatrix}52.0\\-30\end{pmatrix}$.

b



The vector is $\begin{pmatrix}-9.84\\-11.3\end{pmatrix}$.

c



The vector is $\begin{pmatrix}158\\25.0\end{pmatrix}$.

EXERCISE 17E.2

1 **a** $\begin{pmatrix}5\\-4\end{pmatrix}$ **b** $\begin{pmatrix}5\\-4\end{pmatrix}$ **c** $\begin{pmatrix}1\\-4\end{pmatrix}$ **d** $\begin{pmatrix}1\\-4\end{pmatrix}$
e $\begin{pmatrix}0\\-6\end{pmatrix}$ **f** $\begin{pmatrix}0\\-6\end{pmatrix}$ **g** $\begin{pmatrix}4\\-6\end{pmatrix}$ **h** $\begin{pmatrix}3\\-7\end{pmatrix}$

2 **a** $\begin{pmatrix}4\\2\end{pmatrix}$ **b** $\begin{pmatrix}-3\\-4\end{pmatrix}$ **c** $\begin{pmatrix}-1\\2\end{pmatrix}$

3 **a** $\begin{pmatrix}1\\6\end{pmatrix}$ **b** $\begin{pmatrix}-5\\1\end{pmatrix}$ **c** $\begin{pmatrix}-6\\4\end{pmatrix}$ **d** $\begin{pmatrix}-2\\10\end{pmatrix}$
e $\begin{pmatrix}-4\\-2\end{pmatrix}$ **f** $\begin{pmatrix}2\\-10\end{pmatrix}$

4 **a** $\begin{pmatrix}-3\\-3\end{pmatrix}$ **b** $\begin{pmatrix}-4\\9\end{pmatrix}$

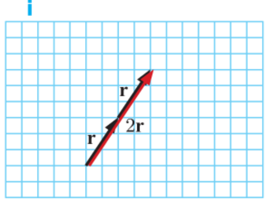
5 **a** $\sqrt{17}$ units **b** 6 units **c** $\sqrt{13}$ units
d $\sqrt{26}$ units **e** $\sqrt{20}$ units **f** $\sqrt{37}$ units

6 **a** **i** $\begin{pmatrix}-2\\-3\end{pmatrix}$ **ii** $\sqrt{13}$ units
b **i** $\begin{pmatrix}5\\-2\end{pmatrix}$ **ii** $\sqrt{29}$ units
c **i** $\begin{pmatrix}-3\\-4\end{pmatrix}$ **ii** 5 units
d **i** $\begin{pmatrix}-12\\5\end{pmatrix}$ **ii** 13 units

7 $\mathbf{a}+\mathbf{b}=\begin{pmatrix}a_1\\a_2\end{pmatrix}+\begin{pmatrix}b_1\\b_2\end{pmatrix}=\begin{pmatrix}a_1+b_1\\a_2+b_2\end{pmatrix}=\begin{pmatrix}b_1+a_1\\b_2+a_2\end{pmatrix}=\mathbf{b}+\mathbf{a}$

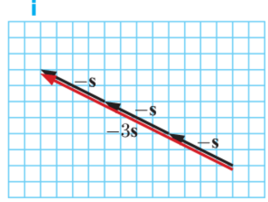
EXERCISE 17F

1 **a** **i**



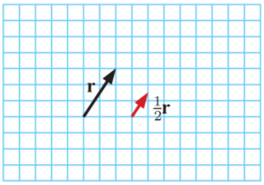
ii $2\mathbf{r}=\begin{pmatrix}4\\6\end{pmatrix}$

b **i**



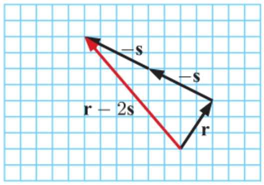
ii $-3\mathbf{s}=\begin{pmatrix}-12\\6\end{pmatrix}$

c **i**



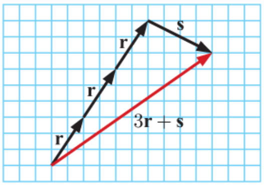
ii $\frac{1}{2}\mathbf{r} = \begin{pmatrix} 1 \\ \frac{3}{2} \end{pmatrix}$

d **i**



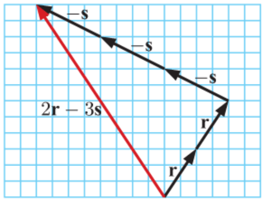
ii $\mathbf{r} - 2\mathbf{s} = \begin{pmatrix} -6 \\ 7 \end{pmatrix}$

e **i**



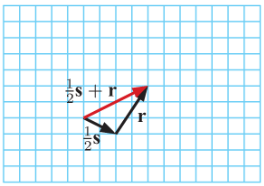
ii $3\mathbf{r} + \mathbf{s} = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$

f **i**



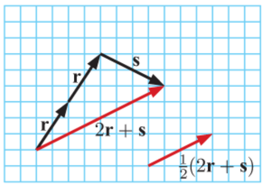
ii $2\mathbf{r} - 3\mathbf{s} = \begin{pmatrix} -8 \\ 12 \end{pmatrix}$

g **i**



ii $\frac{1}{2}\mathbf{s} + \mathbf{r} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

h **i**



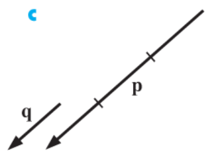
ii $\frac{1}{2}(2\mathbf{r} + \mathbf{s}) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

2 **a**

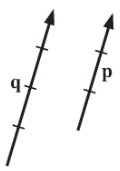
$\mathbf{p} = \mathbf{q}$

b

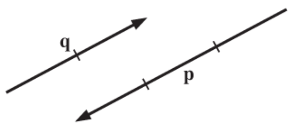
c



d



e



3 **a** $|k\mathbf{a}| = \left|\begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}\right| = \sqrt{(ka_1)^2 + (ka_2)^2}$

$$= \sqrt{k^2a_1^{\,2} + k^2a_2^{\,2}}$$
$$= \sqrt{k^2(a_1^{\,2} + a_2^{\,2})}$$
$$= \sqrt{k^2}\sqrt{a_1^{\,2} + a_2^{\,2}}$$
$$= |k|\,|\mathbf{a}| \qquad \{\text{since } \sqrt{k^2} = |k|\}$$

b $\mathbf{a}$ has $|\mathbf{a}| = \sqrt{a_1^{\,2} + a_2^{\,2}}$ so $\frac{1}{|\mathbf{a}|}\mathbf{a}$ has length $\frac{1}{\sqrt{a_1^{\,2} + a_2^{\,2}}} \bullet \sqrt{a_1^{\,2} + a_2^{\,2}} = 1$ and $\frac{1}{|\mathbf{a}|}\mathbf{a}$ is of the form $k\mathbf{a}$ where $k > 0$ (since $|\mathbf{a}|$ is always positive).

$\therefore$ $\frac{1}{|\mathbf{a}|}\mathbf{a}$ is in the same direction as $\mathbf{a}$.

$\therefore$ $\frac{1}{|\mathbf{a}|}\mathbf{a}$ is a vector of length 1 in the direction of $\mathbf{a}$.

EXERCISE 17G

1 **a** $t = -12$ **b** $t = 6$ **c** $t = 20$ **d** $t = -10$

2 $a = -\frac{12}{5}$ or 2 **3** $t = -1$

4 **a** $\mathbf{p} \parallel \mathbf{q}$ and $|\mathbf{p}| = 2|\mathbf{q}|$ **b** $\mathbf{p} \parallel \mathbf{q}$ and $|\mathbf{p}| = \frac{1}{2}|\mathbf{q}|$
c $\mathbf{p} \parallel \mathbf{q}$ and $|\mathbf{p}| = 3|\mathbf{q}|$ **d** $\mathbf{p} \parallel \mathbf{q}$ and $|\mathbf{p}| = \frac{1}{3}|\mathbf{q}|$

5 $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, $\overrightarrow{SR} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ $\Rightarrow$ $\overrightarrow{PQ} \parallel \overrightarrow{SR}$ and $|\overrightarrow{PQ}| = |\overrightarrow{SR}|$ which is sufficient to deduce that PQRS is a parallelogram.

6 **a** C(2, 5) **b** D(9, 0)

EXERCISE 17H

1 **a** 17 **b** −15 **c** −14 **2** $\begin{pmatrix} 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 8 \\ 2 \end{pmatrix} = 0$

3 **a** **i** 5 **ii** 45° **b** **i** 10 **ii** 0°
c **i** 0 **ii** 90° **d** **i** 5 **ii** $\approx 70.3°$
e **i** 33 **ii** $\approx 59.5°$ **f** **i** −11 **ii** $\approx 138°$

4 **a** $t = 6$ **b** $t = \pm 2\sqrt{3}$ **c** $t = -8$ **d** $t = -\frac{4}{3}$ or 2

5 **a** $\approx 37.9°$ **b** $\approx 121°$ **c** $\approx 14.5°$ **d** $\approx 4.40°$

6 $k = -\frac{11}{4}$

7 **a** $\widehat{P} \approx 8.13°$, $\widehat{Q} \approx 18.4°$, $\widehat{R} \approx 153.4°$
b $\widehat{P} \approx 66.8°$, $\widehat{Q} \approx 31.3°$, $\widehat{R} \approx 81.9°$

EXERCISE 17I

1 **a** $\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} -4 \\ -1 \\ -2 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$

2 **a** $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 6 \\ -9 \end{pmatrix}$ **b** S is $(-1, 5, -13)$

3 **a** $\begin{pmatrix} 8 \\ 0 \\ 12 \end{pmatrix}$ **b** $\begin{pmatrix} 2 \\ -10 \\ 4 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} 3 \\ -5 \\ 5 \end{pmatrix}$
e $\begin{pmatrix} 5 \\ 5 \\ 7 \end{pmatrix}$ **f** $\begin{pmatrix} 7 \\ -25 \\ 13 \end{pmatrix}$

4 **a** 3 units **b** $\sqrt{65}$ units **c** $\sqrt{61}$ units

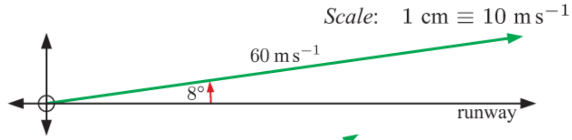
5 $\begin{pmatrix} 6 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 2 \\ -7 \end{pmatrix} = 0$ $\therefore$ $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

6 $t = -2$ **7** $\theta \approx 80.1°$

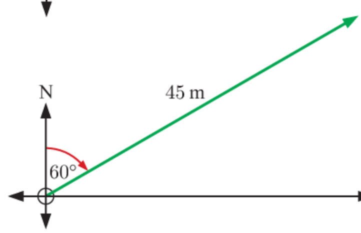
8 $R\widehat{P}Q \approx 57.1°$, $P\widehat{Q}R \approx 84.1°$, $Q\widehat{R}P \approx 38.8°$

REVIEW SET 17A

1 **a** *Scale*: 1 cm ≡ 10 m s^{-1}

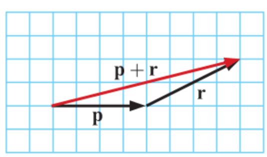


b

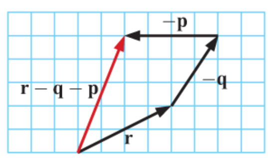


Scale: 1 cm ≡ 10 m

2 **a**



b



3 **a** $\overrightarrow{CA} = -\mathbf{c}$ **b** $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$
c $\overrightarrow{OC} = \mathbf{a} + \mathbf{c}$ **d** $\overrightarrow{BC} = -\mathbf{b} + \mathbf{a} + \mathbf{c}$

4 **a** $\overrightarrow{AD}$ **b** $\overrightarrow{BD}$ **c** $\mathbf{0}$ (the zero vector)

5 Speed is 12.3 km h^{-1} at a bearing of 145°.

6 $\mathbf{a} \parallel \mathbf{b}$ and $|\mathbf{a}| = \frac{1}{3}|\mathbf{b}|$

7 **a** $\begin{pmatrix} 11 \\ 1 \end{pmatrix}$ **b** $\begin{pmatrix} 1 \\ 12 \end{pmatrix}$ **c** $\sqrt{34}$ units

8 **a** $k = -4$ **b** $k = 9$ **9** A$(-2, -4)$

10 **a** -1 **b** 97.1° **11** $\approx 55.6°$

12 **a** $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ -2 \\ -3 \end{pmatrix}$ **b** PQ $= \sqrt{22}$ units

REVIEW SET 17B

1 **a** They have the same length. **b** $\mathbf{p} \parallel \mathbf{q}$ and $|\mathbf{p}| = 2|\mathbf{q}|$

2 $\overrightarrow{BA} = -\overrightarrow{AB}$

3 **a** He must fly in the direction 11.3° south of east.
b ≈ 204 km h^{-1}

4 **a** -23 **b** -21 **5** **a** $\begin{pmatrix} -1 \\ -9 \end{pmatrix}$ **b** $\sqrt{34}$ units

6 **a** $t = \frac{3}{2}$ **b** $t = -\frac{8}{3}$ **7** $t = 6$

8 **a** $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ **b** 5 units **9** $k = 2$ **10** $\approx 26.6°$

11 **a** $2\mathbf{p}$ **b** $\mathbf{p} + \mathbf{q}$ **c** $\mathbf{q} - \mathbf{p}$

12 **a** $\begin{pmatrix} -20 \\ 5 \\ -5 \end{pmatrix}$ **b** $\begin{pmatrix} 8 \\ -15 \\ 7 \end{pmatrix}$ **c** $\approx 123°$

EXERCISE 18A

1 **a**, **c**, **d**, and **f** are exponential functions.

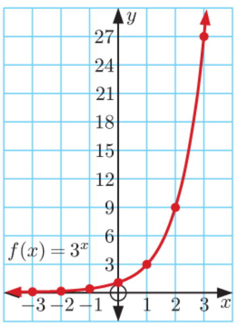
2 **a** 3 **b** 11 **c** $2\frac{1}{3}$ **d** $3^{2x} + 2$

3 **a** -2 **b** $-2\frac{4}{5}$ **c** 22 **d** $5^x - 3$

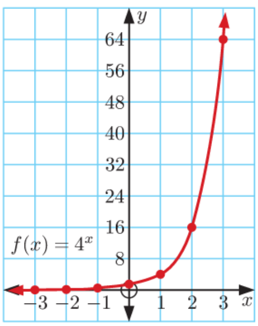
4 **a** $\frac{1}{9}$ **b** 9 **c** $\frac{1}{27}$ **d** 3^{x+3}

EXERCISE 18B.1

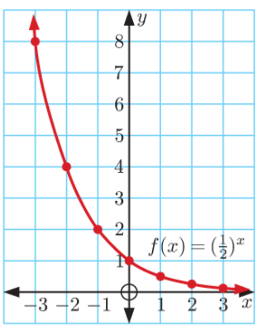
1 **a**



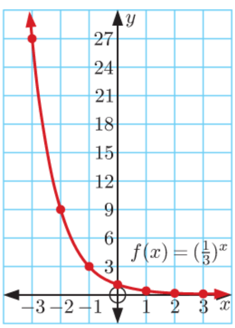
b



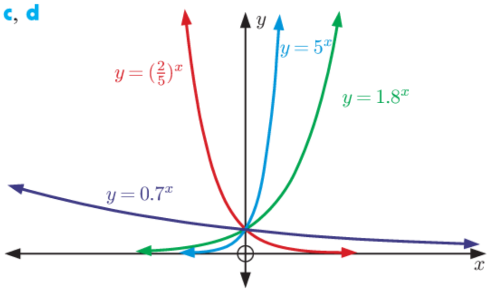
c



d



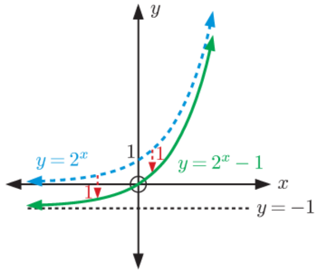
2 **a**, **b**, **c**, **d**



3 **a** D **b** A **c** F **d** C **e** B **f** E

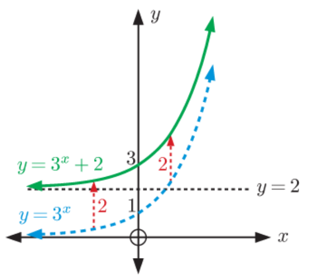
EXERCISE 18B.2

1 **a** **i**



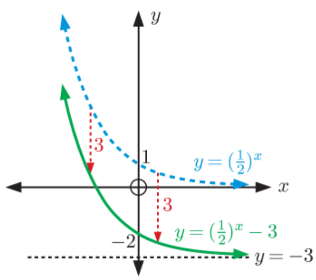
ii $y = -1$
iii $\{y \mid y > -1\}$
iv 0

b **i**



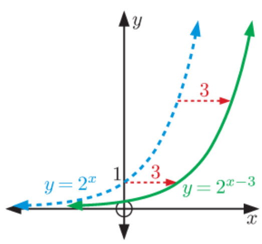
ii $y = 2$
iii $\{y \mid y > 2\}$
iv 3

c **i**

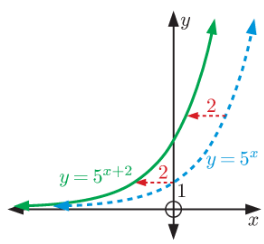


ii $y = -3$
iii $\{y \mid y > -3\}$
iv -2

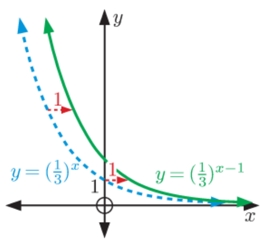
2 **a**



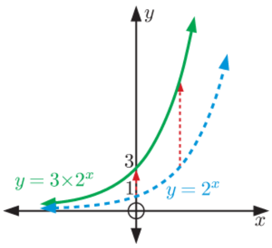
b



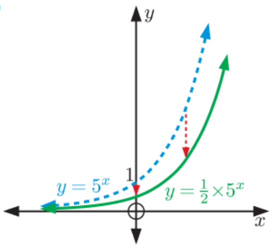
c



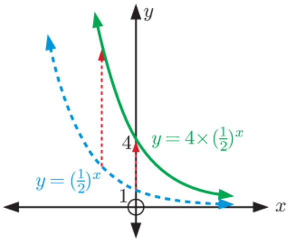
3 **a**



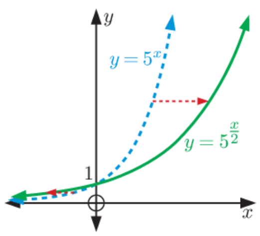
b



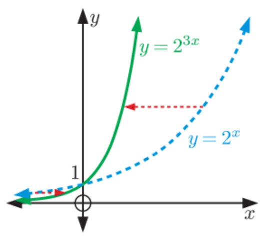
c



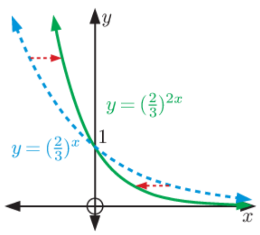
4 **a**



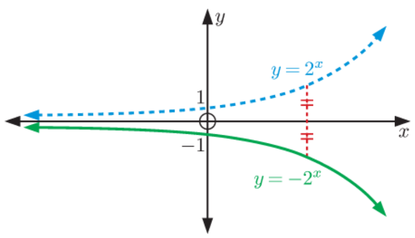
b



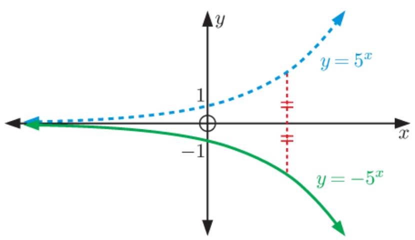
c



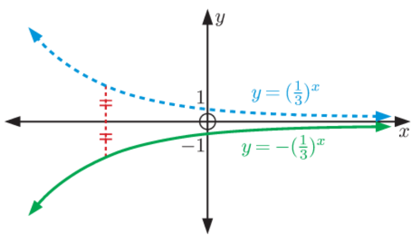
5 **a**



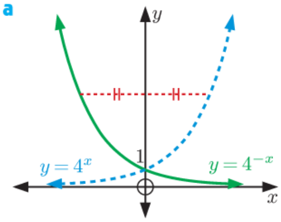
b



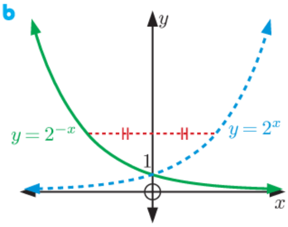
c



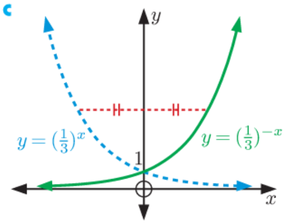
6 **a**



b



c



7 **a** $g(x) = f(x-1) = 3^{x-1} = \frac{1}{3}\times 3^x$

b $h(x) = f(2x) = 3^{2x} = 9^x$

c $j(x) = f(-x) = 3^{-x} = (\frac{1}{3})^x$

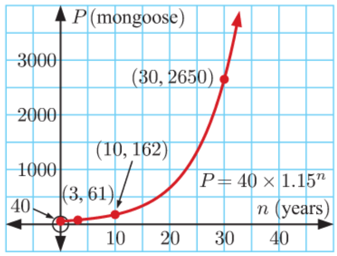
EXERCISE 18C.1

1 **a** 40 mongoose

b **i** $\approx$ 61 mongoose **ii** $\approx$ 162 mongoose

iii $\approx$ 2650 mongoose

c

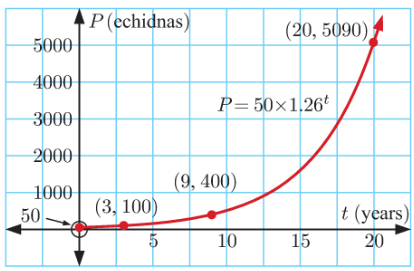


2 **a** 50 echidnas

b **i** $\approx$ 100 echidnas **ii** $\approx$ 400 echidnas

iii $\approx$ 5090 echidnas

c



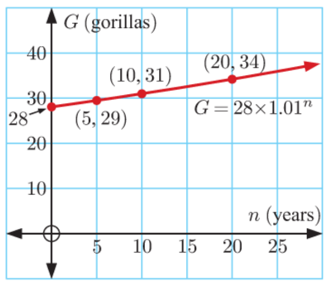
3 **a** $G_0 = 28$

b **i** ≈ 29

ii ≈ 31

iii ≈ 34

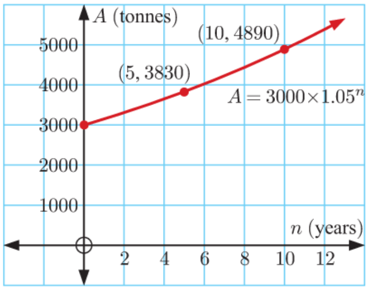
c



4 **a** The initial amount is 3000 tonnes, and the amount each year is 1.05 times the amount from the previous year.

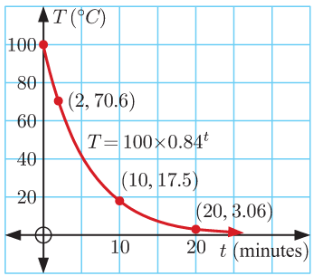
b **i** $\approx$ 3830 tonnes **ii** $\approx$ 4890 tonnes

c

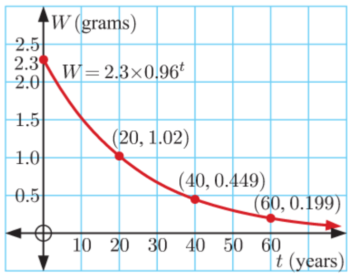


EXERCISE 18C.2

1 **a** 100°C
b **i** ≈ 70.6°C **ii** ≈ 17.5°C **iii** ≈ 3.06°C
c

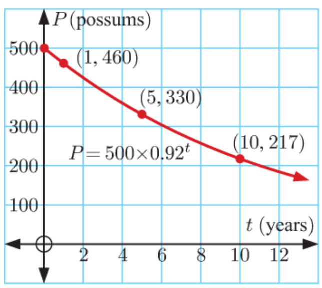


2 **a** 2.3 g
b **i** ≈ 1.02 g **ii** ≈ 0.449 g **iii** ≈ 0.199 g
c



d $\approx 55.8\%$ loss

3 **a** $P = 500 \times 0.92^t$
b **i** 460 **ii** ≈ 330 **iii** ≈ 217
c



EXERCISE 18D

1

After year	*Interest paid*	*Future Value*
0	-	€8000
1	5% of €8000 = €400	€8400
2	5% of €8400 = €420	€8820
3	5% of €8820 = €441	€9261

2 **a** £53 240 **b** £13 240
3 **a** ¥62 985.60 **b** ¥12 985.60 **4** $14 976.01
5 €11 477.02 **6** $\approx 3.95\%$ **7** **a** £1186 **b** $\approx 4.45\%$

EXERCISE 18E

1 $5684.29

2 **a**

Years owned	*Depreciation or annual loss in value*	*Value after n years*
0	-	£8495
1	£1019.40	£7475.60
2	£897.07	£6578.53
3	£789.42	£5789.10
4	£694.69	£5094.41

b £694.69 **c** £3055.10

3 **a** $19 712 **b** $18 788 **4** 11.5%

EXERCISE 18F

1 **a** $x = 1$ **b** $x = 2$ **c** $x = 3$ **d** $x = 0$
e $x = -1$ **f** $x = -1$ **g** $x = -4$ **h** $x = 0$
i $x = -4$ **j** $x = -2$ **k** $x = 6$ **l** $x = 2$
m $x = -\frac{3}{4}$ **n** $x = \frac{7}{2}$ **o** $x = 0$ **p** $x = 4$

2 **a** $x = 3$ **b** $x = 0$ **c** $x = -2$ **d** $x = 3$
e $x = -5$ **f** $x = -2$ **g** $x = \frac{2}{7}$ **h** $x = \frac{1}{3}$
i $x = 1$ **j** $x = 1$ **k** no solution **l** $x = -1$ or 3

3 5 months

4 **a** $x = 1$ **b** $x = 2$ **c** $x = 1$ **d** $x = \frac{5}{4}$
e $x = 2$ **f** $x = -\frac{9}{7}$

EXERCISE 18G.1

1 **a** 2 **b** 5 **c** -2 **d** 1 **e** -4 **f** 0 **g** $\frac{1}{2}$ **h** $\frac{3}{2}$

2 **a** ≈ 1.447 **b** ≈ 0.699 **c** ≈ 2.477 **d** ≈ -0.398
e ≈ 2.903 **f** ≈ 1.954 **g** ≈ -1.155 **h** ≈ 3.602
i undefined

3 **a** $\log 70 \approx 1.845$ **b** $70 \approx 10^{1.845}$

4 10^x is positive for all x.

5 **a** $\log x$ is positive if x is greater than 1.
b $\log x$ is negative if x is between 0 and 1.

EXERCISE 18G.2

1 **a** $\log 30$ **b** $\log 5$ **c** $\log 11$ **d** $\log 12$
e $\log(\frac{5}{4})$ **f** $\log 1 = 0$ **g** $\log 30$ **h** $\log 4$
i $\log(\frac{1}{6})$ **j** $\log(\frac{1}{512})$ **k** $\log 2000$ **l** $\log(\frac{64}{125})$

2 **a** 3 **b** 2 **c** $\frac{5}{6}$ **d** -1 **e** -1 **f** $-\frac{3}{2}$
g $\dfrac{b}{3}$ **h** $\dfrac{2}{a}$

3 **a** $\log \sqrt{5} = \log 5^{\frac{1}{2}} = \frac{1}{2}\log 5$
b $\log \sqrt[3]{2} = \log 2^{\frac{1}{3}} = \frac{1}{3}\log 2$
c $\log(\frac{1}{\sqrt{3}}) = \log 3^{-\frac{1}{2}} = -\frac{1}{2}\log 3$

4 **a** **i** ≈ 0.699 **ii** ≈ 3.699
b $\log 5000 = \log 5 + \log 1000 = \log 5 + 3$

5 $\log 10^x = x\log 10 = x$

6 **a** $\log 600$ **b** $\log 5$ **c** $\log 8$ **d** $\log 250$
e $\log 9$ **f** $\log 14\,000$

7 **a** **i** ≈ 1.114 **ii** ≈ -1.114 **iii** ≈ 2.398
iv ≈ -2.398
b $\log\left(\dfrac{1}{x}\right) = \log x^{-1} = -\log x$

8 3

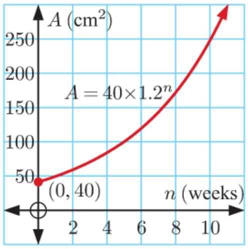
EXERCISE 18G.3

1 **a** $x \approx 1.585$ **b** $x \approx 3.322$ **c** $x \approx 8.644$
d $x \approx -4.454$ **e** $x \approx 4.292$ **f** $x \approx -0.099\,97$
g $x \approx 6.511$ **h** $x \approx 4.923$ **i** $x \approx 49.60$
j $x \approx 4.376$ **k** $x \approx 8.497$ **l** $x \approx 230.7$

2 **a** $x \approx 3.81$ **b** $x \approx 3.54$ **c** $x \approx 1.24$
d $x \approx 6.58$ **e** $x \approx 2.74$ **f** $x \approx 18.1$

3 **a** 15.9 hours **b** 60.6 hours

4 **a** $V = 125 \times (0.85)^t$ cm^3 **b** 90.3 cm^3
c 5.64 minutes

5 **a** no
b The area covered each week is 1.2 times greater than the area covered the previous week, and the initial area covered is 40 cm^2.
c


d 69.12 cm^2
e 5.03 weeks

EXERCISE 18G.4

1 **a** $\log_2 8 = 3$ **b** $\log_3 9 = 2$ **c** $\log_5(\frac{1}{5}) = -1$
d $\log_2 32 = 5$ **e** $\log_7 1 = 0$ **f** $\log_3(\frac{1}{81}) = -4$
g $\log_2(\frac{1}{64}) = -6$ **h** $\log_2(\sqrt{2}) = \frac{1}{2}$

2 **a** $10^3 = 1000$ **b** $2^4 = 16$ **c** $3^{-1} = \frac{1}{3}$
d $4^0 = 1$ **e** $7^{-2} = \frac{1}{49}$ **f** $(\frac{1}{7})^2 = \frac{1}{49}$
g $10^2 = 100$ **h** $(\sqrt{5})^4 = 25$

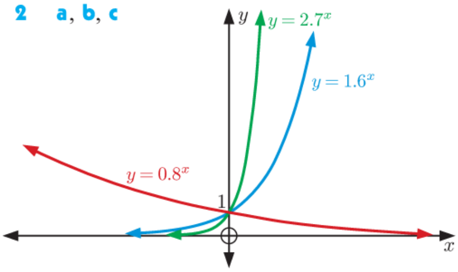
3 $125 = 5^3$, $\log_5 125 = 3$

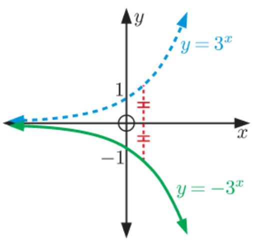
4 $\frac{1}{36} = 6^{-2}$, $\log_6(\frac{1}{36}) = -2$

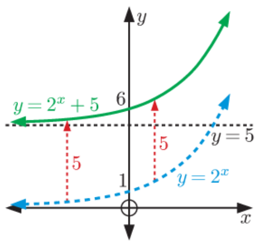
5 **a** 2 **b** 6 **c** $\frac{1}{2}$ **d** $\frac{1}{4}$ **e** $\frac{3}{2}$ **f** 0
g 1 **h** -1 **i** 1 **j** -1 **k** $-\frac{1}{2}$ **l** $-\frac{1}{6}$

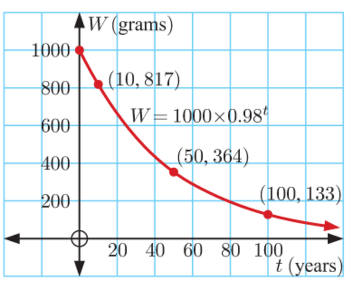
REVIEW SET 18A

1 **a** 0
b 26
c $-\frac{2}{3}$
d $3^{2x} - 1$

2 **a, b, c**


3 **a**


b


4 **a** 1000 g
b **i** ≈ 817 g
ii ≈ 364 g
iii ≈ 133 g
c


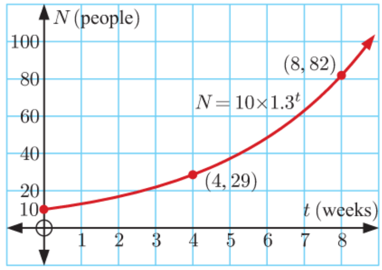
5 **a** £171 475.72 **b** £51 475.72

6 **a** $x = \frac{1}{3}$ **b** $x = \frac{13}{7}$ **c** $x = -\frac{2}{5}$ **d** $x = -2$
e $x = \frac{5}{2}$ **f** $x = \frac{7}{11}$

7 **a** 450 seals **b** 4 years

8 **a** $\log 80 \approx 1.903$ **b** $80 \approx 10^{1.903}$

9 **a** $-\frac{1}{2}$ **b** $\frac{5}{2}$ **10** **a** $\log 8$ **b** $\log 5$ **c** 4 **d** $\frac{2b}{3}$

11 **a** **i** 10 people **ii** ≈ 29 people **iii** ≈ 82 people
b


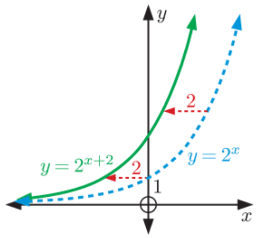
c ≈ 11.4 weeks

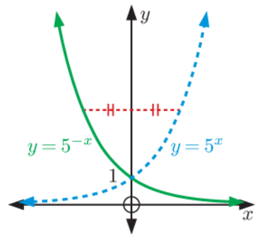
12 **a** $\frac{1}{\sqrt{5}} = 5^{-\frac{1}{2}}$ **b** $\log_5 \frac{1}{\sqrt{5}} = -\frac{1}{2}$

REVIEW SET 18B

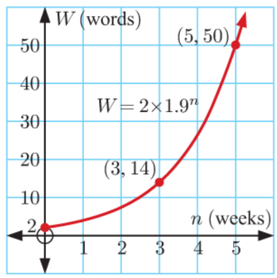
1 **a** The variable x appears in the exponent.
b Translate 2 units to the right.

2 **a** 2 **b** $\frac{2}{9}$ **c** $2 \times 3^{-x-4}$

3 **a**


b


4 $\{y \mid y > -2\}$

5 **a** **i** 2 words
ii ≈ 14 words
iii ≈ 50 words
b


6 \$6273.18 **7** ≈ 2.85

8 **a** $x = -2$ **b** $x = \frac{3}{4}$ **c** $x = \frac{2}{3}$

9 **a** $\log 55$ **b** $\log 400$ **c** $\log 25$

10 **a** 1
b $(1 + \log 5)(1 - \log 5) = 1 - (\log 5)^2$ and
$$\begin{aligned}(1 + \log 5)(1 - \log 5) &= (\log 10 + \log 5)(\log 10 - \log 5)\\ &= \log(10 \times 5) \times \log(\tfrac{10}{5})\\ &= \log 50 \times \log 2\end{aligned}$$

11 **a** $x \approx 5.42$ **b** $x \approx 5.86$ **12** **a** $\frac{1}{2}$ **b** -4

EXERCISE 19A

1 **a** $x = 37$ {angle in a semi-circle, angles in a triangle}
b $x = 30$ {angle in a semi-circle, angles in a triangle}
c $x = 18$ {angle in a semi-circle, angles in a triangle}

2 **a** $a = b = 55$ {tangents from an external point, base angles of isosceles triangle}
b $a = 3$ {chords of a circle}, $b \approx 36.9$
c $a = 60$ {angle in a semi-circle}
$b = 40$ {angles in a triangle, radius-tangent theorem}

3 **a** $x = 40$ {angles in a triangle} **b** ≈ 7.83 cm

4 **Hint:** $AB + CD = (AP + PB) + (DR + CR)$
$= AS + QB + DS + CQ$, etc.

5 $(8 - 4\sqrt{3})$ cm ≈ 1.07 cm {chords of a circle}

6 10.25 cm **7** 1 cm **8** 40 cm

EXERCISE 19B

1 **a** $x = 64$ {angle at the centre}
b $x = 94$ {angle at the centre}
c $x = 70$ {angle at the centre, angles in an isosceles triangle}
d $x = 45$ {angles in an isosceles triangle, angle at the centre}
e $x = 66$ {angle at the centre}
f $x = 25$ {angle at the centre, base angles of isosceles triangle}

2 **a** $x = 46$ {angles subtended by the same arc}
b $x = y = 55$ {angles subtended by the same arc}
c $a = 50$, $b = 40$ {angles subtended by the same arc}
d $a = 55$, $c = 70$ {angles subtended by the same arc}
e $a = 80$ {angles subtended by the same arc}
$b = 200$ {angles at the centre, angles at a point}
f $x = 75$ {angles subtended by the same arc}
$y = 118$ {exterior angle of a triangle}
g $x = 42$ {angles subtended by the same arc, exterior angle of a triangle}
h $x = 25$ {angles in a semi-circle, base angles of isosceles triangle, angles subtended by the same arc}
i $x = 25$ {angle in a semi-circle, angles in a triangle, angles subtended by the same arc}

3 **a** $x = 70$ {angle between a tangent and a chord, angles in an isosceles triangle}
b $x = 40$ {angle between a tangent and a chord, angles in an isosceles triangle}
c $x = 35$ {angles in an isosceles triangle, radius-tangent theorem}

EXERCISE 19C

1 **a** $\triangle OAB$ is isosceles.
b The perpendicular from the centre of a circle to a chord bisects the chord, and bisects the angle at the centre subtended by the chord.

2 **a** [OA], [OP], [OB] are radii of the circle, $\therefore$ are equal.
b **i** $\widehat{APO} = a$ **ii** $\widehat{BPO} = b$
iii $\widehat{AOX} = 2a$ **iv** $\widehat{BOX} = 2b$
v $\widehat{APB} = (a + b)$ **vi** $\widehat{AOB} = (2a + 2b) = 2(a + b)$
c The angle at the centre of a circle is twice the angle on the circle subtended by the same arc.

3 **a** $\widehat{AOB} = 2\alpha$ **b** $\widehat{ACB} = \alpha$ **c** $\widehat{ADB} = \widehat{ACB}$

4 **a** **i** $\widehat{TAX} = 90°$ **ii** $\widehat{ACX} = 90°$
b **i** $90° - \alpha$ **ii** α **iii** α **c** $\widehat{TAC} = \widehat{CBA}$

5 **Hint:** Use alternate angles, 'angle at the centre' theorem, and isosceles triangle.

6 **a** **i** $\widehat{XOY} = \alpha$ **ii** $\widehat{AXO} = 2\alpha$ **iii** $\widehat{XAO} = 2\alpha$
iv $\widehat{XOB} = 4\alpha$ **v** $\widehat{BOY} = 3\alpha$
b $\widehat{BOY} = 3 \times \widehat{YOX}$

7 **a** $\widehat{BXA} = \widehat{BXC} = 90°$ {angles in a semi-circle}
b A, X, and C are collinear. **c** yes
d yes (Repeat **a** to **c** using semi-circles with diameter AD and CD.)

8 **Hint:** Use the 'angle between a tangent and a chord' theorem.

9 **Hint:** Use vertically opposite angles and the 'angles subtended by the same arc' theorem.

10 **Hint:** Use the 'angle in a semi-circle' theorem.

11 **Hint:** Use the 'angle in a semi-circle' theorem.

12 **Hint:** Let [AQ] cut the circle at R. Find $\widehat{ARB}$. Use exterior angle of a triangle.

EXERCISE 19D

1 **a** $\widehat{DOB} = 2a°$, reflex $\widehat{DOB} = 2b°$
b $2a + 2b = 360$ $\therefore$ $a + b = 180$

3 **a** $x = 107$ {opposite angles of a cyclic quadrilateral}
b $x = 60$ {opposite angles of a cyclic quadrilateral}
c $x = 70$ {co-interior angles, opposite angles of a cyclic quadrilateral}
d $x = 90$ {angles on a line, opposite angles of a cyclic quadrilateral}
e $x = 62$ {vertically opposite angles, opposite angles of a cyclic quadrilateral, angle between a tangent and a chord}

4 **a** Yes, one pair of opposite angles are supplementary.
b Yes, AD subtends equal angles at B and C. **c** No
d Yes, opposite angles are supplementary.
e Yes, one pair of opposite angles are supplementary.
f Yes, AD subtends equal angles at B and C.

5 **Hint:** Use co-interior angles and 'opposite angles of a cyclic quadrilateral'.

6 **Hint:** Construct [OD]. Use opposite angles of a parallelogram, corresponding angles, base angles of an isosceles triangle.

7 **Hint:** Use 'chords of a circle theorem'.

8 **Hint:** Use corresponding angles.

9 **Hint:** Use 'one side subtends equal angles at the other two vertices'.

10 **Hint:** Use congruent triangles.

11 **Hint:** Use 'one side subtends equal angles at the other two vertices'.

12 **Hint:** Join [PX] and use 'angles subtended by the same arc'.

13 **Hint:** Use 'opposite angles of a cyclic quadrilateral'.

REVIEW SET 19A

1 **a** $a = 54$ {angle at the centre, angles in an isosceles triangle}
b $a = 62$ {angle in a semi-circle, angles in a triangle}
c $a = 61$ {angles subtended by the same arc, exterior angle of a triangle}
d $a = 80$ {angles on a line, opposite angles of a cyclic quadrilateral}
e $a = 63$ {co-interior angles, opposite angles of a cyclic quadrilateral}
f $a = 45$ {opposite angles of a cyclic quadrilateral}

2 $\alpha + \beta + \gamma = 180°$ {angle between a tangent and a chord}

3 **a** $x = 38$ {angle in a semi-circle, angles in a triangle}
b $x = 140$ {angle at the centre, exterior angle of a triangle}

c $x = 104$ {angle between a tangent and a chord, angles in an isosceles triangle}

4 **a** **Hint:** Use the 'tangents from an external point' theorem.

b **Hint:** Let $\widehat{ACM} = \alpha$ and $\widehat{BCM}$ be β. Use the isosceles triangle theorem to show that $\alpha + \beta = 90°$.

5 **Hint:** Construct [PB] and [XB]. Use 'angle between a tangent and a chord' and 'angles subtended by the same arc' to show that alternate angles are equal.

6 **Further hint:** Use the 'angle at the centre' theorem (twice) and the 'angles in a triangle' theorem.

7 **a** The angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

b **i** $\widehat{PQB} = \alpha$, $\widehat{PQA} = \beta$, $\widehat{AQB} = (\alpha + \beta)$

ii **Hint:** Use the angles in a triangle theorem.

8 **Hint:** Use 'angle between a tangent and a chord', and alternate angles.

REVIEW SET 19B

1 **a** $x = 86$ {angles subtended by the same arc, exterior angle of a triangle}

b $x = \sqrt{34} \approx 5.83$ {chords of a circle, Pythagoras}

c $x = 9$ {radius-tangent, Pythagoras}

2 **a** $x = 55$ {angles at a point, angle at the centre}

b $x = 55$ {angles in an isosceles triangle, radius-tangent, angles in a triangle}

c $x \approx 7.68$ {tangents from an external point, angles in a triangle, radius-tangent}

3 ≈ 6.63 m

4 **Hint:** Use the isosceles triangle theorem and the 'angles subtended by the same arc' theorem.

5 **Hint:** Use the 'angles subtended by the same arc' theorem and the 'angles in a triangle' theorem (on $\triangle$ABQ and $\triangle$APC).

6 **a** **i** $\widehat{PSR} = 2\alpha$ **ii** $\widehat{PQR} = 2\alpha$ **iii** $\widehat{PRQ} = 2\alpha$

iv $\widehat{QPR} = 2\alpha$

b **Hint:** Use the angles in a triangle theorem.

c $\widehat{QRS} = 90°$ **d** [RS] is a diameter of the circle.

7 **Hint:** Use angle in a semi-circle, and 'one side subtends equal angles at the other two vertices'.

8 **a** **Hint:** Let $\widehat{XTA} = \alpha$, $\widehat{XPA} = \beta$. Use 'angle between a tangent and a chord', and sum of angles of a triangle.

b Show $\alpha = \beta$.

EXERCISE 20A

1 **b** and **c** are quadratic functions.

2 **a** $y = 0$ **b** $y = 5$ **c** $y = -15$ **d** $y = 12$

3 **a** no **b** yes **c** yes **d** no **e** no **f** no

4 **a** $x = -3$ **b** $x = -2$ or -3 **c** $x = 1$ or 4

d do not exist

5 **a** $x = 0$ or 1 **b** $x = -1$ or 3 **c** $x = -7$ or $\frac{1}{2}$

d $x = 2$ or 3

6 **a** **i** 75 m **ii** 195 m **iii** 275 m

b **i** At $t = 2$ s and $t = 14$ s.

ii At $t = 0$ s and $t = 16$ s.

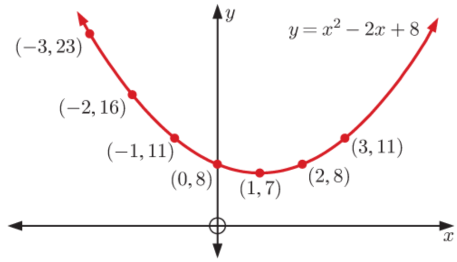
c The object leaves the ground at $t = 0$ s. At $t = 2$ it is rising and at $t = 14$ it is falling. Height 0 m is ground level and the time of flight is 16 seconds.

7 **a** **i** $-\$40$, a loss of $\$40$ **ii** $\$480$ profit

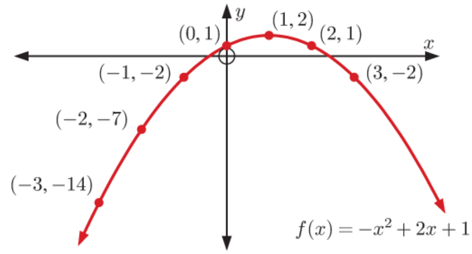
b 10 cakes or 62 cakes

EXERCISE 20B.1

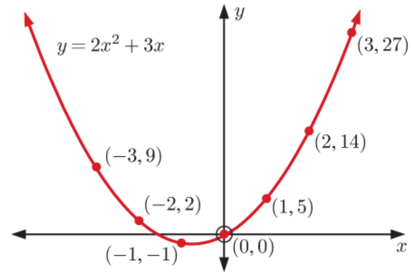
1 **a**



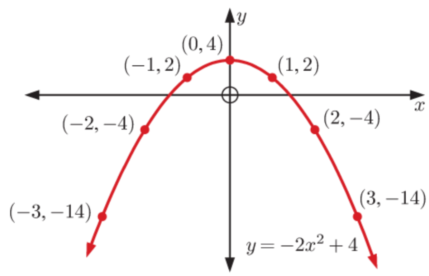
b



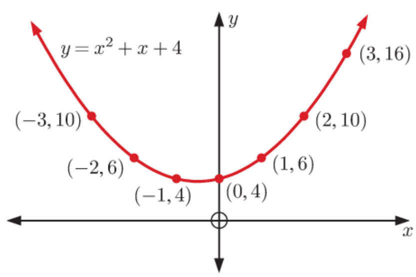
c



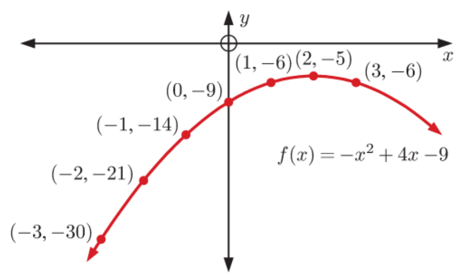
d



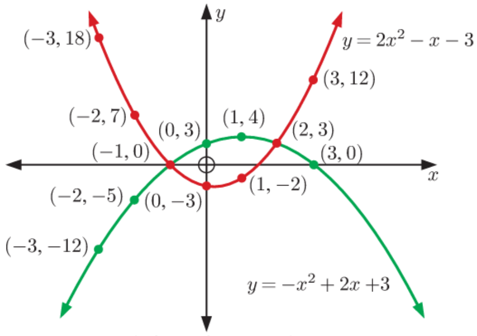
e



f



2 **a**



$x = -1$ or 2 (where they meet)

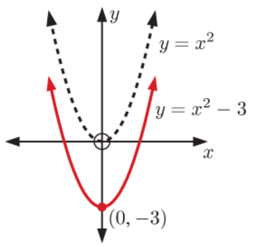
b $2x^2 - x - 3 = -x^2 + 2x + 3$ becomes

$3x^2 - 3x - 6 = 0$

$\therefore\ x^2 - x - 2 = 0,$ etc.

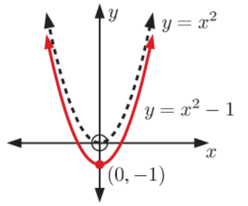
EXERCISE 20B.2

1 **a**



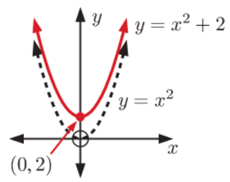
vertex is at $(0, -3)$

b



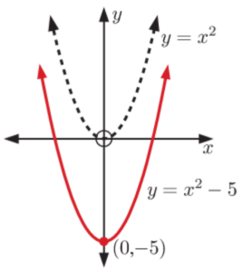
vertex is at $(0, -1)$

c



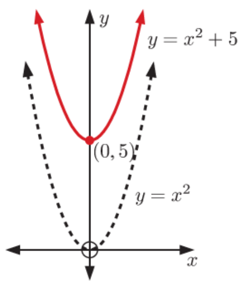
vertex is at $(0, 2)$

d



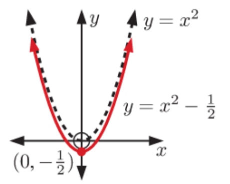
vertex is at $(0, -5)$

e



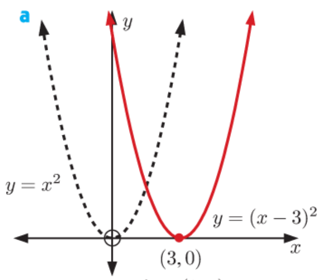
vertex is at $(0, 5)$

f



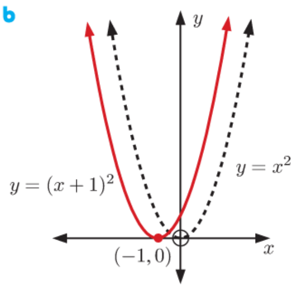
vertex is at $(0, -\frac{1}{2})$

2 **a**



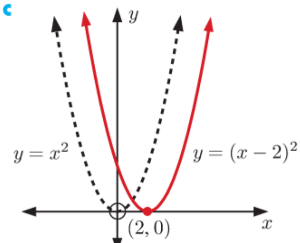
vertex is at $(3, 0)$

b



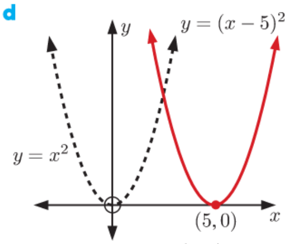
vertex is at $(-1, 0)$

c



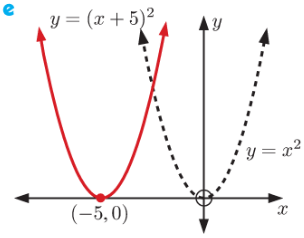
vertex is at $(2, 0)$

d



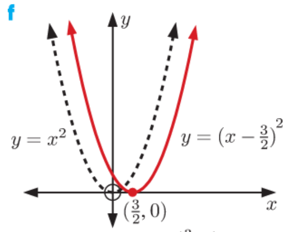
vertex is at $(5, 0)$

e



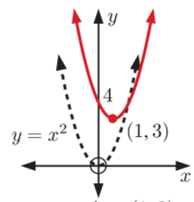
vertex is at $(-5, 0)$

f



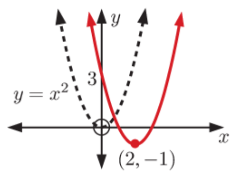
vertex is at $(\frac{3}{2}, 0)$

3 **a** $y = (x-1)^2 + 3$



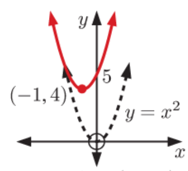
vertex is at $(1, 3)$

b $y = (x-2)^2 - 1$



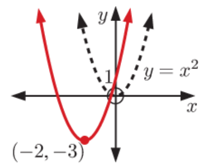
vertex is at $(2, -1)$

c $y = (x+1)^2 + 4$



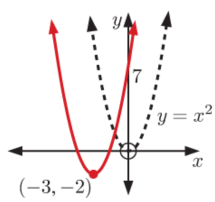
vertex is at $(-1, 4)$

d $y = (x+2)^2 - 3$



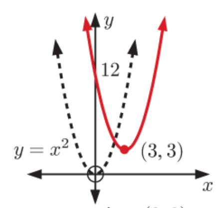
vertex is at $(-2, -3)$

e $y = (x+3)^2 - 2$



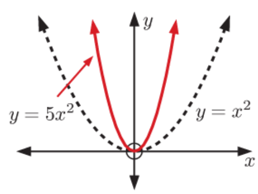
vertex is at $(-3, -2)$

f $y = (x-3)^2 + 3$



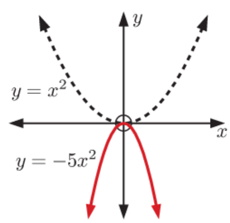
vertex is at $(3, 3)$

4 **a**



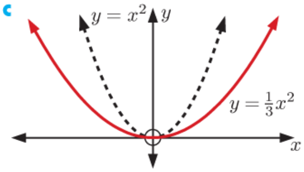
$y = 5x^2$ is 'thinner' than $y = x^2$ and the graph opens upwards.

b



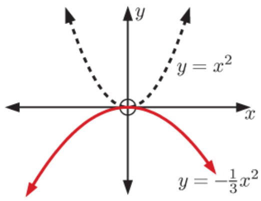
$y = -5x^2$ is 'thinner' than $y = x^2$ and the graph opens downwards.

c



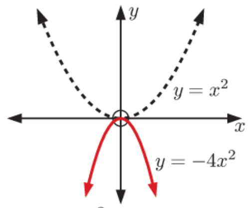
$y=\frac{1}{3}x^2$ is 'wider' than $y=x^2$ and the graph opens upwards.

d



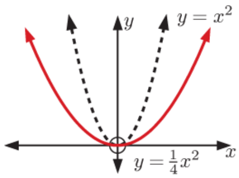
$y=-\frac{1}{3}x^2$ is 'wider' than $y=x^2$ and the graph opens downwards.

e



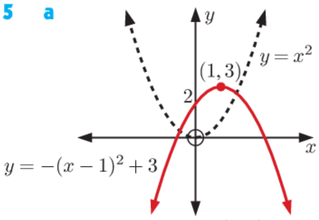
$y=-4x^2$ is 'thinner' than $y=x^2$ and the graph opens downwards.

f



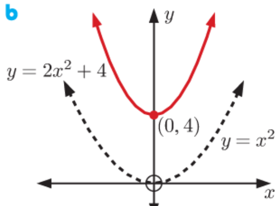
$y=\frac{1}{4}x^2$ is 'wider' than $y=x^2$ and the graph opens upwards.

5 **a**



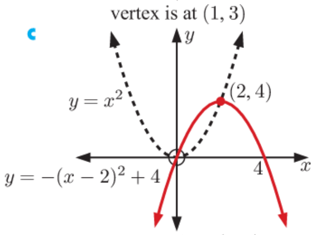
vertex is at $(1,3)$

b



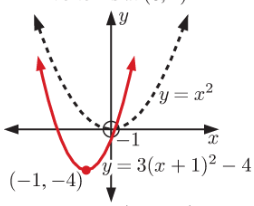
vertex is at $(0,4)$

c



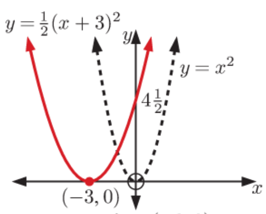
vertex is at $(2,4)$

d



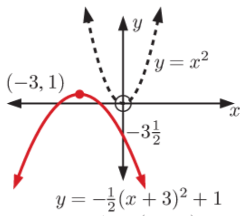
vertex is at $(-1,-4)$

e



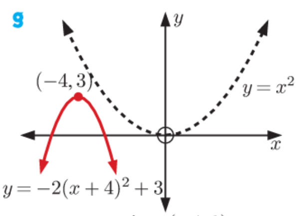
vertex is at $(-3,0)$

f



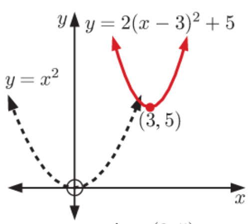
vertex is at $(-3,1)$

g



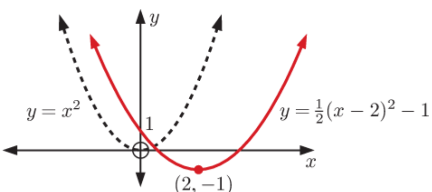
vertex is at $(-4,3)$

h



vertex is at $(3,5)$

i

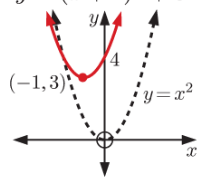


vertex is at $(2,-1)$

6 **a** F **b** C **c** B **d** D **e** E **f** A

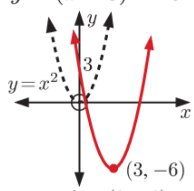
EXERCISE 20B.3

1 **a** $y=(x+1)^2+3$



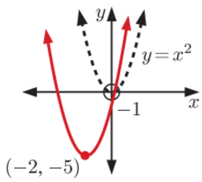
vertex is at $(-1,3)$

b $y=(x-3)^2-6$



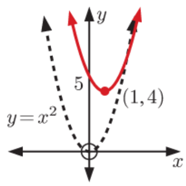
vertex is at $(3,-6)$

c $y=(x+2)^2-5$



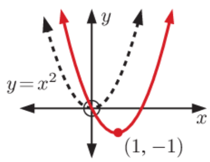
vertex is at $(-2,-5)$

d $y=(x-1)^2+4$



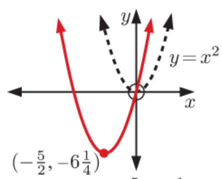
vertex is at $(1,4)$

e $y=(x-1)^2-1$



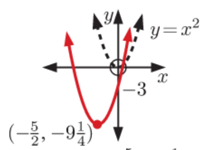
vertex is at $(1,-1)$

f $y=(x+\frac{5}{2})^2-6\frac{1}{4}$



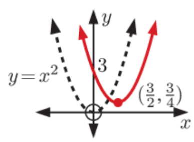
vertex is at $(-\frac{5}{2},-6\frac{1}{4})$

g $y=(x+\frac{5}{2})^2-9\frac{1}{4}$



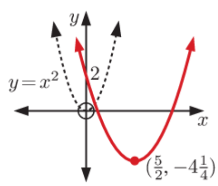
vertex is at $(-\frac{5}{2},-9\frac{1}{4})$

h $y=(x-\frac{3}{2})^2+\frac{3}{4}$



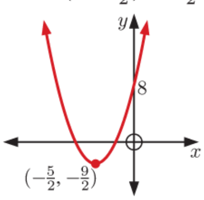
vertex is at $(\frac{3}{2},\frac{3}{4})$

i $y=(x-\frac{5}{2})^2-\frac{17}{4}$



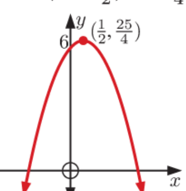
vertex is at $(\frac{5}{2},-4\frac{1}{4})$

2 **a** $y=2(x+\frac{5}{2})^2-\frac{9}{2}$



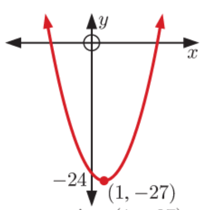
vertex is at $(-\frac{5}{2},-\frac{9}{2})$

b $y=-(x-\frac{1}{2})^2+\frac{25}{4}$



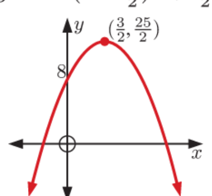
vertex is at $(\frac{1}{2},\frac{25}{4})$

c $y=3(x-1)^2-27$



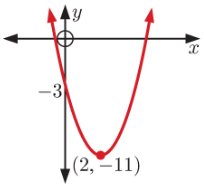
vertex is at $(1,-27)$

d $y=-2(x-\frac{3}{2})^2+\frac{25}{2}$



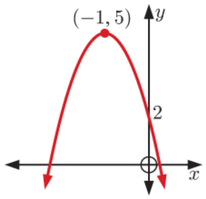
vertex is at $(\frac{3}{2},\frac{25}{2})$

e $y = 2(x-2)^2 - 11$



vertex is at $(2, -11)$

f $y = -3(x+1)^2 + 5$



vertex is at $(-1, 5)$

EXERCISE 20C.1

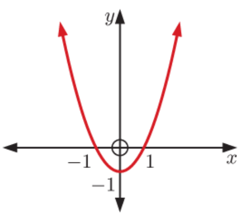
1 **a** 3 **b** 2 **c** -8 **d** 1 **e** 6 **f** 5
g 6 **h** 8 **i** -2

2 **a** 3 and -1 **b** 2 and 4 **c** -3 and -2
d 4 and 5 **e** -3 (touching) **f** 1 (touching)

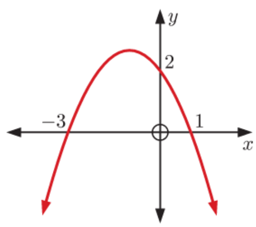
3 **a** -3 and 3 **b** -5 and 5 **c** 0 and 6
d -5 and -2 **e** -4 and 3 **f** 0 and 4
g -2 and -4 **h** -1 (touching) **i** 3 (touching)

4 **a** $2 \pm \sqrt{3}$ **b** $-2 \pm \sqrt{7}$ **c** $3 \pm \sqrt{5}$
d $\dfrac{7 \pm \sqrt{73}}{6}$ **e** $\dfrac{1 \pm \sqrt{41}}{4}$ **f** $\dfrac{9 \pm \sqrt{33}}{8}$

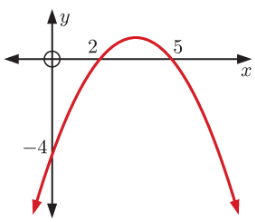
5 **a**



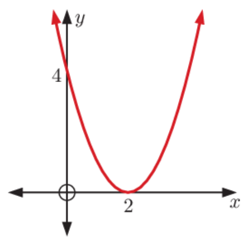
b



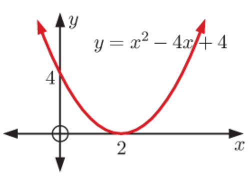
c



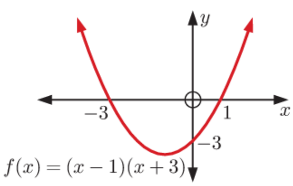
d



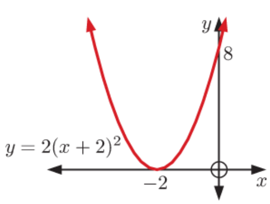
6 **a** **i** $a = 1$
ii 4
iii 2 (touching)



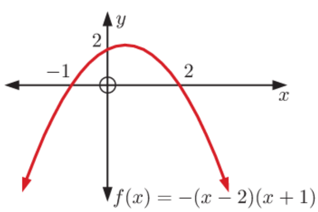
b **i** $a = 1$
ii -3
iii 1 and -3



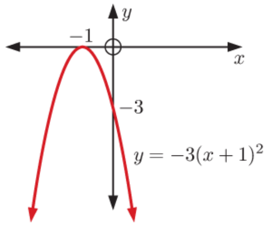
c **i** $a = 2$
ii 8
iii -2 (touching)



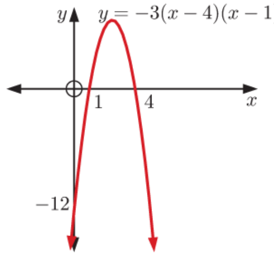
d **i** $a = -1$
ii 2
iii 2 and -1



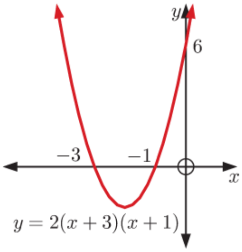
e **i** $a = -3$ **ii** -3
iii -1 (touching)



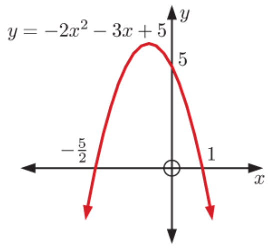
f **i** $a = -3$ **ii** -12
iii 4 and 1



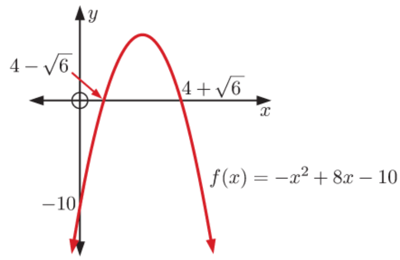
g **i** $a = 2$ **ii** 6
iii -3 and -1



h **i** $a = -2$ **ii** 5
iii $-\frac{5}{2}$ and 1



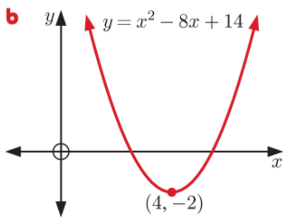
i **i** $a = -1$
ii -10
iii $4 \pm \sqrt{6}$



EXERCISE 20C.2

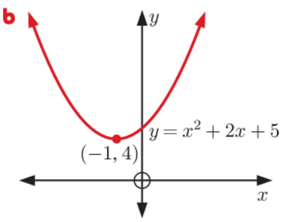
1 **a** $\Delta = 32$, $\Delta > 0$ $\therefore$ graph cuts x-axis twice.
b $\Delta = -39$, $\Delta < 0$ $\therefore$ graph does not cut the x-axis.
c $\Delta = 8$, $\Delta > 0$ $\therefore$ graph cuts x-axis twice.
d $\Delta = 0$, $\therefore$ graph touches x-axis.
e $\Delta = -24$, $\Delta < 0$ $\therefore$ graph does not cut the x-axis.
f $\Delta = 0$, $\therefore$ graph touches x-axis.

2 **a** $y = (x-4)^2 - 2$
b



c two x-intercepts
d $\Delta = 8$ which is > 0

3 **a** $y = (x+1)^2 + 4$
b

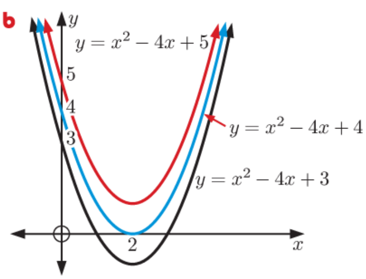


c no x-intercepts
d $\Delta = -16$ which is < 0

4 **a** C **b** F **c** A **d** B **e** E **f** D

5 **a** **i** $\Delta > 0$, cuts the x-axis twice
ii $\Delta = 0$, touches the x-axis
iii $\Delta < 0$, does not cut the x-axis

b



6 **a** If $a > 0$ the graph opens upwards. 

A negative value for c means the graph has a negative y-intercept. $\therefore$ the graph cuts the x-axis twice.

b If $a > 0$ and $c < 0$, then $-4ac > 0$. We know that $b^2 \geqslant 0$ and so $b^2 - 4ac > 0$. $\therefore$ the graph cuts the x-axis twice.

EXERCISE 20D

1 **a** $x = 2$ **b** $x = 1$ **c** $x = -2$ **d** $x = \frac{3}{2}$ **e** $x = -\frac{5}{2}$ **f** $x = 2$

2 **a** $x = 3$ **b** $x = 2$ **c** $x = 0$ **d** $x = -\frac{5}{2}$ **e** $x = -4$ **f** $x = \frac{3}{2}$

3 **a** $x = -2$ **b** $x = \frac{3}{2}$ **c** $x = -\frac{2}{3}$ **d** $x = -2$ **e** $x = \frac{5}{4}$ **f** $x = 10$ **g** $x = -6$ **h** $x = 12\frac{1}{2}$ **i** $x = 150$

EXERCISE 20E

1 **a** **i** V(2, -2) **ii** minimum **iii** $\{y \mid y \geqslant -2\}$

b **i** V(-1, -4) **ii** minimum **iii** $\{y \mid y \geqslant -4\}$

c **i** V(0, 4) **ii** minimum **iii** $\{y \mid y \geqslant 4\}$

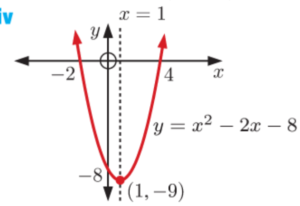
d **i** V(0, 1) **ii** maximum **iii** $\{y \mid y \leqslant 1\}$

e **i** V(-2, 0) **ii** maximum **iii** $\{y \mid y \leqslant 0\}$

f **i** V($\frac{5}{2}$, $-\frac{19}{2}$) **ii** minimum **iii** $\{y \mid y \geqslant -9\frac{1}{2}\}$

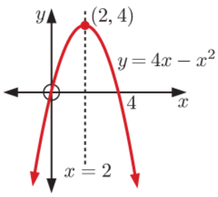
2 **a** **i** x-intercepts -2 and 4, y-intercept -8

ii $x = 1$ **iii** V(1, -9)

iv 

b **i** x-intercepts 0 and 4, y-intercept 0

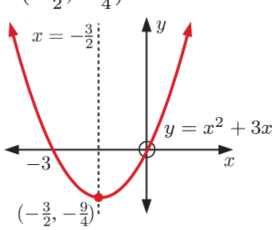
ii $x = 2$ **iii** V(2, 4)

iv 

c **i** x-intercepts 0 and -3, y-intercept 0

ii $x = -\frac{3}{2}$

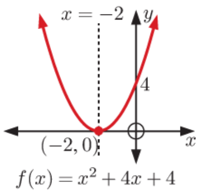
iii V($-\frac{3}{2}$, $-\frac{9}{4}$)

iv 

d **i** x-intercept -2, y-intercept 4

ii $x = -2$

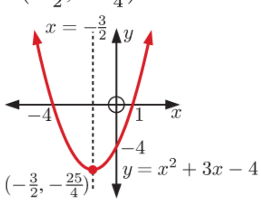
iii V(-2, 0)

iv 

e **i** x-intercepts -4 and 1, y-intercept -4

ii $x = -\frac{3}{2}$

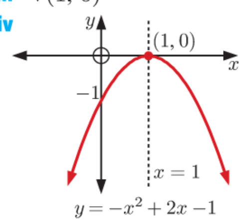
iii V($-\frac{3}{2}$, $-6\frac{1}{4}$)

iv 

f **i** x-intercept 1, y-intercept -1

ii $x = 1$

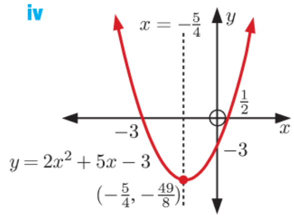
iii V(1, 0)

iv 

g **i** x-intercepts -3 and $\frac{1}{2}$, y-intercept -3

ii $x = -\frac{5}{4}$

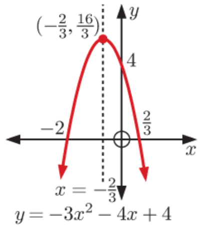
iii V($-\frac{5}{4}$, $-\frac{49}{8}$)

iv 

h **i** x-intercepts -2 and $\frac{2}{3}$, y-intercept 4

ii $x = -\frac{2}{3}$

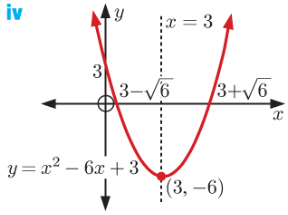
iii V($-\frac{2}{3}$, $\frac{16}{3}$)

iv 

i **i** x-intercepts $3 \pm \sqrt{6}$, y-intercept 3

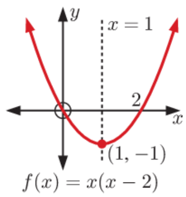
ii $x = 3$

iii V(3, -6)

iv 

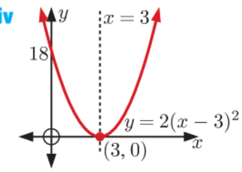
3 **a** **i** x-intercepts 0 and 2, y-intercept 0

ii $x = 1$ **iii** V(1, -1)

iv 

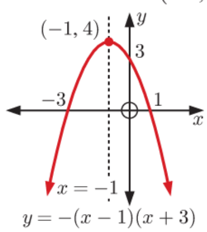
b **i** x-intercept 3, y-intercept 18

ii $x = 3$ **iii** V(3, 0)

iv 

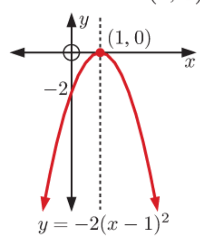
c **i** x-intercepts -3 and 1, y-intercept 3

ii $x = -1$ **iii** V(-1, 4)

iv 

d **i** x-intercept 1, y-intercept -2

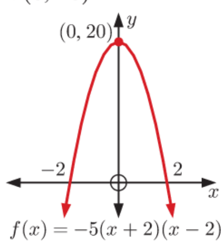
ii $x = 1$ **iii** V(1, 0)

iv 

e **i** x-intercepts -2 and 2, y-intercept 20

ii $x = 0$

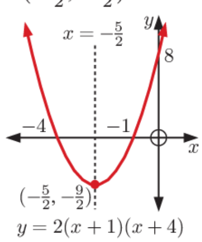
iii V(0, 20)

iv 

f **i** x-intercepts -1 and -4, y-intercept 8

ii $x = -\frac{5}{2}$

iii V($-\frac{5}{2}$, $-\frac{9}{2}$)

iv 

EXERCISE 20F

1 **a** 2 seconds **b** 20 m **c** 4 seconds

2 **a** 25 bicycles **b** €425

c €200 (Due to fixed daily costs such as wages and electricity.)

3 **a** 60 km h^{-1} (when $t = 0$) **b** $t = 1$ s **c** 66 km h^{-1}

4 **a** 30 taxis **b** \$1600/hour **c** \$200

5 **a** 25°C **b** 7:00 am the next day **c** -11°C

6 **a** 2 m **b** 3.8 m **c** 1 s

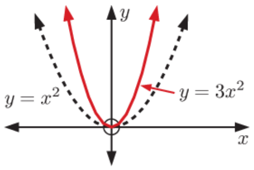
7 **a** **Hint:** [FB] $\parallel$ [EC] **b** $\dfrac{\text{BF}}{2} = \dfrac{1-x}{1}$ etc.

c Area $= x(2(1-x))$ etc. **d** **i** $x = \frac{1}{2}$ **ii** $\frac{1}{2}$ cm^2

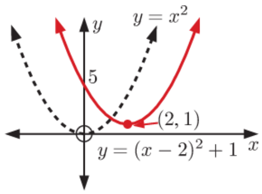
REVIEW SET 20A

1 **a** $y = -11$ **b** $x = 6$ or -3

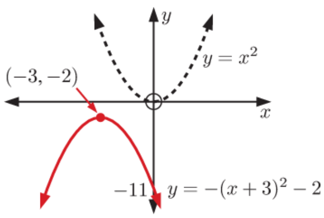
2 **a**



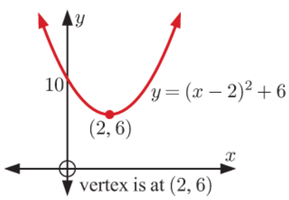
b



c

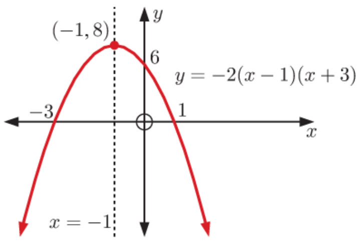


3 **a** $y = (x-2)^2 + 6$ **b**



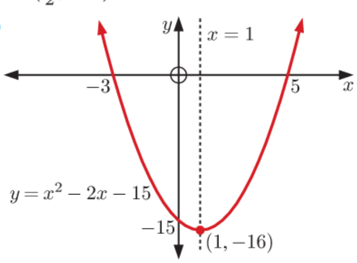
4 **a** 0 and -4 **b** -7 and 4 **5** **a** $x = -\frac{3}{2}$ **b** $x = 2$

6 **a** **i** downwards **ii** 6 **iii** -3 and 1 **iv** $x = -1$ **b**



7 **a** V(4, -19) **b** V($\frac{1}{2}$, -2)

8 **a** **i** -15 **ii** -3 and 5 **iii** $x = 1$ **iv** V(1, -16) **b**



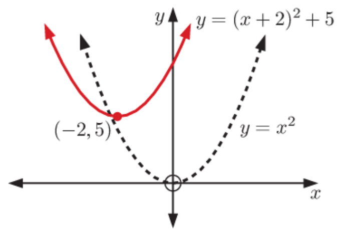
9 **a** unmarked side $= (40 - 2x)$ m
$\therefore$ $A = x(40 - 2x)$ m^2, etc. **b** $x = 10$ **c** 200 m^2

10 **a** $\Delta < 0$, does not cut the x-axis
b $\Delta > 0$, cuts the x-axis twice
c $\Delta = 0$, touches the x-axis

REVIEW SET 20B

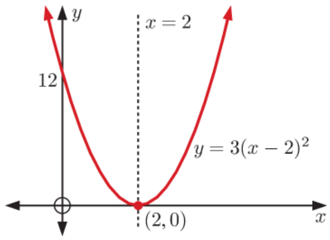
1 $x = -7$ or 6 **2** No, as $f(2) = 4 - 6 + 8 = 6 \neq 5$.

3



4 **a** $\dfrac{1 \pm \sqrt{61}}{6}$ **b** $1 \pm \sqrt{7}$

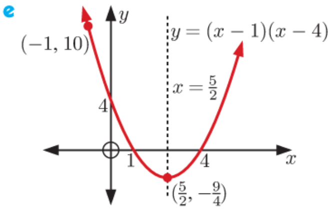
5



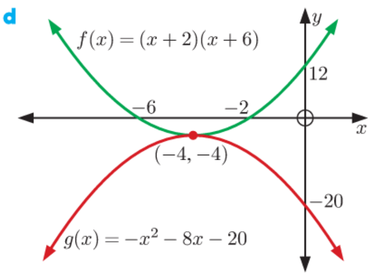
6 **a** $x = 1$ **b** $x = \frac{5}{6}$

7 **a** V(2, -3) **b** $\{y \mid y \leqslant -3\}$

8 **a** $f(-1) = 10$
b x-intercepts 1 and 4, y-intercept 4
c $x = \frac{5}{2}$
d V($\frac{5}{2}$, $-\frac{9}{4}$)
e



9 **a** $f(x)$ has x-intercepts -6 and -2 and its y-intercept is 12. $g(x)$ does not cut the x-axis and its y-intercept is -20.
b Both $f(x)$ and $g(x)$ have axis of symmetry $x = -4$. Both $f(x)$ and $g(x)$ have vertex $(-4, -4)$.
c Range of $f(x)$ is $\{y \mid y \geqslant -4\}$. Range of $g(x)$ is $\{y \mid y \leqslant -4\}$.
d



10 **a** If $c = 0$, the y-intercept of the graph is 0. This means the graph passes through the origin, or touches the x-axis at (0, 0). So the graph cuts the x-axis at least once (it cuts twice if $b \neq 0$).
b If $c = 0$, then $-4ac = 0$ also. That means the discriminant $b^2 - 4ac = b^2$, which is always $\geqslant 0$. Hence, the graph cuts the x-axis at least once. (It touches the x-axis if $b^2 = 0$.)

EXERCISE 21A

1 **a** $\frac{\pi}{6}$ **b** $\frac{\pi}{2}$ **c** $\frac{\pi}{3}$ **d** $\frac{\pi}{4}$ **e** $\frac{2\pi}{3}$
f $\frac{3\pi}{4}$ **g** $\frac{\pi}{18}$ **h** $\frac{5\pi}{4}$ **i** $\frac{7\pi}{6}$ **j** $\frac{7\pi}{4}$

2 **a** $\approx 1.15^c$ **b** $\approx 0.401^c$ **c** $\approx 0.122^c$ **d** $\approx 1.99^c$
e $\approx 3.80^c$

3 **a** 45° **b** 30° **c** 36° **d** 72° **e** 10°
f 50° **g** 540° **h** 20° **i** 100° **j** 720°
k 240° **l** 300° **m** 420° **n** 330° **o** 675°

4 **a** $\approx 57.3°$ **b** $\approx 172°$ **c** $\approx 28.6°$ **d** $\approx 44.7°$
e $\approx 123°$

5 **a**

Deg	0	45	90	135	180	225	270	315	360
Rad	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Deg	0	30	60	90	120	150	180	210
Rad	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$

Deg	240	270	300	330	360
Rad	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

6 **a** **i** $l = \left(\frac{\theta}{2\pi}\right) \times 2\pi r$ **ii** $A = \left(\frac{\theta}{2\pi}\right) \times \pi r^2$

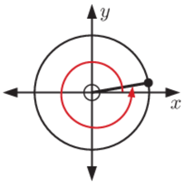
b 30 cm, 150 cm^2 **c** 17.5 cm^2

EXERCISE 21B

1 **a** A$(\cos 67°, \sin 67°)$, B$(\cos 148°, \sin 148°)$, C$(\cos 281°, \sin 281°)$, D$(\cos(-24°), \sin(-24°))$

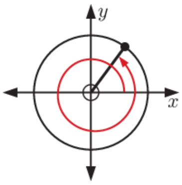
b A$(0.391, 0.921)$, B$(-0.848, 0.530)$, C$(0.191, -0.982)$, D$(0.914, -0.407)$

2 **a**



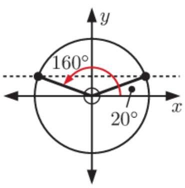
same point on unit circle
$\therefore$ same x-coordinate
$\therefore$ $\cos 380° = \cos 20°$

b



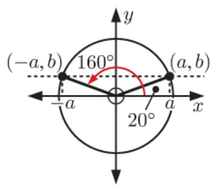
same point on unit circle
$\therefore$ same y-coordinate
$\therefore$ $\sin 413° = \sin 53°$

c



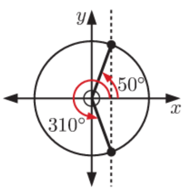
same y-coordinate on unit circle
$\therefore$ $\sin 160° = \sin 20°$

d



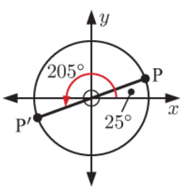
$\cos 160° = -a$
$= -\cos 20°$

e



same x-coordinate on unit circle
$\therefore$ $\cos 310° = \cos 50°$

f



[OP] and [OP′] have the same gradient
$\therefore$ $\tan 25° = \tan 205°$

3 **a** $\cos 0° = 1$, $\sin 0° = 0$ **b** $\cos 90° = 0$, $\sin 90° = 1$
c $\cos 2\pi = 1$, $\sin 2\pi = 0$ **d** $\cos 450° = 0$, $\sin 450° = 1$
e $\cos(-\frac{\pi}{2}) = 0$, $\sin(-\frac{\pi}{2}) = -1$
f $\cos(-180°) = -1$, $\sin(-180°) = 0$

4 **a** **i** ≈ -0.454 **ii** ≈ -0.891 **iii** ≈ 0.809 **iv** ≈ -0.588
b **i** ≈ 1.96 **ii** ≈ -0.727

5 **a** $\cos 49°$ is positive, $\sin 49°$ is positive, $\tan 49°$ is positive.
b $\cos 158°$ is negative, $\sin 158°$ is positive, $\tan 158°$ is negative.
c $\cos 207°$ is negative, $\sin 207°$ is negative, $\tan 207°$ is positive.
d $\cos 296°$ is positive, $\sin 296°$ is negative, $\tan 296°$ is negative.
e $\cos \frac{3\pi}{4}$ is negative, $\sin \frac{3\pi}{4}$ is positive, $\tan \frac{3\pi}{4}$ is negative.
f $\cos \frac{\pi}{6}$ is positive, $\sin \frac{\pi}{6}$ is positive, $\tan \frac{\pi}{6}$ is positive.
g $\cos \frac{5\pi}{3}$ is positive, $\sin \frac{5\pi}{3}$ is negative, $\tan \frac{5\pi}{3}$ is negative.
h $\cos \frac{7\pi}{6}$ is negative, $\sin \frac{7\pi}{6}$ is negative, $\tan \frac{7\pi}{6}$ is positive.

6 **a**

Quadrant	*Degree*	*Radian measure*
1	$0° < \theta < 90°$	$0 < \theta < \frac{\pi}{2}$
2	$90° < \theta < 180°$	$\frac{\pi}{2} < \theta < \pi$
3	$180° < \theta < 270°$	$\pi < \theta < \frac{3\pi}{2}$
4	$270° < \theta < 360°$	$\frac{3\pi}{2} < \theta < 2\pi$

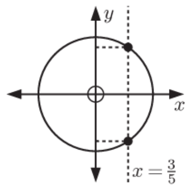
Quadrant	$\cos\theta$	$\sin\theta$	$\tan\theta$
1	+ ve	+ ve	+ ve
2	− ve	+ ve	− ve
3	− ve	− ve	+ ve
4	+ ve	− ve	− ve

b **i** 3 and 4 **ii** 1 and 4 **iii** 3 **iv** 4

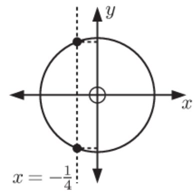
7 $\tan(180° - \theta) = \dfrac{\sin(180° - \theta)}{\cos(180° - \theta)} = \dfrac{\sin\theta}{-\cos\theta} = -\tan\theta$

EXERCISE 21C

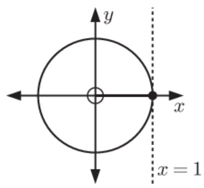
1 **a** $\sin\theta = \pm\frac{4}{5}$



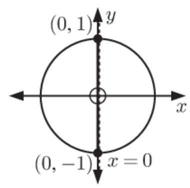
b $\sin\theta = \pm\frac{\sqrt{15}}{4}$



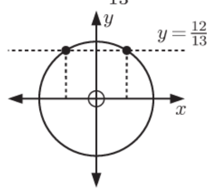
c $\sin\theta = 0$



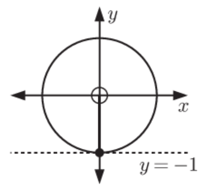
d $\sin\theta = \pm 1$



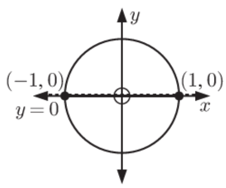
2 **a** $\cos\theta = \pm\frac{5}{13}$



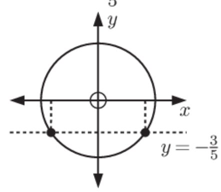
b $\cos\theta = 0$



c $\cos\theta = \pm 1$



d $\cos\theta = \pm\frac{4}{5}$



3 **a** $\cos\theta = \frac{\sqrt{5}}{3}$ **b** $\cos\theta = -\frac{3}{5}$ **c** $\cos\theta = -\frac{2\sqrt{2}}{3}$
d $\cos\theta = \frac{12}{13}$

4 **a** $\sin\theta = \frac{4}{5}$ **b** $\sin\theta = -\frac{\sqrt{15}}{4}$ **c** $\sin\theta = \frac{\sqrt{7}}{4}$
d $\sin\theta = -\frac{12}{13}$

5 **a** $\cos\theta = -\frac{\sqrt{55}}{8}$ **b** $\tan\theta = \frac{3}{\sqrt{55}}$

EXERCISE 21D

1 **a** $\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1$ **c** $0, 1, 0$
d $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -1$ **e** $1, 0,$ undefined **f** $\frac{\sqrt{3}}{2}, -\frac{1}{2}, -\sqrt{3}$
g $-1, 0,$ undefined **h** $0, -1, 0$
i $-\frac{1}{2}, -\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$ **j** $-\frac{\sqrt{3}}{2}, -\frac{1}{2}, \sqrt{3}$ **k** $-\frac{1}{2}, \frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{3}}$
l $0, 1, 0$ **m** $-\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3}$ **n** $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -1$
o $1, 0,$ undefined **p** $0, -1, 0$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{3\sqrt{3}}{8}$ **e** $\frac{1}{4}$ **f** 3

3 **a** $\frac{5\pi}{6}$ **b** $\frac{5\pi}{4}$ **c** $\frac{\pi}{6}$ **d** $\frac{7\pi}{4}$

EXERCISE 21E.1

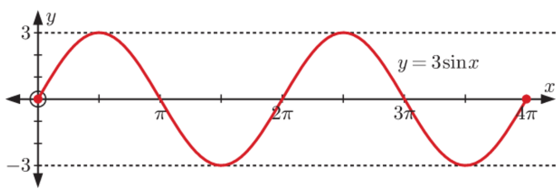
1 **a** 0
b **i** $\theta = 0, \pi, 2\pi, 3\pi, 4\pi$ **ii** $\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$
iii $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ **iv** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$
c $\theta \approx \frac{\pi}{12}, \approx \frac{11\pi}{12}, \approx \frac{25\pi}{12}, \approx \frac{35\pi}{12}$
d **i** $0 < \theta < \pi, \quad 2\pi < \theta < 3\pi$ **e** $\{y \mid -1 \leqslant y \leqslant 1\}$
ii $\pi < \theta < 2\pi, \quad 3\pi < \theta < 4\pi$

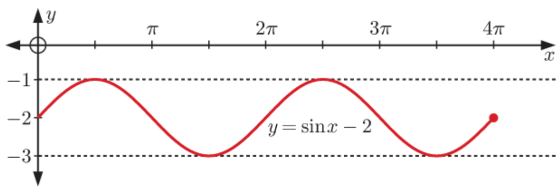
2 **a** 1
b **i** $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ **ii** $\theta = 0, 2\pi, 4\pi$
iii $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ **iv** $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$
c $\theta \approx \frac{5\pi}{12}, \approx \frac{19\pi}{12}, \approx \frac{29\pi}{12}, \approx \frac{43\pi}{12}$
d **i** $0 \leqslant \theta < \frac{\pi}{2}, \quad \frac{3\pi}{2} < \theta < \frac{5\pi}{2}, \quad \frac{7\pi}{2} < \theta \leqslant 4\pi$
ii $\frac{\pi}{2} < \theta < \frac{3\pi}{2}, \quad \frac{5\pi}{2} < \theta < \frac{7\pi}{2}$
e $\{y \mid -1 \leqslant y \leqslant 1\}$

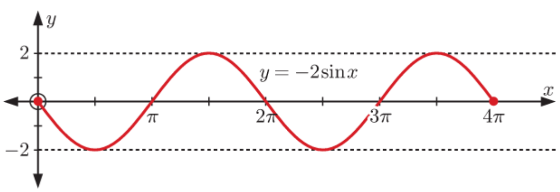
EXERCISE 21E.2

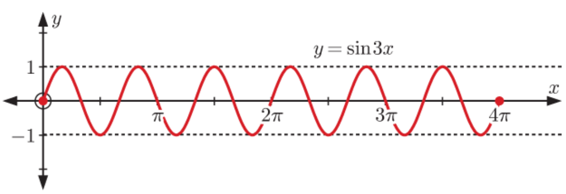
1 **a** 4 **b** 2 **c** $\frac{1}{3}$ **2** **a** $\frac{2\pi}{3}$ **b** $\frac{\pi}{2}$ **c** 4π

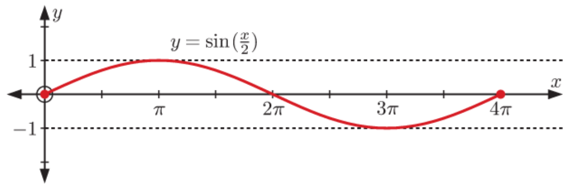
3 **a** $y = -3$ **b** $y = 5$ **c** $y = 0$

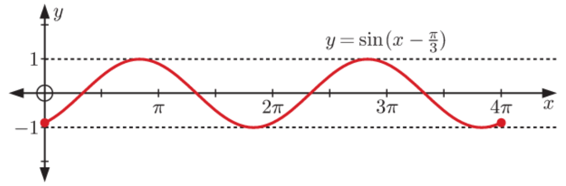
4 **a**


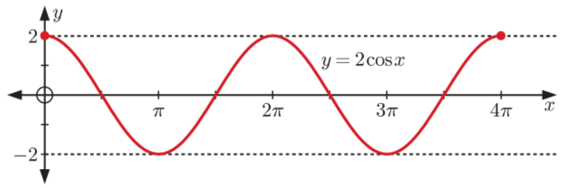
b


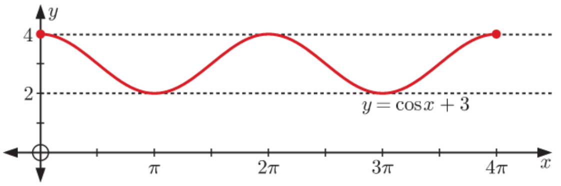
c


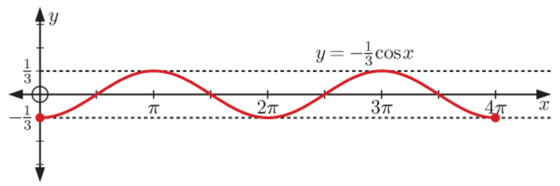
d


e


f


5 **a**


b


c


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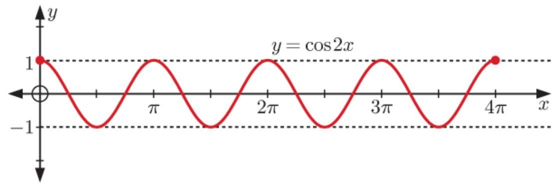
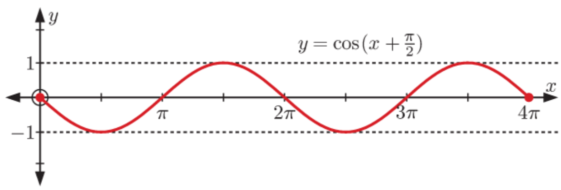
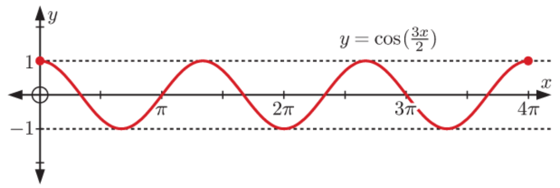
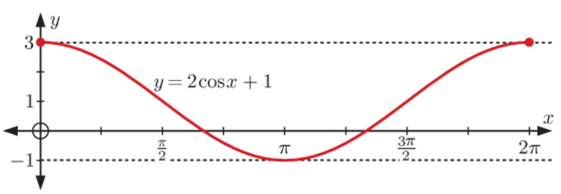
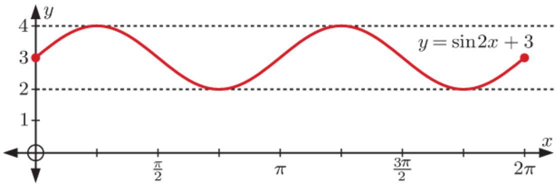


e


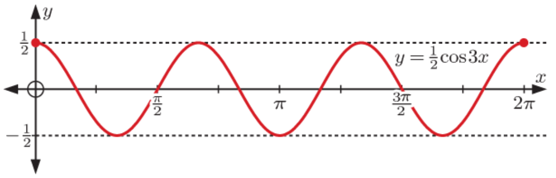
f


6 **a**


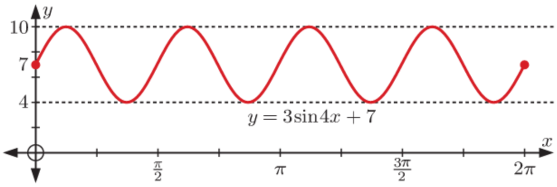
b



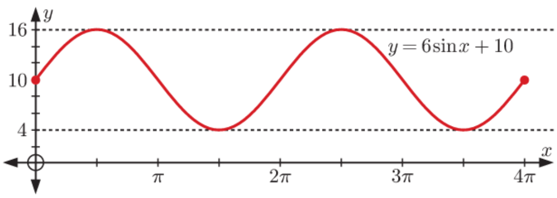
c



d



7 **a**

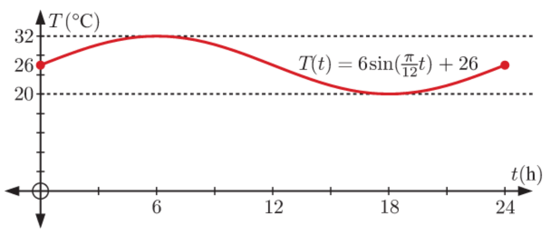


b $y = 13$ **c** maximum value $= 16$, when $x = \frac{\pi}{2}$ and $\frac{5\pi}{2}$

d minimum value $= 4$, when $x = \frac{3\pi}{2}$ and $\frac{7\pi}{2}$

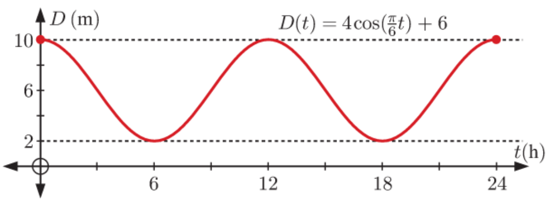
8 **a** **i** 6 **ii** $T = 26$ **iii** 24

b



c **i** 26°C **ii** 29°C **d** 32°C, at 6 pm

9 **a**



b highest $= 10$ m, at midnight, midday, and midnight the next day

lowest $= 2$ m, at 6 am and 6 pm

c no (water height is 4 m)

EXERCISE 21F

1 **a** $2\cos\theta$ **b** $7\sin\theta$ **c** $3\sin\theta$ **d** $2\sin\theta$
e $-3\cos\theta$ **f** $5\cos\theta$

2 **a** 5 **b** -3 **c** -1 **d** 3 **e** $7\cos^2\theta$
f $6\sin^2\theta$ **g** $-\cos^2\theta$ **h** $-6\sin^2\theta$ **i** $-7\cos^2\theta$

3 **a** $\cos^2\theta - 5\cos\theta$ **b** $4\sin\theta - \sin^2\theta$
c $\cos^2\theta + 2\cos\theta - 3$ **d** $-\cos^2\theta$
e $4 + 4\sin\theta + \sin^2\theta$ **f** $\sin^2\alpha - 6\sin\alpha + 9$
g $\cos^2\alpha - 8\cos\alpha + 16$ **h** $1 - 2\sin\phi\cos\phi$
i $-1 + 2\cos\alpha - \cos^2\alpha$

4 **a** $(1+\sin\phi)(1-\sin\phi)$ **b** $(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)$
c $(\cos\beta+1)(\cos\beta-1)$ **d** $\sin\beta(3\sin\beta-1)$
e $3\cos\phi(2+\cos\phi)$ **f** $2\sin\theta(2\sin\theta-1)$
g $(\sin\theta+2)(\sin\theta+4)$ **h** $(\cos\theta+6)(\cos\theta+1)$
i $(4\cos\alpha-1)(2\cos\alpha+1)$

5 **a** 1 **b** $2\sin\theta$ **c** $-\sin\theta$
d $1+\cos\alpha$ **e** $\sin\theta - 1$ **f** $\dfrac{1}{\cos\alpha+\sin\alpha}$
g $\cos\theta - \sin\theta$ **h** $\dfrac{1}{\cos\phi - \sin\phi}$ **i** $2\cos\theta$

EXERCISE 21G

1 **a** $\theta = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ **b** $\theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ **c** $\theta = \frac{\pi}{3}$ or $\frac{4\pi}{3}$
d $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ **e** $\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$ **f** $\theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$
g $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ **h** $\theta = \pi$ **i** $\theta = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

2 **a** $\theta = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$, or $\frac{15\pi}{4}$ **b** $\theta = \frac{\pi}{2}$ or $\frac{5\pi}{2}$
c $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$, or $\frac{11\pi}{3}$ **d** $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
e $\theta = \frac{5\pi}{6}$ or $-\frac{5\pi}{6}$ **f** $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}, \frac{19\pi}{4}$, or $\frac{23\pi}{4}$

3 **a** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$
c $\theta = \frac{\pi}{2}, \pi$, or $\frac{3\pi}{2}$ **d** $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$, or 2π
e $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$, or 2π **f** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$

EXERCISE 21H

1 **a** $\sin\theta$ **b** $-4\sin\theta$ **c** $6\cos\theta$ **d** $11\sin\theta$
e $\cos^2\alpha$ **f** $\sin^2\alpha$ **g** 1

3 **a** $2\cos\theta$ **b** $-3\cos\theta$ **c** $2\sin\theta$ **d** $-9\sin\theta$
e $7\sin\theta$ **f** $7\cos\theta$

4 $\tan(90° - \theta) \times \tan\theta = 1$

5 This proof is acceptable for θ being an acute angle only. Another method is required for a general proof.

EXERCISE 21I

2 **a** $\cos\alpha$ **b** $\sin\beta$ **c** $\sin A$ **d** $-\sin\alpha$ **e** $-\sin\beta$
f $-\sin\theta$ **g** $\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$ **h** $\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha$
i $\frac{1}{\sqrt{2}}(\sin\theta + \cos\theta)$ **j** $\frac{1}{\sqrt{2}}(\cos\phi - \sin\phi)$
k $\frac{1}{\sqrt{2}}(\cos C - \sin C)$ **l** $\frac{\sqrt{3}}{2}\cos A + \frac{1}{2}\sin A$

3 **a** $\cos(A-B)$ **b** $\sin(\alpha-\beta)$ **c** $\cos(M+N)$
d $\sin(C+D)$ **e** $2\sin(\alpha-\beta)$ **f** $-\cos(\alpha+\beta)$

4 **a** Use the expansions of $\sin(A+B)$ and $\cos(A+B)$ letting B be A.
b **i** $-\frac{24}{25}$ **ii** $\frac{7}{25}$ **iii** $-\frac{24}{7}$

5 $\dfrac{1+\sqrt{3}}{2\sqrt{2}}$

6 **a** $\overrightarrow{OP} = \begin{pmatrix}\cos B \\ \sin B\end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix}\cos A \\ \sin A\end{pmatrix}$ **b** Use $\overrightarrow{OP} \bullet \overrightarrow{OQ}$.

REVIEW SET 21A

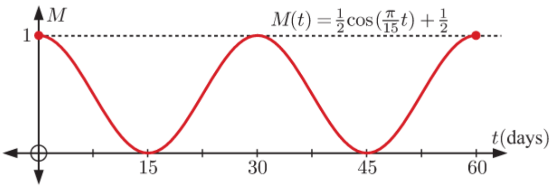
1 **a** $\frac{\pi}{12}$ **b** $\frac{\pi}{8}$ **2** **a** 18° **b** 140°

3 **a** $\sin\theta = \pm\frac{\sqrt{5}}{3}$ **b** $\sin\theta = \pm\frac{2}{\sqrt{13}}$

4 **a** $\frac{1}{2}$ **b** 3 **5** **a** $5, \pi$ **b** $\frac{1}{4}, 2\pi$ **c** $4, 4\pi$

6 **a** $4\sin^2\theta$ **b** $\dfrac{\cos\theta}{\cos\theta + 2}$ **c** $\dfrac{1}{\cos\theta\sin\theta}$

7 **a**



b **i** 0.75 **ii** 0.25 **iii** ≈ 0.835 **iv** ≈ 0.165

c Once every 30 days. **d** January 16, February 15

8 **a** $\sin^2\theta - 3\sin\theta$ **b** $\cos^2\theta + 10\cos\theta + 25$

9 **a** $\theta = \frac{2\pi}{3}$ or $\frac{5\pi}{3}$ **b** $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

c $\theta = \frac{\pi}{6}$ or $\frac{11\pi}{6}$

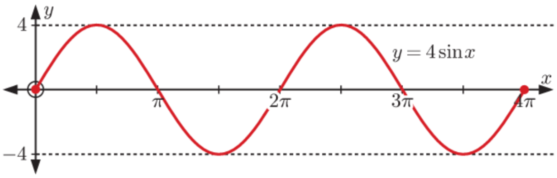
REVIEW SET 21B

1 **a** $P(\cos 237°, \sin 237°)$ **b** $P(-0.545, -0.839)$

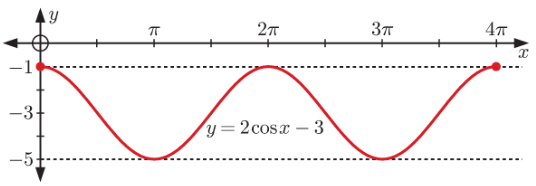
2 $\cos\frac{7\pi}{4}$ is positive, $\sin\frac{7\pi}{4}$ is negative, $\tan\frac{7\pi}{4}$ is negative

3 **a** $\cos\theta = -\frac{1}{\sqrt{5}}$ **b** $\cos\theta = \frac{2\sqrt{10}}{7}$ **4** $150°$

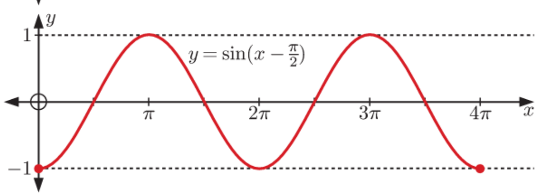
5 **a**



b

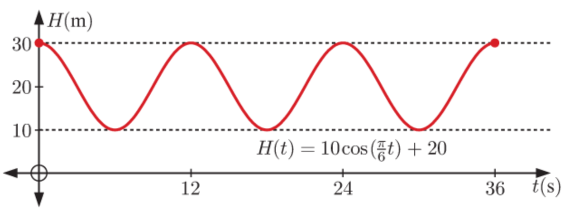


c



6 **a** $4\sin\theta(\sin\theta - 3)$ **b** $(\cos\theta - 6)(\cos\theta + 3)$

7 **a**



b 20 m **c** 10 m **d** 12 seconds

8 **a** $3\cos\theta$ **b** $-\sin^3\phi$ **c** $3\sin\theta$

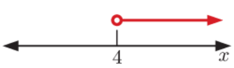
9 **a** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$, or $\frac{10\pi}{3}$ **b** $\theta = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$, or $\frac{19\pi}{6}$

c $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{5\pi}{2}, \frac{7\pi}{2}$, or $\frac{11\pi}{3}$

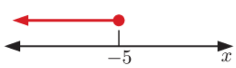
10 **a** $-\cos\theta$ **b** $-\cos\phi$ **c** $\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

EXERCISE 22A

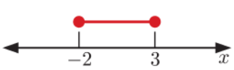
1 **a**



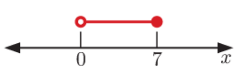
b



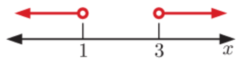
c



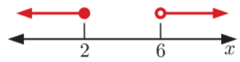
d



e



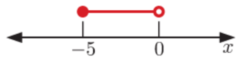
f



g

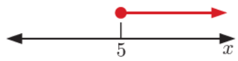
h

i

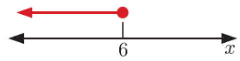


j

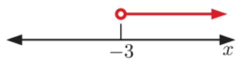
k



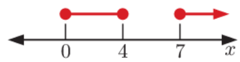
l



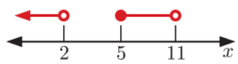
m



n



o



2 **a** $\{x \mid x \geqslant 6\}$ **b** $\{x \mid x < 3\}$ **c** $\{x \mid -2 \leqslant x < 1\}$

d $\{x \mid x \leqslant 0$ or $x > 1\}$ **e** $\{x \mid x < -1$ or $x > 1\}$

f $\{x \mid 0 < x \leqslant 4\}$ **g** $\{x \mid x < -2$ or $1 < x \leqslant 4\}$

h $\{x \mid -2 \leqslant x \leqslant 1$ or $x > 3\}$

3 **a** $[-1, 6]$ **b** $]0, 5[$ **c** $]-4, 7]$ **d** $[4, 8[$

e $]-7, \infty[$ **f** $]-\infty, 0]$ **g** $]-\infty, 2] \cup [5, \infty[$

h $]-\infty, -3[\cup]4, \infty[$ **i** $]-1, 1] \cup [2, \infty[$

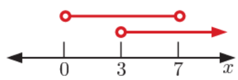
j $]-\infty, -4[\cup [2, 7[$ **k** $[3, 8[$ **l** $[-2, 7[$

m $[5, \infty[$ **n** $]-\infty, -2[$ **o** $]-\infty, 2] \cup]3, 5[$

p $[-4, 1] \cup]4, \infty[$ **q** $]-\infty, -2] \cup]2, \infty[$

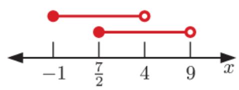
r $]-\infty, -3[\cup]10, \infty[$

4 **a**



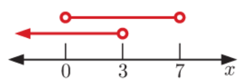
$\{x \mid x > 0\}$

b



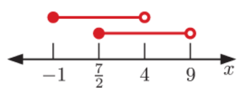
$\{x \mid -1 \leqslant x < 9\}$

c



$\{x \mid 0 < x < 3\}$

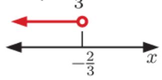
d



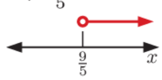
$\{x \mid \frac{7}{2} \leqslant x < 4\}$

EXERCISE 22B

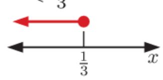
1 **a** $x < -\frac{2}{3}$



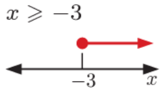
b $x > \frac{9}{5}$



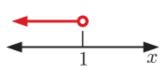
c $x \leqslant \frac{1}{3}$



d $x \geqslant -3$

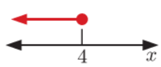


e $x < 1$

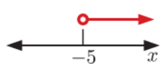


f $x \leqslant -\frac{1}{5}$

2 **a** $x \leqslant 4$

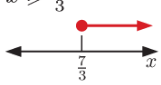


b $x > -5$

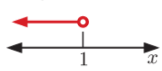


c $x > -4$

d $x \geqslant \frac{7}{3}$

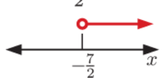


e $x < 1$

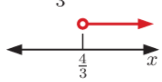


f $x \leqslant \frac{3}{5}$

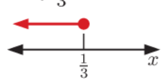
3 **a** $x > -\frac{7}{2}$



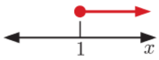
b $x > \frac{4}{3}$



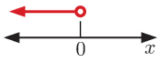
c $x \leqslant \frac{1}{3}$



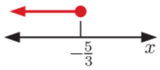
d $x \geqslant 1$



e $x < 0$



f $x \leqslant -\frac{5}{3}$



EXERCISE 22C

1 **a** $-\ \underset{3}{|}\ +\quad x$

b $+\ \underset{0}{|}\ -\ \underset{1}{|}\ +\quad x$

c $-\ \underset{-2}{|}\ +\ \underset{5}{|}\ -\quad x$

d $-\ \underset{-2}{|}\ +\ \underset{0}{|}\ -\ \underset{2}{|}\ +\quad x$

e $-\ \underset{-3}{|}\ -\quad x$

f $+\ \underset{2}{|}\ +\quad x$

g $+\ \underset{-2}{|}\ +\ \underset{4}{|}\ -\quad x$

h $-\ \underset{0}{\vdots}\ +\quad x$

i $+\ \underset{-5}{|}\ -\ \underset{1}{\vdots}\ +\quad x$

2 **a** $+\ \underset{-3}{|}\ -\ \underset{1}{|}\ +\quad x$

b $+\ \underset{0}{|}\ -\ \underset{4}{|}\ +\quad x$

c $+\ \underset{-5}{|}\ -\ \underset{0}{|}\ +\quad x$

d $-\ \underset{-2}{|}\ +\ \underset{3}{|}\ -\quad x$

e $-\ \underset{\frac{1}{3}}{|}\ +\ \underset{4}{|}\ -\quad x$

f $+\ \underset{\frac{1}{2}}{|}\ -\ \underset{3}{|}\ +\quad x$

g $+\ \underset{-4}{|}\ -\ \underset{4}{|}\ +\quad x$

h $-\ \underset{-1}{|}\ +\ \underset{1}{|}\ -\quad x$

i $-\ \underset{0}{|}\ +\ \underset{2}{|}\ -\quad x$

j $+\ \underset{1}{|}\ -\ \underset{3}{|}\ +\quad x$

k $-\ \underset{-\frac{1}{3}}{|}\ +\ \underset{\frac{1}{3}}{|}\ -\quad x$

l $-\ \underset{-4}{|}\ +\ \underset{6}{|}\ -\quad x$

m $+\ \underset{-\frac{1}{3}}{|}\ -\ \underset{3}{|}\ +\quad x$

n $-\ \underset{-1}{|}\ +\ \underset{\frac{1}{2}}{|}\ -\quad x$

o $-\ \underset{-\frac{3}{5}}{|}\ +\ \underset{\frac{3}{2}}{|}\ -\quad x$

3 **a** $+\ \underset{1}{|}\ +\quad x$

b $+\ \underset{-4}{|}\ +\quad x$

c $-\ \underset{-3}{|}\ -\quad x$

d $-\ \underset{2}{|}\ -\quad x$

e $+\ \underset{\frac{1}{3}}{|}\ +\quad x$

f $+\ \underset{3}{|}\ +\quad x$

g $-\ \underset{-2}{|}\ -\quad x$

h $-\ \underset{1}{|}\ -\quad x$

i $-\ \underset{-\frac{1}{2}}{|}\ -\quad x$

4 **a** $+\ \underset{-1}{|}\ -\ \underset{2}{\vdots}\ +\quad x$

b $+\ \underset{0}{\vdots}\ -\ \underset{1}{|}\ +\quad x$

c $-\ \underset{-\frac{5}{2}}{|}\ +\ \underset{2}{\vdots}\ -\quad x$

d $-\ \underset{\frac{1}{3}}{|}\ +\ \underset{4}{\vdots}\ -\quad x$

e $+\ \underset{-4}{|}\ -\ \underset{\frac{2}{3}}{\vdots}\ +\quad x$

f $-\ \underset{0}{|}\ +\ \underset{1}{\vdots}\ -\quad x$

g $-\ \underset{-1}{\vdots}\ +\ \underset{2}{|}\ +\quad x$

h $-\ \underset{-3}{\vdots}\ -\ \underset{0}{|}\ +\quad x$

i $+\ \underset{-2}{|}\ -\ \underset{0}{|}\ +\ \underset{4}{\vdots}\ -\quad x$

j $+\ \underset{-4}{|}\ -\ \underset{2}{\vdots}\ +\ \underset{3}{|}\ -\quad x$

k $+\ \underset{-1}{\vdots}\ -\ \underset{2}{\vdots}\ +\ \underset{5}{|}\ -\quad x$

l $+\ \underset{-1}{|}\ -\ \underset{0}{\vdots}\ +\ \underset{1}{|}\ -\quad x$

m $-\ \underset{-3}{|}\ +\ \underset{-1}{\vdots}\ -\ \underset{1}{|}\ +\quad x$

n $+\ \underset{1}{\vdots}\ +\quad x$

o $+\ \underset{-1}{\vdots}\ -\ \underset{\frac{3}{2}}{\vdots}\ +\ \underset{3}{|}\ +\quad x$

5 **a** **i** $\{x \mid -3 < x < 6\}$ **ii** $\{x \mid -3 \leqslant x \leqslant 6\}$
iii $\{x \mid x < -3 \text{ or } x > 6\}$ **iv** $\{x \mid x \leqslant -3 \text{ or } x \geqslant 6\}$

b **i** $\{x \mid x < -1 \text{ or } x > 2\}$ **ii** $\{x \mid x \leqslant -1 \text{ or } x > 2\}$
iii $\{x \mid -1 < x < 2\}$ **iv** $\{x \mid -1 \leqslant x < 2\}$

c **i** $\{x \mid -4 < x < 0 \text{ or } 0 < x < 5\}$
ii $\{x \mid -4 < x \leqslant 5\}$ **iii** $\{x \mid x < -4 \text{ or } x > 5\}$
iv $\{x \mid x < -4 \text{ or } x = 0 \text{ or } x \geqslant 5\}$

EXERCISE 22D

1 **a** $+\ \underset{-3}{|}\ -\ \underset{2}{|}\ +\quad x$

b **i** $x < -3$ or $x > 2$ **ii** $x \leqslant -3$ or $x \geqslant 2$
iii $-3 < x < 2$ **iv** $-3 \leqslant x \leqslant 2$

2 **a** $1 \leqslant x \leqslant 3$ **b** $-\frac{3}{2} < x < 4$ **c** $x < -1$ or $x > 2$
d no solutions **e** $x \leqslant 0$ or $x \geqslant 2$ **f** $-\frac{1}{2} < x < 0$
g $-4 < x < 4$ **h** $-2 \leqslant x \leqslant 2$ **i** $x < -5$ or $x > 1$
j $-1 \leqslant x \leqslant 2$ **k** no solutions **l** $-7 \leqslant x \leqslant 4$
m $x \in \mathbb{R},\ x \neq 1$ **n** $-1 \leqslant x \leqslant \frac{5}{2}$ **o** $x \leqslant -\frac{4}{3}$ or $x \geqslant 2$
p $x < -2$ or $x > \frac{2}{5}$ **q** $\frac{4}{3} < x < \frac{3}{2}$ **r** $x = \frac{2}{3}$

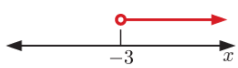
3 **a** $-2 < x < 3$ **b** $x < -3$ or $x > 2$
c $x \leqslant -4$ or $x > \frac{1}{2}$ **d** $0 < x \leqslant 3$
e $-3 < x < 0$ or $x > 4$ **f** $x < -1$ or $0 \leqslant x < 1$
g $x < -3$ or $x \geqslant -1$ **h** $x < \frac{3}{2}$ or $x > 5$
i $0 < x < \frac{1}{3}$ **j** $x < -\frac{1}{3}$ or $x \geqslant -\frac{2}{7}$
k $-\frac{1}{2} < x < 1$ **l** $-2 \leqslant x < 0$ or $x \geqslant 2$

4 **a** $\{x \mid x \leqslant 0 \text{ or } x \geqslant 2\}$ **b** $\{x \mid x \leqslant 0 \text{ or } x \geqslant 3\}$
c $\{x \mid x \leqslant -1 \text{ or } x \geqslant 3\}$ **d** $\{x \mid x < -2 \text{ or } x > 1\}$
e $\{x \mid x \leqslant -3 \text{ or } x > 1\}$
f $\{x \mid -5 \leqslant x < -2 \text{ or } x > 2\}$

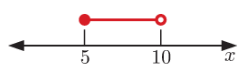
5 **a** $-9 < m < -1$ **b** $m = -9$ or $m = -1$
c $m < -9$ or $m > -1$ and $m \neq 0$

REVIEW SET 22A

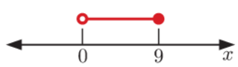
1 **a**



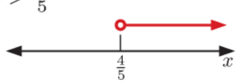
b



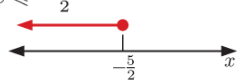
c



2 **a** $x > \frac{4}{5}$



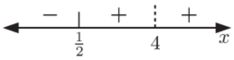
b $x \leqslant -\frac{5}{2}$



3 **a** $x > \frac{10}{3}$ **b** $x \geqslant -\frac{7}{9}$

4 **a** $+\ \underset{-4}{|}\ -\ \underset{1}{|}\ +\quad x$

b $-\ \underset{-3}{|}\ +\ \underset{5}{|}\ -\quad x$

c 

5 **a** $x < -1$ or $x > 6$ **b** $x \leqslant -1$ or $x > 6$
c $-1 < x < 6$ **d** $-1 \leqslant x < 6$

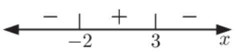
6 **a** $-6 < x < -2$ **b** $x \leqslant -9$ or $x \geqslant 4$
c $-\frac{3}{2} < x < 5$

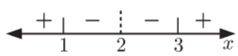
7 **a** $-4 < x < 0$ **b** $x > -2$ **c** $-1 < x < \frac{1}{3}$

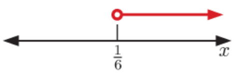
8 $-3 < x < 3$

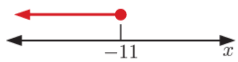
REVIEW SET 22B

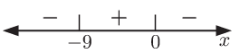
1 **a** $[-2, 8[$ **b** $]-\infty, 1] \cup]4, 5]$ **c** $]-3, 1[$
d $]-\infty, -7] \cup]3, \infty[$

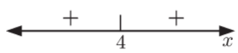
2 **a** 

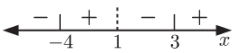
b 

3 **a** $x > \frac{1}{6}$ 

b $x \leqslant -11$ 

4 **a** 

b 

c 

5 **a** $x \geqslant \frac{8}{5}$ **b** $-7 < x < 5$ **c** $x < -3$ or $x > 2$

6 **a** $1 < x < 3$ **b** $1 < x \leqslant 3$
c $x < -6$ or $-6 < x < 1$ or $x > 3$
d $x < 1$ or $x \geqslant 3$

7 **a** $x \leqslant -5$ or $x \geqslant 6$ **b** $x < -5$ or $x > 0$
c $x < -\frac{7}{2}$ or $x > -1$

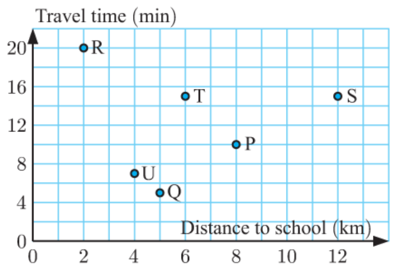
8 **a** $\{x \mid x \leqslant 0$ or $x \geqslant 4\}$ **b** $\{x \mid x < -2$ or $x > \frac{1}{3}\}$

EXERCISE 23A

1 **a** D **b** C **c** E and H

2 **a** E **b** A **c** B and C
d Yes, generally more hours of training results in higher productivity (from the graph).

3 **a** D **b** F **c** 2 **d** 3

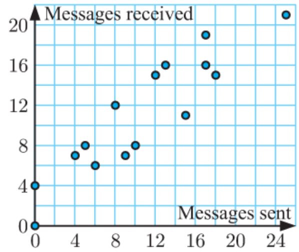
4 **a** 

b S
c Q
d Student R. They live the shortest distance from school, yet take the most time to get there.

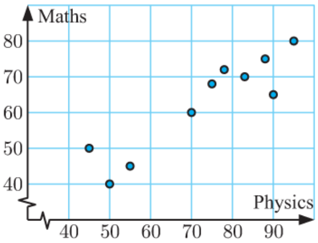
EXERCISE 23B

1 **a** **i** negative association **ii** linear **iii** strong
b **i** no association **ii** not linear
c **i** positive association **ii** linear **iii** moderate
d **i** positive association **ii** linear **iii** weak
e **i** positive association **ii** not linear **iii** moderate
f **i** negative association **ii** not linear **iii** strong

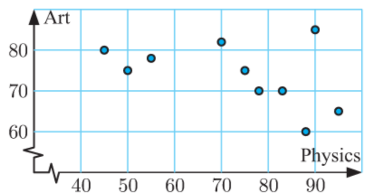
2 **a** "...... as x increases, y increases"
b "...... as T increases, d decreases"
c "...... randomly scattered"

3 **a** 

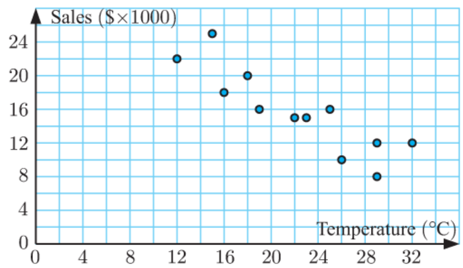
b A moderate, positive, linear correlation.

4 **a** **i** 

ii A moderate, positive, linear correlation.

b 

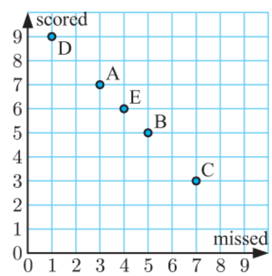
A very weak, negative, linear correlation (virtually zero correlation).

5 **a** 

b A moderate, linear, negative correlation.

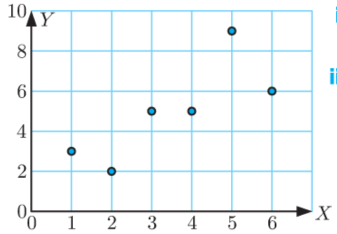
EXERCISE 23C

1 **a** positive correlation as $r > 0$ **b** $r^2 \approx 0.659$
c moderate positive linear correlation, $0.50 \leqslant r^2 < 0.75$

2 **a** 

b $r = -1$
c perfect negative linear correlation
The number of shots scored is directly related to those missed. Out of 10 shots, each time the number of goals scored increases by 1, the number of misses decreases by 1.

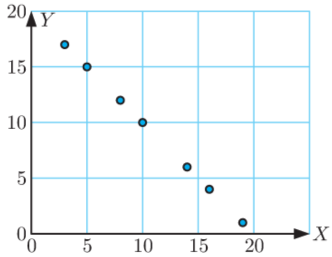
3 **a** **i**



ii $r \approx 0.786$, $r^2 \approx 0.617$

iii moderate positive linear correlation

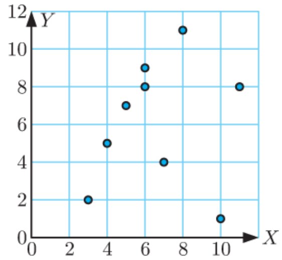
b **i**



ii $r = -1$, $r^2 = 1$

iii perfect negative linear correlation

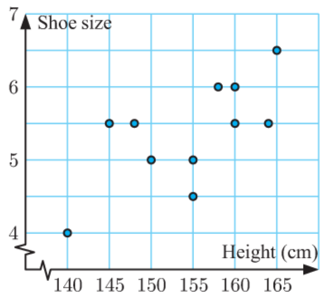
c **i**



ii $r \approx 0.146$, $r^2 \approx 0.0215$

iii very weak positive linear correlation

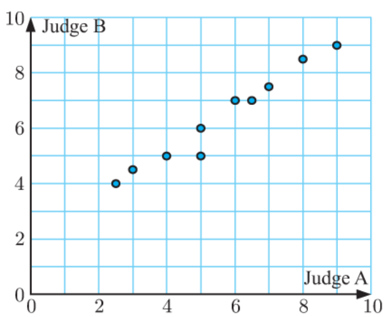
4 **a**



b $r \approx 0.673$ $r^2 \approx 0.452$

c A weak, positive, linear correlation exists between *Shoe size* and *Height*.

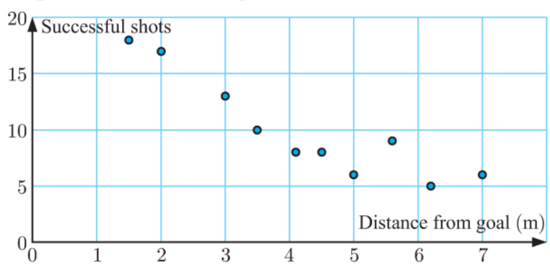
5 **a**



b $r \approx 0.981$, $r^2 \approx 0.962$

c A very strong, positive, linear correlation exists between Judge B's scores and Judge A's scores.

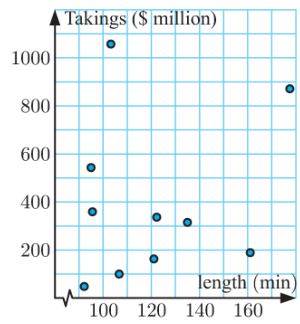
6 **a**



b negative **c** $r \approx -0.911$, $r^2 \approx 0.830$

d There is a strong, negative correlation between the number of successful shots and the distance from goal.

7 **a**



b No, there appears to be almost no correlation between the variables.

c $r \approx 0.191$, $r^2 \approx 0.037$

d There is a very weak positive correlation (almost no correlation) between movie length and box office performance.

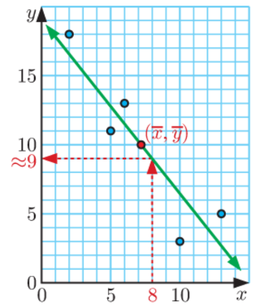
EXERCISE 23D.1

1 **a** $\overline{x} = 7.2$, $\overline{y} = 10$

c negatively correlated

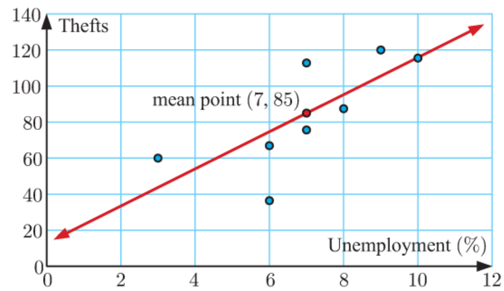
e $x = 8$, $y \approx 9$

b, d



2 **a** $\overline{x} = 7$, $\overline{y} = 85$ **b** $r \approx 0.739$

c, d, e

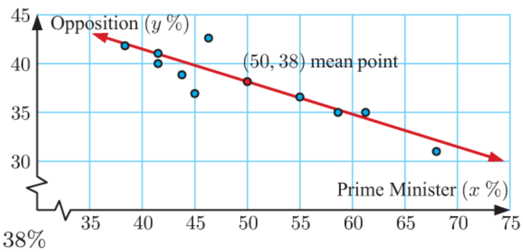


f **i** ≈ 168 thefts

ii Unreliable as it is an extrapolation well beyond the upper pole.

3 **a** $\overline{x} = 50$, $\overline{y} = 38$

b

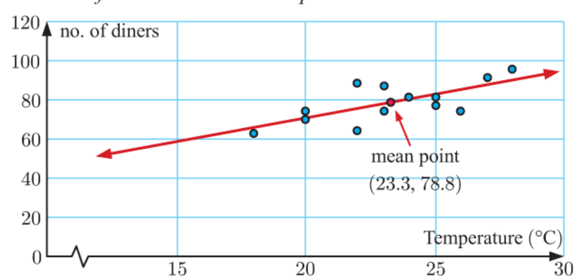


c 38%

4 **a** $\overline{x} = 23.3$, $\overline{y} = 78.8$

b $r^2 \approx 0.509$ There is a moderate correlation between *number of diners* and *noon temperature*.

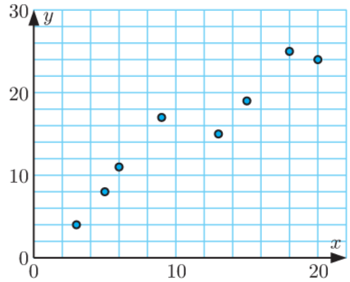
c



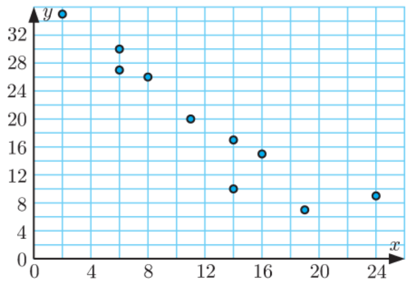
d **i** 73 diners **ii** 56 diners

e We expect the first estimate to be reliable as it is an interpolation, but the second may not be reliable as it is an extrapolation.

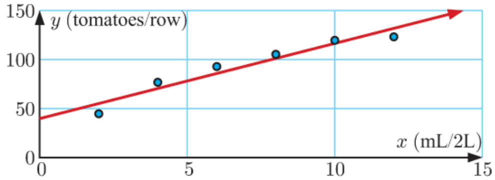
EXERCISE 23D.2

1 **a** **i** 

ii $y \approx 1.12x + 2.90$

b **i** 

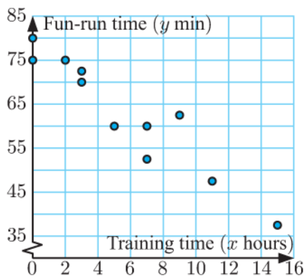
ii $y \approx -1.34x + 35.7$

2 **a** 

b $y \approx 7.66x + 40.1$

c The y-intercept indicates that a row can be expected to produce on average 40.1 tomatoes when no spray is applied.

d 94 tomatoes/row. Even though this is an interpolation, this seems low compared with the yield at 6 and 8 mL/2 L. Looking at the graph it would appear that the relationship is not linear.

3 **a** 

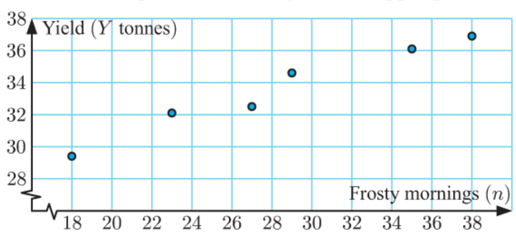
b $r \approx -0.952$, $r^2 \approx 0.907$

c $y \approx -2.69x + 77.9$

d The gradient indicates that an increase in training time of 1 hour will decrease the fun-run time by ≈ 2.7 minutes.

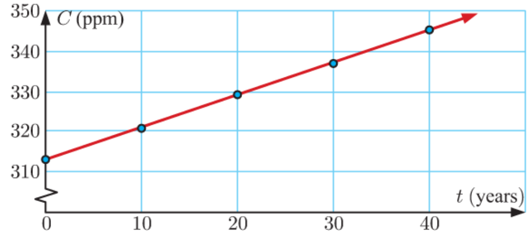
e **i** -2.8 minutes

ii This value is clearly absurd as one cannot record a negative time for a fun-run. It is very unreliable as it is an extrapolation well beyond the upper pole.

4 **a** 

b $Y \approx 0.371n + 23.1$ **c** ≈ 34.6 tonnes

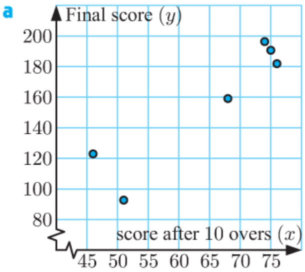
d "...... , the *greater* the yield of cherries."

5 **a** 

b $r = 1$, perfect positive linear correlation

c $C \approx 0.8t + 313$ **d** 335 parts per million

e 361 parts per million

6 **a** 

b $r \approx 0.922$, $r^2 \approx 0.851$, strong positive linear correlation

c $y \approx 2.90x - 30.9$

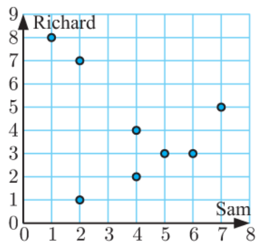
d ≈ 143 runs. This should be fairly reliable as it is interpolated.

REVIEW SET 23A

1 There is a weak, negative correlation between the variables.

2 **a** E **b** A

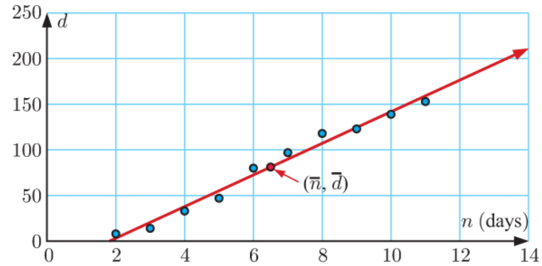
c The greater the time spent on player fitness, the smaller the chance of player injury.

3 **a** 

b $r \approx -0.334$

$r^2 \approx 0.112$

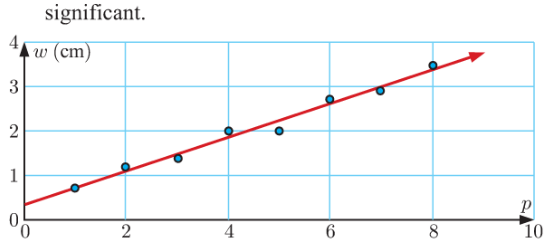
There is a very weak, negative correlation between the variables.

4 **a, b** 

b $\overline{n} = 6.5$, $\overline{d} = 81.2$

c **i** About 210 diagnosed cases.

ii Very unreliable as 14 is outside the poles. The medical team have probably isolated those infected at this stage and there could be a downturn which may be very significant.

5 **a** 

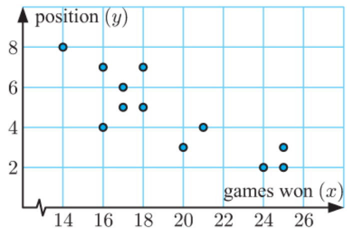
b $r \approx 0.990$ **c** $w \approx 0.381p + 0.336$

d **i** ≈ 5.7 cm

ii As $p = 14$ is outside the poles, this prediction could be unreliable.

6 **a** Negatively correlated. The more games won, the higher the team's position on the ladder (and so the value for *Position* is smaller).

b



c $r \approx -0.843$, $r^2 \approx 0.711$

d $y \approx -0.454x + 13.4$ **e** 3rd

REVIEW SET 23B

1 **a** J **b** E **c** D and G **d** C

2 **a** **i** D **ii** H

b As the number of pages increases, the number of chapters generally increases.

3 **a** **i** Positive, as numbers get larger they have more possible factors.

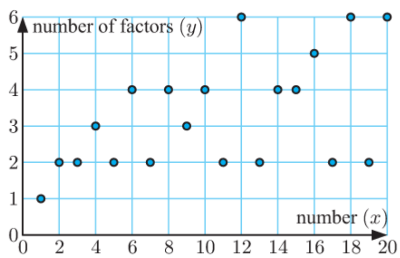
ii Weak, the larger the number, the more prime, square, cubic, etc. numbers you encounter that have very small numbers of factors.

b

Number	1	2	3	4	5	6	7	8	9	10
Number of factors	1	2	2	3	2	4	2	4	3	4

Number	11	12	13	14	15	16	17	18	19	20
Number of factors	2	6	2	4	4	5	2	6	2	6

c

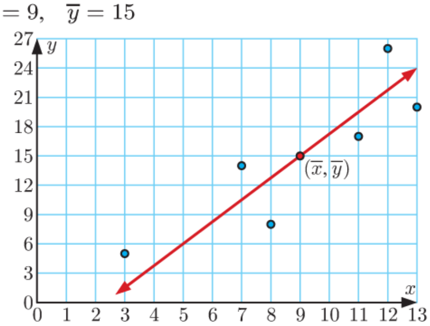


d $r \approx 0.531$, $r^2 \approx 0.282$

e weak, positive, linear correlation

4 **a** $\overline{x} = 9$, $\overline{y} = 15$

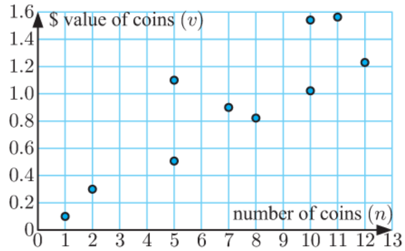
b, d



c positively correlated

e $y \approx 6$. Reasonably accurate by interpolation.

5 **a**



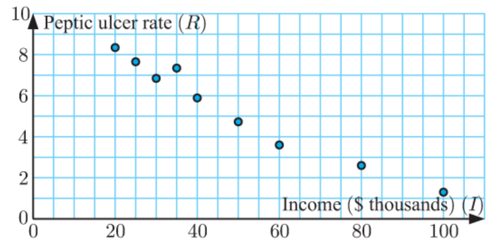
b $v \approx 0.114n + 0.100$

c Each coin added to a wallet or purse increases the value by 11.4 cents on average.

d **i** $\approx \$2.38$

ii 20 coins is too far beyond the upper pole to extrapolate with any reliable accuracy.

6 **a**



b $R \approx -0.0907I + 9.79$ **c** when $I = 55$, $R \approx 4.80$

d \$120 000 gives a rate of -1.09 which is meaningless. So, \$120 000 is outside the data range of this model.

e **i** The point $(35, 7.3)$, as it does not follow the general trend of the data in that area.

ii $R \approx -0.0887I + 9.61$; when $I = 55$, $R \approx 4.73$

EXERCISE 24A

1 **a** polynomial **b** polynomial

c not a polynomial, one term has x as the index

d polynomial

e not a polynomial, the index in the second term is not an integer

f not a polynomial, the last term has a negative index

2 **a** 4 **b** 2 **c** 5 **d** 2 **e** 3 **f** 4

3 **a** -4 **b** 6 **c** -12 **d** 0

EXERCISE 24B.1

1 **a** **i** $3x^2 + 3x - 3$ **ii** $-x^2 - 3x + 5$

b **i** $3x^3 - 5x^2 + x + 7$ **ii** $-x^3 - 3x^2 + 3x - 5$

c **i** $-5x^4 + x^3 - 9x^2 - x - 4$

ii $-5x^4 - x^3 + 7x^2 + 3x + 4$

d **i** $2x^4 - 4x^3 + 2x + 5$ **ii** $2x^4 + 2x^3 + 12x - 11$

2 **a** $3x^2 + 9x - 6$ **b** $-8x^3 + 4x + 16$

c $-x^2 - 3x + 2$ **d** $-8x^3 + x^2 + 7x + 14$

e $6x^3 + 2x^2 + 3x - 16$ **f** $-4x^3 - x^2 - x + 10$

3 **a** $3x^2 + 5x - 2$ **b** $2x^3 - 9x^2 + x + 12$

c $4x^5 - 21x^4 + 21x^3 - 2x^2 - 10x + 8$

d $-2x^7 - x^5 + 17x^4 + x^3 + 2x^2 - 35x$

4 **a** $3x^4 - 9x^3 + 12x - 3$ **b** $-10x^3 + 15x^2 - 30$

c $2x^4 - 9x^2 + 8x + 16$

d $-x^4 + 11x^3 - 12x^2 - 4x + 25$

e $4x^4 - 16x^3 + 6x^2 + 16x - 16$

f $2x^7 - 9x^6 + 9x^5 + 14x^4 - 32x^3 + 3x^2 + 24x - 6$

EXERCISE 24B.2

1 **a** quotient is $x + 1$, remainder is 2

b quotient is $2x+1$, remainder is 3
c quotient is $x-4$, remainder is 14
d quotient is x^2+x+3, remainder is 5
e quotient is x^2+3x-4, remainder is -5
f quotient is x^2-2, remainder is 11

2 a $x-1+\frac{2}{x-3}$ b $3x-2-\frac{3}{x+1}$ c $x+2+\frac{13}{x-2}$
d $x^2-5x-14-\frac{63}{x-4}$ e $x^2-2x+9-\frac{22}{x+2}$
f $2x^3-x^2+4x-10+\frac{26}{x+3}$

EXERCISE 24B.3

1 a, b $3x+8+\frac{11}{x-2}$

2 a $x-2+\frac{3}{x-1}$ b $x-3+\frac{13}{x+3}$
c $2x^2+3x+10+\frac{17}{x-2}$ d $3x^2-2x+2-\frac{4}{x+1}$
e $2x^2-10x+42-\frac{200}{x+5}$
f $x^3-2x^2+3x+2+\frac{6}{x-3}$

EXERCISE 24C

1 a 11 b -10 c 46 2 5 ✓
3 a $a=-2$ b $a=4$ c $a=5$
4 a $a=-4$, $b=7$ b $a=-5$, $b=6$ c $a=2$, $b=4$

EXERCISE 24D

1 a factor b not a factor c factor d not a factor
2 a $c=2$ b $c=-2$ c $b=3$
3 a $a=1$, $b=10$ b $p=\frac{12}{7}$, $q=\frac{75}{7}$
4 a $f(3)=0$ b x^2+x-20
c $f(x)=(x-3)(x^2+x-20)$
d $f(x)=(x-3)(x-4)(x+5)$

REVIEW SET 24A

1 a polynomial
b not a polynomial, the last term has a negative index
c not a polynomial, the index in the second term is not an integer
2 a $8x^2+6x+3$ b $7x^2-9x+9$
c $15x^4+32x^3+29x-4$
3 a 1 b -53
4 a $x^3-2x^2-9x+10$ b $-10x^2-20x+15$
c $2x^5+4x^4-13x^3-6x^2+43x-21$
5 a quotient is $2x+5$, remainder is 3
b quotient is x^2-4x+2, remainder is -5
6 a not a factor b factor 7 $k=6$ 8 $a=\frac{8}{7}$, $b=\frac{174}{7}$
9 a i $f(3)=-1$ ii 2 iii x^3-x^2+4x-6
b quotient is x^2-x, remainder is -1
By the Remainder Theorem, if $P(x)$ divided by $(x-k)$ has remainder R, then $P(k)=R$. In this case, $f(3)=-1$, and $f(x)$ divided by $(x-3)$ has remainder -1.
10 a $f(-5)=0$ b x^2-4x+3
c $f(x)=(x+5)(x^2-4x+3)$
d $f(x)=(x+5)(x-1)(x-3)$

REVIEW SET 24B

1 3 2 a $x^3+x^2-10x+6$ b x^3-x^2+2x+4
3 $x^3-2x^2+x+4-\frac{6}{x-4}$ 4 a -5 b 24 5 -103
6 a $5x^3+25x-10$
b $2x^7+7x^5-4x^4-14x^3+6x^2+5x-2$
7 $c=3$ 8 not a factor 9 $a=4$, $b=-1$
10 a $a=6$ b $f(2)=0$ d $f(x)=(x+1)(x-2)^3$

EXERCISE 25A.1

1 a -2 b -4 c -3 2 a -4 b -1 c ≈ -0.4
3 a ≈ 1.4 b ≈ 2.8 c ≈ 0.5
4 a ≈ 0.5 b ≈ 0.25 c ≈ 0.2

EXERCISE 25A.2

1 a 4 b 6 2 a -2 b -4 c -6
3 a 3 b 2 c -4 d $-\frac{3}{2}$
4 $y=x^3$ is not a quadratic function.

EXERCISE 25B.1

1 a 11 b 6 c 5 d -3 e 4 f 2
2 a 2 b -5 c 3 d 4 e -1 f 3
g -4 h -2 i 8

EXERCISE 25B.2

1 a $(3+h)^2$ b $\frac{(3+h)^2-9}{(3+h)-3}=6+h$ for $h\neq 0$
c i 7 ii 6.5 iii 6.1 iv 6.01 d 6
2 a 8 b 3 c -1 d 4
3 a 2 b
c $2a$

x-coordinate	Gradient of tangent
1	2
2	4
3	6
4	8

EXERCISE 25C

1 $f'(x)=1$
2 a $f'(x)=3x^2$ b $f'(2)=12$, gradient of tangent to $f(x)=x^3$ at $x=2$ is 12.
3 a $f'(x)=-\frac{1}{x^2}$
b $f'(-1)=-1$, gradient of tangent to $f(x)=\frac{1}{x}$ at $x=-1$ is -1.
$f'(3)=-\frac{1}{9}$, gradient of tangent to $f(x)=\frac{1}{x}$ at $x=3$ is $-\frac{1}{9}$.
4 a i $f'(x)=0$ ii $f'(x)=0$ b $f'(x)=0$
5 a $f'(x)=10x$ b $f'(x)=5g'(x)$
c $$f'(x)=\lim_{h\to 0}\frac{cg(x+h)-cg(x)}{h}=\lim_{h\to 0}c\left(\frac{g(x+h)-g(x)}{h}\right)=c\left(\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right)=cg'(x)$$
6 a i $f'(x)=4x$ ii $f'(x)=9$ iii $f'(x)=4x+9$
b $$f'(x)=\lim_{h\to 0}\frac{(g(x+h)+h(x+h))-(g(x)+h(x))}{h}$$
$$=\lim_{h\to 0}\frac{(g(x+h)-g(x))+(h(x+h)-h(x))}{h}$$
$$=\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}+\lim_{h\to 0}\frac{h(x+h)-h(x)}{h}$$
$$=g'(x)+h'(x)$$

7 **a**

$f(x)$	$f'(x)$
x^1	1
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
x^{-1}	$-x^{-2}$

b If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

EXERCISE 25D

1 **a** $6x^5$ **b** $9x^8$ **c** $15x^2$ **d** $-\dfrac{2}{x^3}$ **e** $5-2x$

f $-\frac{1}{2}x^{-\frac{3}{2}}$ **g** $3x^2+4$ **h** $2x^3$ **i** $2x$ **j** $1-\dfrac{1}{x^2}$

k $\dfrac{2}{x^2}$ **l** $-\dfrac{1}{x^2}$ **m** $1-\dfrac{5}{x^2}$ **n** $-\dfrac{20}{x^3}$ **o** $\dfrac{3}{\sqrt{x}}$

p $2x-1$ **q** $\dfrac{8}{x^3}$ **r** $-\dfrac{2}{x\sqrt{x}}$ **s** $9x^2-2+\dfrac{4}{x^3}$

t $-1+\dfrac{1}{x\sqrt{x}}$ **u** $3x^2+4x+1$ **v** $8x-4$

2 **a** $3x^2+4x-3$ **b** 11, 17 **c** ...... at $(2, 11)$ is 17.

3 **a** -6 **b** $-\frac{3}{2}$ **c** 5 **d** 3 **e** $\frac{3}{4}$ **f** $-\frac{1}{4}$

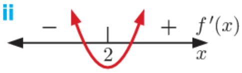
4 **a** -3 **b** $\frac{3}{4}$ **c** 0

5 **a** $(-2, 1)$ **b** $(1, 1)$ **c** $(-1, -1)$ and $(\frac{1}{3}, -\frac{23}{27})$
d $(2, 3)$ and $(-2, -1)$ **e** $(0, 7)$ and $(4, -25)$

EXERCISE 25E

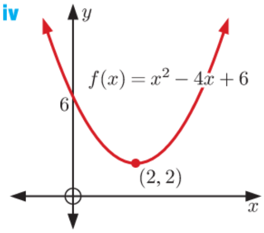
1 **a** **i** $f'(x) = 2x-4$

ii



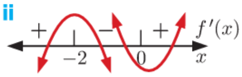
iii minimum turning point at $(2, 2)$

iv



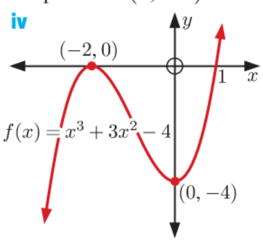
b **i** $f'(x) = 3x^2+6x$

ii



iii maximum turning point at $(-2, 0)$
minimum turning point at $(0, -4)$

iv

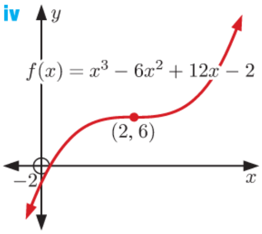


c **i** $f'(x)$
$= 3x^2-12x+12$

ii

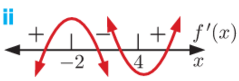
iii stationary inflection at $(2, 6)$

iv



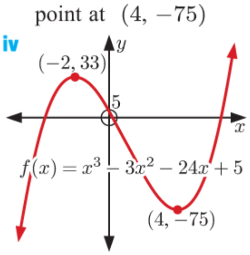
d **i** $f'(x)$
$= 3x^2-6x-24$

ii



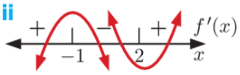
iii maximum turning point at $(-2, 33)$
minimum turning point at $(4, -75)$

iv



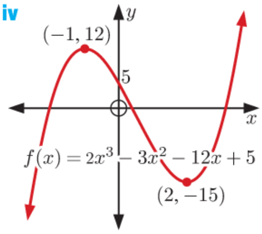
e **i** $f'(x)$
$= 6x^2-6x-12$

ii



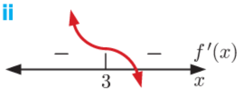
iii maximum turning point at $(-1, 12)$
minimum turning point at $(2, -15)$

iv



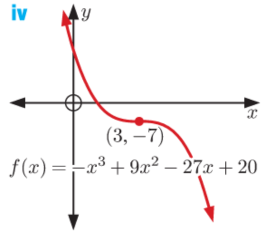
f **i** $f'(x)$
$= -3x^2+18x-27$

ii



iii stationary inflection at $(3, -7)$

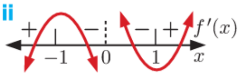
iv



g **i** $f'(x) = 1-\dfrac{1}{x^2}$

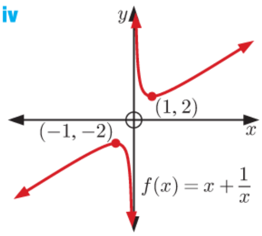
$= \dfrac{x^2-1}{x^2}$

ii



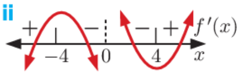
iii maximum turning point at $(-1, -2)$
minimum turning point at $(1, 2)$

iv



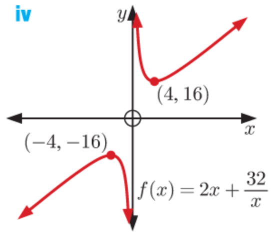
h **i** $f'(x)$
$= 2-\dfrac{32}{x^2}$
$= \dfrac{2(x+4)(x-4)}{x^2}$

ii



iii maximum turning point at $(-4, -16)$
minimum turning point at $(4, 16)$

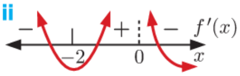
iv



i **i** $f'(x) = -\dfrac{1}{x^2}-\dfrac{2}{x^3}$

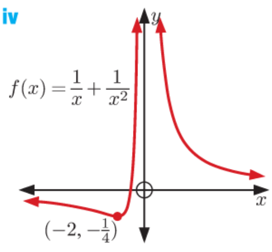
$= \dfrac{-(x+2)}{x^3}$

ii



iii minimum turning point at $(-2, -\frac{1}{4})$

iv



2 **a** $f'(x) = 2ax+b$ $\therefore$ $f'(x) = 0$ when $x = \dfrac{-b}{2a}$.

b **i** $a < 0$  **ii** $a > 0$ 

3 250, when both numbers are 5

4 **a** $V(x) = x(30-2x)(20-2x)$
$= 4x^3-100x^2+600x$ mL

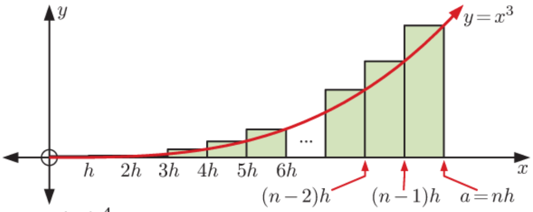
b Squares with sides about 3.92 cm.

5 **a** P is $(a, 9-a^2)$ **b** $0 < a < 3$ **c** $A = 18a-2a^3$
d $12\sqrt{3}$ units^2 when $a = \sqrt{3}$

EXERCISE 25F

1 **a** $A = a$ units^2 **b** $A = \dfrac{a^2}{2}$ units^2

2 **a**



b $S = \left(\frac{a}{n}\right)^4 \left(1^3 + 2^3 + 3^3 + 4^3 + ... + n^3\right)$

c $S = \frac{a^4}{n^4}\left[\frac{n^2(n+1)^2}{4}\right] = \frac{a^4}{4}\left[\frac{n^2(n+1)^2}{n^4}\right]$ etc.

d as $n \to \infty$, $\frac{1}{n} \to 0$ $\therefore$ $\left(1 + \frac{1}{n}\right)^2 \to 1$

$\therefore$ $\lim\limits_{n\to\infty} S = \frac{a^4}{4}$ So, the area is $\frac{a^4}{4}$ units^2.

3 **a** $A = \frac{a^5}{5}$ units^2 **b** $A = \frac{a^6}{6}$ units^2

EXERCISE 25G.1

1 **a** $F'(x) = x$ $\therefore$ $\int x\,dx = \frac{x^2}{2} + c$

b $F'(x) = x^4$ $\therefore$ $\int x^4\,dx = \frac{x^5}{5} + c$

c $F'(x) = 8x^7$ $\therefore$ $\int x^7\,dx = \frac{x^8}{8} + c$

d $F'(x) = -2x^{-3}$ $\therefore$ $\int x^{-3}\,dx = -\frac{x^{-2}}{2} + c$

e $F'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $\therefore$ $\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}} + c$

2 **a** $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ **b** no, cannot divide by 0

3 **a** $F'(x) = 5$ $\therefore$ $\int 5\,dx = 5x + c$ **b** $\int k\,dx = kx + c$

4 **a** $F'(x) = 6x^2 + 10x$

$\therefore$ $\int (6x^2 + 10x)\,dx = 2x^3 + 5x^2 + c$

b $6\int x^2\,dx + 10\int x\,dx = 6\left(\frac{x^3}{3} + c_1\right) + 10\left(\frac{x^2}{2} + c_2\right)$

$= 2x^3 + 5x^2 + c$

c $\int kf(x)\,dx = k\int f(x)\,dx$, k is constant

EXERCISE 25G.2

1 **a** $\frac{x^{11}}{11} + c$ **b** $\frac{1}{-3x^3} + c$ **c** $x^3 + c$ **d** $\frac{x^3}{3} + \frac{3x^2}{2} - 2x + c$

e $\frac{x^4}{4} - \frac{x^2}{2} + c$ **f** $\frac{10}{3}x^{\frac{3}{2}} + c$ **g** $\frac{x^3}{3} + 2\sqrt{x} + c$

h $\frac{x^4}{2} + \frac{3x^2}{2} + \frac{4}{x} + c$ **i** $\frac{x^2}{2} + \frac{3}{x} - \frac{2}{x^2} + c$

j $\frac{2}{5}x^2\sqrt{x} - 4\sqrt{x} + c$ **k** $\frac{x^3}{3} - 2x^2 + 4x + c$

l $\frac{4}{3}x^3 + 2x^2 + x + c$

2 **a** $\frac{x^4}{4} + 2x^3 + 6x^2 + 8x + c$

b $\frac{x^5}{5} - x^4 + 2x^3 - 2x^2 + x + c$

EXERCISE 25H

1 **a** $10\frac{1}{2}$ **b** 16 **c** 12 **d** $-7\frac{1}{2}$ **e** $4\frac{2}{3}$ **f** 8

2 **a** $2\frac{2}{3}$ **b** $52\frac{2}{3}$ **c** 3 **d** 0 **e** $-30\frac{1}{2}$ **f** $43\frac{3}{4}$

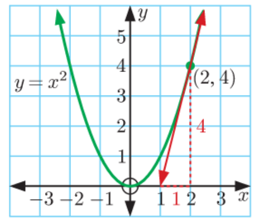
3 **a** $8\frac{2}{3}$ units^2 **b** $4\frac{2}{3}$ units^2 **c** 96.8 units^2 **d** 6 units^2

e $57\frac{1}{3}$ units^2 **f** $1\frac{3}{5}$ units^2 **g** 8 units^2

4 $9 - 4\sqrt{2}$

REVIEW SET 25A

1 **a**



b 4

c $f'(x) = 2x$

$\therefore$ $f'(2) = 4$

2 **a** 21 **b** 3 **c** -4 **3** -1 **4** $f'(x) = 2x - 2$

5 **a** $f'(x) = 21x^2$ **b** $f'(x) = 6x - 3x^2$

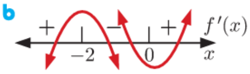
c $f'(x) = 8x - 12$ **d** $f'(x) = 7 + 4x$

6 **a** $f'(x) = 2x - \frac{1}{x^2}$ **b** $f'(1) = 1$, the gradient of $y = f(x)$ at $x = 1$ is 1.

7 $(-2, -1)$, $(2, 1)$

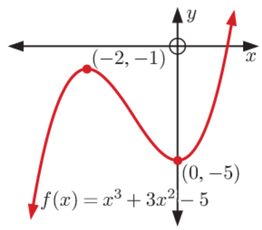
8 **a** $f'(x) = 3x^2 + 6x$

b



c maximum turning point at $(-2, -1)$
minimum turning point at $(0, -5)$

d



9 $F'(x) = \frac{1}{2\sqrt{x}}$ $\therefore$ $\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$

10 **a** $\frac{1}{2x^2} + c$ **b** $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$ **c** $\frac{x^5}{5} + \frac{10}{3}x^3 + 25x + c$

11 **a** 6 **b** $\frac{7}{2} - 2\sqrt{2}$ **c** $\frac{94\sqrt{2} - 56}{15}$ **12** $32\frac{2}{3}$ units^2

REVIEW SET 25B

1 **a** -3 **b** $f'(x) = 2x + 1$ $\therefore$ $f'(-2) = -3$

2 **a** -3 **b** -2 **c** 5

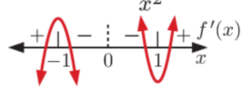
3 **a** $f'(x) = \frac{3}{\sqrt{x}}$ **b** $f'(x) = -\frac{1}{x^2} + \frac{8}{x^3}$

c $f'(x) = x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$ **d** $f'(x) = 24x^2 - 24x + 6$

4 $9x - y = 11$ **5** $(\frac{1}{2}, 2\sqrt{2})$ **6** **a** 0 **b** -3

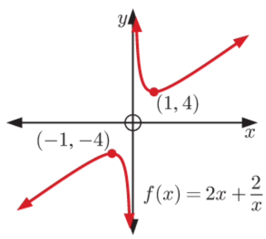
7 **a** $f'(x) = 2 - \frac{2}{x^2}$

b



c maximum turning point at $(-1, -4)$
minimum turning point at $(1, 4)$

d



8 **a** $-3x + c$ **b** $\frac{x^4}{2} - \frac{x^3}{3} + c$ **c** $x - \frac{3x^2}{2} + x^3 - \frac{x^4}{4} + c$

9 **a** $P = 2r + \pi r + 2x$ m **b** $x = 100 - r - \frac{\pi r}{2}$

c $A = 2xr + \frac{1}{2}\pi r^2 = 2r\left(100 - r - \frac{\pi r}{2}\right) + \frac{1}{2}\pi r^2$ etc.

d $r = 28.0$ m, rectangle has length 56.0 m, width 28.0 m

10 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $12\frac{2}{3}$ **11** $13\frac{3}{4}$ units^2

12 $k \approx 3.832$ (to 3 decimal places)