Title: Packaging and Geometrical shapes

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RQ- Analysing the Surface area of 1 litre packages

<u>Introduction</u>

Most of the common items found in a fridge are water bottles, vegetables, fruits, cheese etc, some of the most common items found in my fridge are milk, water, and juice. When I saw these three items I thought that all three of these items are in different shaped packages but each have a volume of 1 litre, so that is when an idea came to my mind- "what is the ideal shape to hold a volume of 1 litre?". In this investigation, I am going to calculate the ideal shape to hold a volume of 1-liter. For this investigation, I have chosen 1-liter package of- Apple Juice, Water, and Milk, as all of them are shaped differently packaging with different sized faces.

First I will be taking the measurements of all three packages. I shall calculate their actual surface area and volume using simple mathematics then I will be using calculus to find the dimensions for the least surface area for all three packages. After finding actual surfaces I will be finding the minimum surface area and find the percentage difference to see which of the packages is closest to the minimum. To validate my results and calculations, I will model an equation for surface area and edge lengths. Further I will also be drawing a graph for the equation of the surface area using an online colored Graphical Display Calculator(GDC) and find the minimum value from the graph.

<u>Aim</u>

The Aim of this investigation is to find the ideal packaging surface area for 1-liter of Water, Milk, and Apple Juice.

<u>Data Collection</u>

To find the ideal packaging surface area for 1-liter, I took 1-liter packages of Milk, Water and Apple Juice. After the collection of these items I measured the height, length and the width of apple juice and milk packages and the base radius and height of the water bottle. The measurements were collected in centimeters(cm). (The measurements are given below)

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Measurements

I will be measuring in centimeters(cm), the height, length and the width of milk and apple juice packages and the base radius and height of the water bottle.

Apple Juice-

Height-19.5 cm = b

width - 6 cm = a

Length- 9 cm = 1.5 a

Water bottle-

Height -24.7 cm = H

Radius- 3.7cm = r

Milk-

Height- 18.5 cm = h

Width- 7 cm = $x = y$

Length- 7 cm = $x = y$

(Water) (Apple Juice) (Milk)

This is an Apple juice Package. Ignoring the top cap of the juice package I am assuming this as a cuboid. I will be using a to represent the width of the base and b to represent the height of the package as the length of the rectangular base is 1.5 times bigger than its width.

So the formula for the Volume, V, is:
V = a (1.5a) b

To find the actual Volume of the Juice Package I substitute the values to obtain -
$Volume\ of\ cuboid = length \times width \times height$

$\Rightarrow V = 6 \times 9 \times 19.5 = 1053\ cm^3$

This is 53 ml more than is stated on the juice package, the reason behind this could be because I ignored the top part of the juice package.

After finding the volume of the juice package, I found the formula for the surface area ,C, for a cuboid-
$\Rightarrow C = 2a\,(1.5a) + 2ab + 2(1.5a)b$ … equation 1

Using the fact that the actual volume of the package is 1000 ml, I get-
$1000 = 1.5a^2 b$

After that, I rearranged the formula and got-

$b = \frac{1000}{1.5a^2}$ … equation 2

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I Substituted this equation 2 into the equation 1 to obtain -

$C = 3a^2 + 2a \times \frac{1000}{1.5a^2} + 3a \times \frac{1000}{1.5a^2}$

Further solving this equation I got-

$C = 3a^2 + 1333.3a^{-1} + 2000a^{-1}$

I Differentiated C with respect to a .

$\frac{dC}{da} = 6a - 1333.3a^{-2} - 2000a^{-2}$

Since, at maximum or minimum values, $\frac{dC}{da} = 0$

Using the fact, we have

$6a - 3333.3a^{-2} = 0$

$\Rightarrow 6a^3 = 3333.3$ [multiplying throughout by non-zero quantity a^2]

$\Rightarrow a^3 = \frac{3333.3}{6}$

$\Rightarrow a^3 = 555.55$

$\Rightarrow a = \sqrt[3]{555.55}$

$\Rightarrow a = 8.220679512\ cm$

$\Rightarrow a \approx 8.22\ \cancel{cm}$ [correct to 3 significant figures]

Using the method I ~~also found~~ the value for b

$\Rightarrow b = 9.87\ cm$

So, the best dimensions that I got were 8.22 cm by 9.87 cm

After finding the best dimensions, these values gave the surface area of-

$C = 2a(1.5a) + 2ab + 2(1.5a)b$

We substitute the values and obtain :

$C = 2(1.5)(8.22)^2 + 2(8.22)(9.87) + 2(1.5)(8.22)(9.87)$

$\Rightarrow C = (202.70) + (162.26) + (243.39)$

$\Rightarrow C = 608.36$

Rounding it off to three significant figures I obtain -

$C = 608\ cm^2$

After finding the surface area of the Juice package with the best dimensions, I found the actual surface area with the measurements that I took-

$C = 2\,(length \times width + width \times height + height \times length)$ [1]

$\Rightarrow C = 2(6)(9) + 2(6)(19.5) + 2(9)(19.5)$

$\Rightarrow C = 108 + 234 + 351$

$\Rightarrow C = 693cm^2$

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[1] ("Surface Area of a Cuboid - Web Formulas")

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The difference between these values $\triangle C$ -

$\triangle C = (693 - 608)\ cm^2$

$= 85cm^2$

After finding the difference, I found the percentage difference-

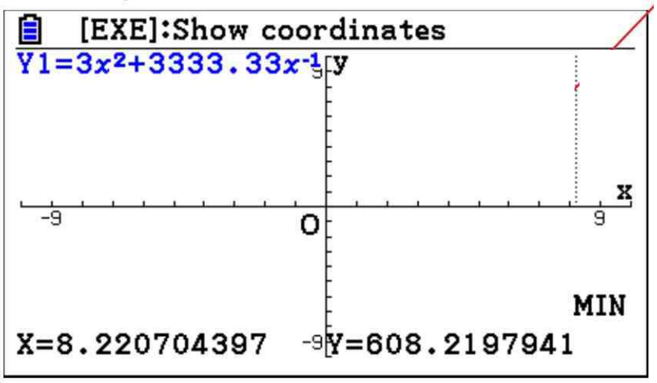
$\frac{85}{693} \times 100$

$= 12.3\%$

These dimensions could give a cuboid where the height was less than the width. As the width is larger than the height this package would not be very practical for everyday use, as it would be difficult to hold and travel with eg- from the grocery store to home and to store it.

To validate my findings I had also drawn the graph for the equation of the surface area and found the minimum value from the graph-

- **Graph 1**



As the minimum values on the graph was (8.22,608.21) which agrees to my calculations, thus the validity of my findings are high.

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This is a bottle of water, again ignoring the top part of the bottle I assumed it as a cylinder and measured the radius and the height of the bottle. I will be using H to represent the height of the bottle and r to represent the radius of the circle.

So, the formula for the volume of a cylinder is-

$V = \pi r^2 H$

Then, I calculated the volume using measurements of water which is present in the bottle-

$V = \pi(3.7)^2 \times 24.7 = 1062.307565cm^3$

Rounding it up a whole number I got-

$V = 1062cm^3 \Rightarrow 1062ml$

This is 62ml more than is stated on the water bottle, however the reason behind this could be due to assuming this water bottle as a perfect cylinder by not considering the top and the bottom.

After finding the volume of the water bottle, I found the formula for the surface area ,C, for a cylinder-

$C = 2\pi r^2 + 2\pi rH$

Further using the formula for volume of the cylinder and the fact that the water bottle contains 1000 ml, I got -

$1000 = \pi r^2 H$

And rearranging this formula for H I got-

$H = \frac{1000}{\pi r^2}$ ...equation 3

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I Substituted this expression for H into the equation for the surface area, I got-

$C = 2\pi r^2 + 2\pi r(\frac{1000}{\pi r^2})$

$C = 2\pi r^2 + 2000r^{-1}$

After substituting I differentiated C with respect to r and then equated this expression to zero in order to find the maximum or minimum value for r -

$\frac{dC}{dr} = 4\pi r - 2000r^{-2} = 0$

After getting the $\frac{dC}{dr}$ I further multiplied it by non-zero quantity r^2 and obtain : -

$4\pi r^3 - 2000 = 0$

$\Rightarrow r^3 = \frac{2000}{4\pi} = 159.15$

$\Rightarrow r = 5.42cm$

$\Rightarrow H = 10.84cm$ [using equation 3

The findings show that base radius of 5.42cm and height of 10.84 cm will produce the minimum surface area of the bottle of water.

So the minimum surface area of the bottle, C, is-

$\Rightarrow C = 2\pi(5.42)^2 + 2\pi(5.42)(10.84)$

$\Rightarrow C = (184.57) + (369.15)$

$\Rightarrow C = 553.73cm^2$

Rounding it off to a whole number-

$C = 554cm^2$

And the Actual surface area is-

$C = 2\pi(3.7)^2 + 2\pi(3.7)(24.7)$

$\Rightarrow C = 86.01 + 574.22$

$\Rightarrow C = 660.23\ cm^2$

The Difference is-

$C = 660.23 - 554$

$\Rightarrow C = 106.23cm^2$

$\Rightarrow C \approx 106\ cm^2$

The Percentage is-

$\Rightarrow \frac{106}{660.23} \times 100$

$= 16.06\% \approx 16\%$

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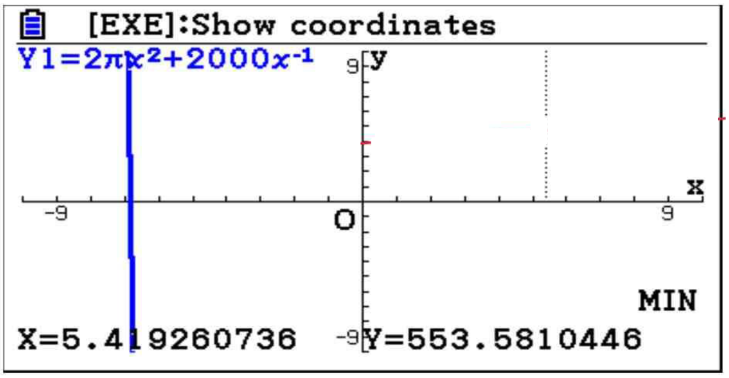
The final results show that these measurements would form a cylinder, where the diameter of the base was equal to the height. However these measurements would have some disadvantages, Like- It would be quite difficult to fit this bottle in the fridge doors and would be difficult to travel with. These measurements also shows that the dimensions that gives the minimum surface area are not always the best.

Again to validate my findings I had also drawn the graph for the equation of the surface area and found the minimum value from the graph-

- **Graph 2**



As the minimum values on the graph was (5.42,554) which agrees to my calculations, thus the validity of my calculations is high.

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This is a milk package. The actual shape of this package is a cuboid with the width x and the height h. The base of this milk package is a square.

So, the formula for the volume of a cuboid is-

$V = x^2 h$

The actual volume, U, is-

$7^2 \times (18.5)$

$= 906.5cm^3$

The result shows that the volume is less than what is stated in the package. However when I picked it up the package was bulging, so I am assuming that the milk package contains the right amount of volume- which is 1-liter.

So, the formula of the surface area, C, of this cuboid is-

$C = 2x^2 + 4xh$ ... equation 4

Now using the fact that the milk package contains 1000 ml, we get-

$1000 = x^2 h$

And now rearranging this formula for h, I got-

$\rightarrow h = \frac{1000}{x^2}$ ...equation 5

After getting the rearranged formula I substituted it in the formula using equation 4 to obtain :-

$C = 2x^2 + 4x \times \frac{1000}{x^2}$

Further solving it, I got-

$C = 2x^2 + 4000x^{-1}$ ...equation 6

Again after substituting I differentiated C with respect to x and then equated this expression to zero in order to find the maximum or minimum values for x-

$C = 2x^2 + 4000x^{-1}$ (equation 5)

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$\Rightarrow \frac{dC}{dx} = 4x - 4000x^{-2}$

After getting the $\frac{dC}{dx}$ I further multiplied it by non zero quantity x^2

$\Rightarrow 4x^3 - 4000 = 0$

$4x^3 = 4000$

$\Rightarrow x^3 = \frac{4000}{4}$

$\Rightarrow x^3 = 1000$

$\Rightarrow x = \sqrt[3]{1000}$

$\Rightarrow x = 10\ cm$

Substituting the value of x in equation 5, we obtain : -

$h = 10cm$

So the results show that the best dimensions are 10cm by 10 cm by 10cm- which shows that this a cube(because all sides have the same length in a cube).

The surface, C, is-

$C = 2(10)^2 + 4(10)(10)$

$\Rightarrow C = 200 + 400$

$\Rightarrow C = 600cm^2$

And the Actual surface area, C, is-

$C = 2(7)^2 + 4(7)(18.5)$

$\Rightarrow C = 98 + 518$

$\Rightarrow C = 616cm^2$

The difference is-

$(616 - 600)\ cm^2$

$\Rightarrow 16cm^2$

The percentage difference is-

$\frac{16}{616} \times 100$

$2.597402597...\% \approx 2.60\%$ (correct to three significant figures)

So the results show that milk package has the smallest percentage from all three, wastes the least amount of packaging and is closest to the ideal shape.

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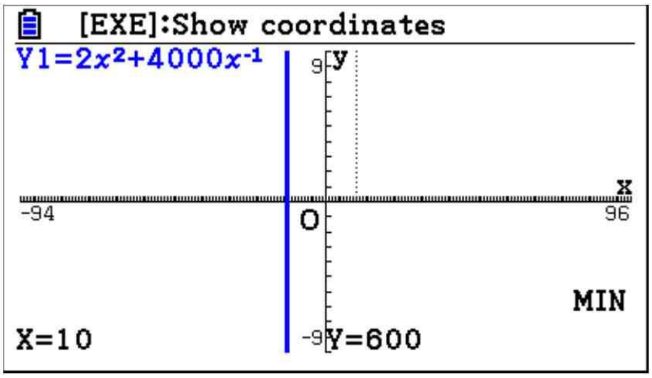
Now, to validate my findings I had also drawn the graph for the equation of the surface area and found the minimum value from the graph-

- **Graph 3**



As the minimum values on the graph was (10,600) which agrees to my calculations, thus the validity of my calculations is high.

Limitations

I didn't consider some parts of the design and assumed as it was a perfect shapes while constructing the formula for the three packages which would have affected my results. Also, I only used a ruler while measuring the packages, which could have been a little bit inaccurate and could also change the final results. I investigated only three packaging products which cannot be generalized for every single packaged products. To mitigate this limitation I could have investigated more than three packaged products.

Areas for further Research

As mentioned earlier I only investigated three different packaging products which cannot be generalized for every single packaged products. So to mitigate this limitation and for further research I could have investigated more than three packaged products. I also could have analysed more number of geometrical shapes in order to get a varied number results.

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Conclusion

After analysing all three packages with different shapes and results, It is evident that Milk package is closest to ideal shape for the surface area and water is closest in terms of ideal package for volume but is the most distant in surface area. Using differentiation was an efficient method for me to find the minimum or the maximum values and then sketching the graph of the expression helped me confirm that it was indeed a minimum value and also helped me in validating my results. However none of the three packages can be categorised as an ideal packaging shape.

The companies of these packages can calculate the best size of packages. However, they should produce the size of packages which is consumer friendly, easy to produce in mass and safe to transport and help them to create more profit.

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Bibliography

"Percentage difference." *Math is fun*. 27 Feb. 2011. Website. 3 Nov. 2016.

In-text citations:

- ("Percentage Difference")

"Surface area of a Cuboid - web formulas." *Web Formulas*. 18 Mar. 2011. Website. 6 Nov. 2016.

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