

Probability Practice 2 (Discrete & Continuous Distributions)

1. A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.
- (a) In eight such selections, what is the probability that a black disc is selected
- (i) exactly once? (3)
- (ii) at least once? (3)
- (b) The process of selecting and replacing is carried out 400 times. What is the expected number of black discs that would be drawn? (2)

(Total 8 marks)

2. A fair coin is tossed eight times. Calculate
- (a) the probability of obtaining exactly 4 heads; (2)
- (b) the probability of obtaining exactly 3 heads; (1)
- (c) the probability of obtaining 3, 4 or 5 heads. (3)

(Total 6 marks)

3. A factory makes calculators. Over a long period, 2% of them are found to be faulty. A random sample of 100 calculators is tested.

- (a) Write down the expected number of faulty calculators in the sample.
- (b) Find the probability that three calculators are faulty.
- (c) Find the probability that more than one calculator is faulty.

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(Total 6 marks)

4. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X

X	0	1	2
$P(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

- (a) Calculate $E(X)$, the mean number of red balls chosen. (3)

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

- (b) (i) Draw a tree diagram to represent the above information, including the probability of each event.
- (ii) Hence find the probability distribution for Y , where Y is the number of red balls chosen. (8)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

- (c) Calculate the probability that two red balls are chosen. (5)
- (d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die. (3)

(Total 19 marks)

5. The probability distribution of the discrete random variable X is given by the following table.

x	1	2	3	4	5
$P(X = x)$	0.4	p	0.2	0.07	0.02

- Find the value of p .
- Calculate the expected value of X .

[illegible]

(Total 6 marks)

6. Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a 0, 3, 4, or 9 printed on it.

- (a) Kim states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$P(X = x)$	0.3	0.45	0.2	0.35

Explain why Kim is incorrect.

(2)

- (b) Ching Li correctly states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$P(X = x)$	0.4	k	$2k$	0.3

Find the value of k .

(2)

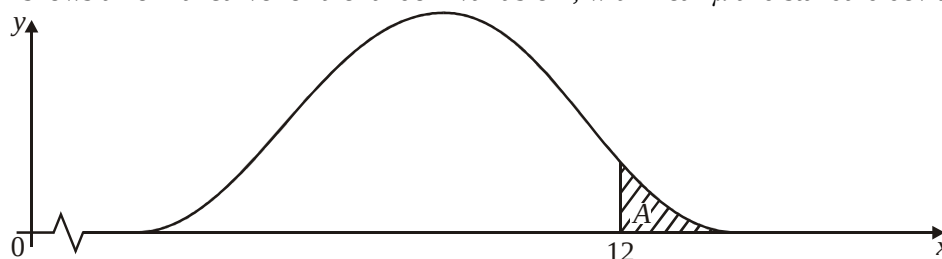
- (c) Jonathan correctly states that the probability distribution for his pack of cards is given by $P(X = x) = \frac{x+1}{20}$. One card is drawn at random from his pack.

- (i) Calculate the probability that the number on the card drawn is 0.
- (ii) Calculate the probability that the number on the card drawn is greater than 0.

(4)

(Total 8 marks)

7. The graph shows a normal curve for the random variable X , with mean μ and standard deviation σ .



It is known that $p(X \geq 12) = 0.1$.

- (a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A .

(1)

It is also known that $p(X \leq 8) = 0.1$.

- (b) Find the value of μ , explaining your method in full.
 (c) Show that $\sigma = 1.56$ to an accuracy of three significant figures.
 (d) Find $p(X \leq 11)$.

(5)

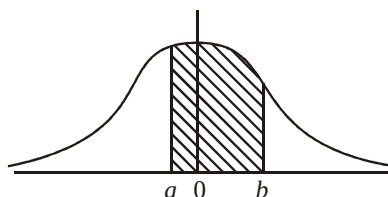
(5)

(5)

(Total 16 marks)

8. The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

- (a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



- (i) Write down the values of a and b .
 (ii) Find the probability that the lifespan of an insect of this species is
 (a) more than 55 hours;
 (b) between 55 and 60 hours.
 (b) 90% of the insects die after t hours.
 (i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%.
 (ii) Find the value of t .

(2)

(1)

(2)

(2)

(3)

(Total 10 marks)

9. An urban highway has a speed limit of 50 km h^{-1} . It is known that the speeds of vehicles travelling on the highway are normally distributed, with a standard deviation of 10 km h^{-1} , and that 30% of the vehicles using the highway exceed the speed limit.

- (a) Show that the mean speed of the vehicles is approximately 44.8 km h^{-1} .

(3)

The police conduct a “Safer Driving” campaign intended to encourage slower driving, and want to know whether the campaign has been effective. It is found that a sample of 25 vehicles has a mean speed of 41.3 km h^{-1} .

- (b) Given that the null hypothesis is
 H_0 : the mean speed has been unaffected by the campaign
 State H_1 , the alternative hypothesis.

(1)

- (c) State whether a one-tailed or two-tailed test is appropriate for these hypotheses, and explain why.

(2)

- (d) Has the campaign had significant effect at the 5% level?

(4)

(Total 10 marks)

10. Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.
- (a) What percentage of the population has an IQ between 90 and 125? (2)
- (b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125? (3)
- (c) The mean IQ of a random group of 25 persons suffering from a certain brain disorder was found to be 95.2. Is this sufficient evidence, at the 0.05 level of significance, that people suffering from the disorder have, on average, a lower IQ than the entire population? State your null hypothesis and your alternative hypothesis, and explain your reasoning. (4)
- (Total 9 marks)**
11. Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.
- (a) What is the probability a bag selected at random will weigh less than 25.0 kg? (2)
- In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.
- (b) Show that the increased mean is 26.0 kg. (3)
- It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.
- (c) Calculate the mean and standard deviation that satisfy these requirements. (3)
- The cost of the new machine is \$5000. Cement sells for \$0.80 per kg.
- (d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment? (3)
- (Total 11 marks)**
12. The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.
- (a) Find the probability that a packet chosen at random has mass
- (i) less than 740 g;
- (ii) at least 780 g;
- (iii) between 740 g and 780 g. (5)
- (b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g? (2)
- (c) The mass of 70% of the packets is more than x grams. Find the value of x . (2)
- (Total 9 marks)**
13. In a country called *Tallopia*, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.
- (a) What percentage of adults in *Tallopia* have a height greater than 197 cm? (3)
- (b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in

Tallopia. Give your answer to the nearest cm.

(4)

(Total 7 marks)

14. A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

(a) What proportion of television sets break down in less than 72 months?

(2)

(b) (i) Calculate the proportion of sets which have a lifetime between 72 months and 90 months.

(ii) Illustrate this proportion by appropriate shading in a sketch of a normal distribution curve.

(5)

(c) If a set breaks down in less than x months, the company replace it free of charge. They replace 4% of the sets. Find the value of x .

(3)

(Total 10 marks)

15. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.

(2)

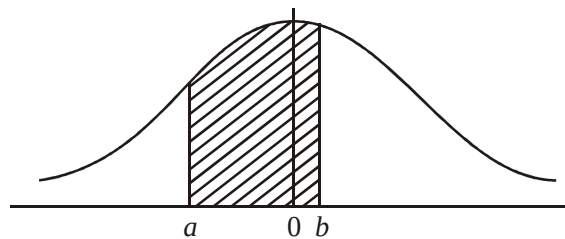
(b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b .

(3)

(Total 5 marks)

16. Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds.

(a) The graph below is that of the **standard** normal curve. The shaded area represents the probability that the reaction time of a person chosen at random is between 0.70 and 0.79 seconds.



(i) Write down the value of a and of b .

(ii) Calculate the probability that the reaction time of a person chosen at random is

(a) greater than 0.70 seconds;

(b) between 0.70 and 0.79 seconds.

(6)

Three percent (3%) of the population have a reaction time less than c seconds.

(b) (i) Represent this information on a diagram similar to the one above. Indicate clearly the area representing 3%.

(ii) Find c .

(4)

(Total 10 marks)

Probability Practice 2 (Discrete & Continuous Distributions)

1. $p(\text{Red}) = \frac{35}{40} = \frac{7}{8}$ $p(\text{Black}) = \frac{5}{40} = \frac{1}{8}$
- (a) (i) $p(\text{one black}) = \binom{8}{1} \left(\frac{1}{8}\right)^1 \left(\frac{7}{8}\right)^7$ (M1)(A1)
 $= 0.393$ to 3 sf (A1) 3
- (ii) $p(\text{at least one black}) = 1 - p(\text{none})$ (M1)
 $= 1 - \binom{8}{0} \left(\frac{1}{8}\right)^0 \left(\frac{7}{8}\right)^8$ (A1)
 $= 1 - 0.344$
 $= 0.656$ (A1) 3
- (b) 400 draws: expected number of blacks $= \frac{400}{8}$ (M1)
 $= 50$ (A1) 2

[8]

2. (a) $p(4 \text{ heads}) = \binom{8}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{8-4}$ (M1)
 $= \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \times \left(\frac{1}{2}\right)^8$
 $= \frac{70}{256} \approx 0.273$ (3 sf) (A1) 2
- (b) $p(3 \text{ heads}) = \binom{8}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{8-3} = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \left(\frac{1}{2}\right)^8$
 $= \frac{56}{256} \approx 0.219$ (3 sf) (A1) 1
- (c) $p(5 \text{ heads}) = p(3 \text{ heads})$ (by symmetry) (M1)
 $p(3 \text{ or } 4 \text{ or } 5 \text{ heads}) = p(4) + 2p(3)$ (M1)
 $= \frac{70 + 2 \times 56}{256} = \frac{182}{256}$
 ≈ 0.711 (3 sf) (A1) 3

[6]

3. (a) $X \sim B(100, 0.02)$
 $E(X) = 100 \times 0.02 = 2$ A11
- (b) $P(X = 3) = \binom{100}{3} (0.02)^3 (0.98)^{97}$ (M1)
 $= 0.182$ A12
- (c) **METHOD 1**
 $P(X > 1) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1))$ M1
 $= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ (M1)
 $= 0.597$ A12
- METHOD 2**
 $P(X > 1) = 1 - P(X \leq 1)$ (M1)
 $= 1 - 0.40327$ (A1)
 $= 0.597$ A12

Note: Award marks as follows for finding $P(X > 1)$,

if working shown.

$$P(X \geq 1)$$

$$= 1 - P(X < 2) = 1 - 0.67668$$

$$= 0.323$$

A0

M1(ft)

A1(ft) 2

[6]

4. (a) Using $E(X) = \sum_0^2 x P(X = x)$

(M1)

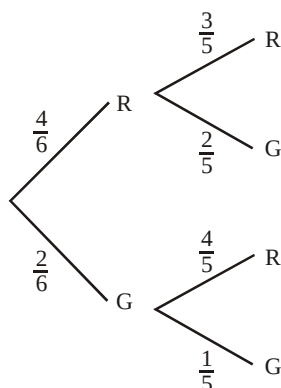
$$\text{Substituting correctly } E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$$

A1

$$= \frac{8}{10} \quad (0.8)$$

A13

(b) (i)



A1A1A1 3

Note: Award (A1) for each complementary pair of probabilities,ie $\frac{4}{6}$ and $\frac{2}{6}$, $\frac{3}{5}$ and $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$.

(ii) $P(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$

A1

$$P(Y = 1) = P(RG) + P(GR) \left(= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \right)$$

M1

$$= \frac{16}{30}$$

A1

$$P(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$$

(A1)

For forming a distribution

M1 5

y	0	1	2
P(Y = y)	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

(c) $P(\text{Bag A}) = \frac{2}{6} \left(= \frac{1}{3} \right)$

(A1)

$$P(\text{Bag A} | B) = \frac{4}{6} \left(= \frac{2}{3} \right)$$

(A1)

For summing $P(A \cap RR)$ and $P(B \cap RR)$

(M1)

$$\text{Substituting correctly } P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30}$$

A1

$$= \frac{27}{90} \left(\frac{3}{10}, 0.3 \right)$$

A15

(d) For recognising that $P(1 \text{ or } 6 | RR) = P(A | RR) = \frac{P(A \cap RR)}{P(RR)}$

(M1)

$$= \frac{1}{30} \div \frac{27}{90}$$

A1

$$= \frac{3}{27} \left(\frac{1}{9}, 0.111 \right)$$

A13

[19]

5. (a) For using $\sum p=1$ $(0.4 + p + 0.2 + 0.07 + 0.02 = 1)$

(M1)

$$p = 0.31$$

A1N2

- (b) For using $E(X) = \sum xP(X=x)$

(M1)

$$E(X) = 1(0.4) + 2(0.31) + 3(0.2) + 4(0.07) + 5(0.02) \\ = 2$$

A1

A2N2

[6]

6. (a) Adding probabilities

(M1)

Evidence of knowing that sum = 1 for probability distribution
eg Sum greater than 1, sum = 1.3, sum does not equal 1

R1

N2

- (b) Equating sum to 1 $(3k + 0.7 = 1)$

M1

$$k = 0.1$$

A1N1

- (c) (i) $P(X=0) = \frac{0+1}{20}$

(M1)

$$= \frac{1}{20}$$

A1N2

- (ii) Evidence of using $P(X > 0) = 1 - P(X = 0)$

$$\left(\text{or } \frac{4}{20} + \frac{5}{20} + \frac{10}{20} \right)$$

(M1)

$$= \frac{19}{20}$$

A1N2

[8]

7. (a) Area $A = 0.1$

(A1) 1

- (b) **EITHER** Since $p(X \geq 12) = p(X \leq 8)$,
then 8 and 12 are symmetrically disposed around the
mean.

(M1)

(M1)(R1)

$$\text{Thus mean} = \frac{8+12}{2}$$

(M1)

$$= 10$$

(A1)

Notes: If a candidate says simply “by symmetry $\mu = 10$ ” with no
further explanation award [3 marks] (M1, A1, R1). As a full
explanation is requested award an additional (A1) for saying since
 $p(X < 8) = p(X > 12)$ and another (A1) for saying that the normal
curve is symmetric.

OR

$$p(X \geq 12) = 0.1 \Rightarrow p\left(Z \geq \frac{12-\mu}{\sigma}\right) = 0.1$$

(M1)

$$\Rightarrow p\left(Z \leq \frac{12-\mu}{\sigma}\right) = 0.9$$

$$p(X \leq 8) = 0.1 \Rightarrow p\left(Z \leq \frac{8-\mu}{\sigma}\right) = 0.1$$

$$\Rightarrow p\left(Z \leq \frac{\mu-8}{\sigma}\right) = 0.9$$

(A1)

$$\text{So } \frac{12 - \mu}{\sigma} = \frac{\mu - 8}{\sigma}$$

(M1)

$$\Rightarrow 12 - \mu = \mu - 8$$

(M1)

$$\Rightarrow \mu = 10$$

(A1) 5

$$(c) \quad \Phi\left(\frac{12-10}{\sigma}\right) = 0.9$$

(A1)(M1)(A1)

Note: Award (A1) for $\left(\frac{12-10}{\sigma}\right)$, (M1) for standardizing, and (A1) for 0.9.

$$\Rightarrow \frac{2}{\sigma} = 1.282 \text{ (or 1.28)}$$

(A1)

$$\sigma = \frac{2}{1.282} \left(\text{or } \frac{2}{1.28} \right)$$

(A1)

$$= 1.56 \text{ (3 sf)}$$

(AG) 5

Note: Working backwards from $\sigma = 1.56$ to show it leads the given data should receive a maximum of [3 marks] if done correctly.

$$(d) \quad p(X \leq 11) = p\left(Z \leq \frac{11-10}{1.561}\right) \text{ (or 1.56)}$$

(M1)(A1)

Note: Award (M1) for standardizing and (A1) for $\left(\frac{11-10}{1.561}\right)$.

$$= p(Z \leq 0.6407) \text{ (or 0.641 or 0.64)}$$

(A1)

$$= \Phi(0.6407)$$

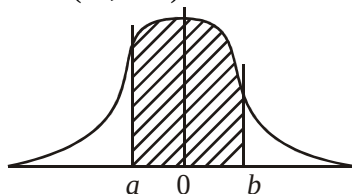
(M1)

$$= 0.739 \text{ (3 sf)}$$

(A1) 5

[16]

8. (a) Let X be the lifespan in hours
 $X \sim N(57, 4.4^2)$



$$(i) \quad a = -0.455 \text{ (3 sf)}$$

(A1)

$$b = 0.682 \text{ (3 sf)}$$

(A1)

$$(ii) \quad (a) \quad P(X > 55) = P(Z > -0.455)$$

$$= 0.675$$

(A1)

$$(b) \quad P(55 \leq X \leq 60) = P\left(\frac{2}{4.4} \leq Z \leq \frac{3}{4.4}\right)$$

$$\approx P(0.455 \leq Z \leq 0.682)$$

$$\approx 0.6754 + 0.752 - 1$$

(A1)

$$= 0.428 \text{ (3sf)}$$

(A1)

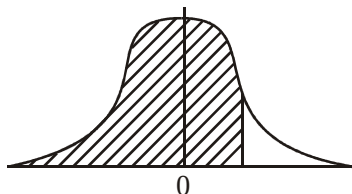
OR

$$P(55 \leq X \leq 60) = 0.428 \text{ (3 sf)}$$

(G2) 5

- (b) 90% have died $\Rightarrow$ shaded area = 0.9

(M1)



(A1)

Hence $t = 57 + (4.4 \times 1.282)$ (M1)
 $= 57 + 5.64$ (A1)
 $= 62.6$ hours (A1)
OR $t = 62.6$ hours (G3) 5

[10]

9. (a) **Note:** Candidates using tables may get slightly different answers, especially if they do not interpolate. Accept these answers.

$$P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\frac{50 - \mu}{10}\right) \quad (\text{A1})$$

$$\text{Hence, } \frac{50 - \mu}{10} = \Phi^{-1}(0.7) \quad (\text{M1})$$

$$\mu = 50 - 10\Phi^{-1}(0.7) \quad (\text{M1})$$

$$= 44.75599 \dots = 44.8 \text{ km/h (3 sf) (accept 44.7)} \quad (\text{AG}) \quad 3$$

(b) H_1 : “the mean speed has been reduced by the campaign”. (A1) 1

(c) One-tailed; because H_1 involves only “<”. (A2) 2

(d) For a one-tailed test at 5% level, critical region is
 $Z < \mu_m - 1.64\sigma_m$ (accept $-1.65\sigma_m$) (M1)

$$\text{Now, } \mu_m = \mu = 44.75\dots; \sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2 \text{ (allow ft)} \quad (\text{A1})$$

$$\text{So test statistic is } 44.75\dots - 1.64 \times 2 = 41.47 \quad (\text{A1})$$

$$\text{Now } 41.3 < 41.47 \text{ so reject } H_0, \text{ yes.} \quad (\text{A1}) \quad 4$$

[10]

10. (a) Let X be the random variable for the IQ.

$$X \sim N(100, 225) \quad (\text{M1})$$

$$P(90 < X < 125) = P(-0.67 < Z < 1.67) \quad (\text{M1})$$

$$= 0.701$$

$$70.1 \text{ percent of the population (accept 70 percent).} \quad (\text{A1})$$

OR
 $P(90 < X < 125) = 70.1\% \quad (\text{G2}) \quad 2$

(b) $P(X \geq 125) = 0.0475$ (or 0.0478) (M1)

$$P(\text{both persons having IQ} \geq 125) = (0.0475)^2 \text{ (or } (0.0478)^2) \quad (\text{M1})$$

$$= 0.00226 \text{ (or } 0.00228) \quad (\text{A1}) \quad 3$$

(c) Null hypothesis (H_0): mean IQ of people with disorder is 100 (M1)

Alternative hypothesis (H_1): mean IQ of people with disorder is less than 100 (M1)

$$P(\bar{X} < 95.2) = P\left(Z < \frac{95.2 - 100}{\frac{15}{\sqrt{25}}}\right) = P(Z < -1.6) = 1 - 0.9452$$

$$= 0.0548 \quad (\text{A1})$$

The probability that the sample mean is 95.2 and the null hypothesis true is $0.0548 > 0.05$. Hence the evidence is not sufficient. (R1) 4

[9]

11. (a) $Z = \frac{25 - 25.7}{0.50} = -1.4$ (M1)
- $P(Z < -1.4) = 1 - P(Z < 1.4)$
 $= 1 - 0.9192$
 $= 0.0808$ (A1)
- OR**
 $P(W < 25) = 0.0808$ (G2) 2
- (b) $P(Z < -a) = 0.025 \Rightarrow P(Z < a) = 0.975$
 $\Rightarrow a = 1.960$ (A1)
- $\frac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96(0.50)$ (M1)
 $= 25 + 0.98 = 25.98$ (A1)
 $= 26.0$ (3 sf) (AG)
- OR**
 $\frac{25.0 - 26.0}{0.50} = -2.00$ (M1)
 $P(Z < -2.00) = 1 - P(Z < 2.00)$
 $= 1 - 0.9772 = 0.0228$ (A1)
 ≈ 0.025 (A1)
- OR**
 $\mu = 25.98$ (G2)
 $\Rightarrow \text{mean} = 26.0$ (3 sf) (A1)(AG) 3
- (c) Clearly, by symmetry $\mu = 25.5$ (A1)
- $Z = \frac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma$ (M1)
 $\Rightarrow \sigma = 0.255$ kg (A1) 3
- (d) On average, $\frac{\text{cement saving}}{\text{bag}} = 0.5$ kg (A1)
- $\frac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = \0.40 (M1)
- To save \$5000 takes $\frac{5000}{0.40} = 12500$ bags (A1) 3
12. (a) (These answers may be obtained from a calculator or by finding z in each case and the corresponding area.)
- $M \sim N(750, 625)$
- (i) $P(M < 740 \text{ g}) = 0.345$ (G2)
- OR**
 $z = -0.4 \quad P(z < -0.4) = 0.345$ (A1)(A1)
- (ii) $P(M > 780 \text{ g}) = 0.115$ (G2)
- OR**
 $z = 1.2 \quad P(z > 1.2) = 1 - 0.885 = 0.115$ (A1)(A1)
- (iii) $P(740 < M < 780) = 0.540$ (G1)
- OR**
 $1 - (0.345 + 0.115) = 0.540$ (A1) 5

[11]

- (b) Independent events
Therefore, $P(\text{both} < 740) = 0.345^2$
 $= 0.119$ (M1)
(A1) 2
- (c) 70% have mass < 763 g (G1)
Therefore, 70% have mass of at least $750 - 13$
 $x = 737$ g (A1) 2

[9]

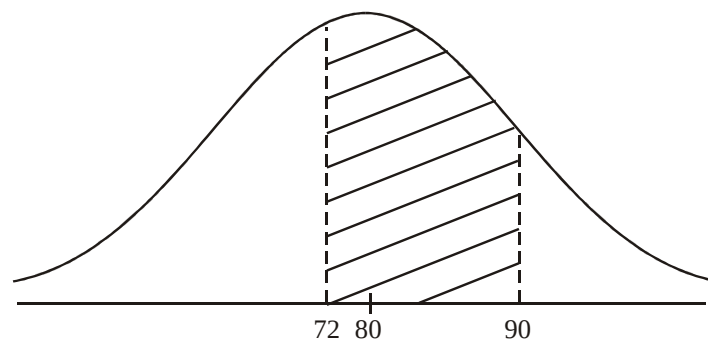
13. **Note:** Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf

- (a) $z = \frac{197 - 187.5}{9.5} = 1.00$ (M1)
 $P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$
 $= 0.159$ (3 sf) (A1)
 $= 15.9\%$ (A1)
- OR**
 $P(H > 197) = 0.159$ (G2)
 $= 15.9\%$ (A1) 3
- (b) Finding the 99th percentile
 $\Phi(a) = 0.99 \Rightarrow a = 2.327$ (accept 2.33) (A1)
 $\Rightarrow 99\%$ of heights under $187.5 + 2.327(9.5) = 209.6065$ (M1)
 $= 210$ (3 sf) (A1)
- OR**
99% of heights under $209.6 = 210$ cm (3 sf) (G3)
Height of standard doorway $= 210 + 17 = 227$ cm (A1) 4

[7]

14. $X \sim N(80, 8^2)$

- (a) $P(X < 72) = P(Z < -1)$ (M1)
 $= 1 - 0.8413$
 $= 0.159$ (A1)
- OR**
 $P(X < 72) = 0.159$ (G2) 2
- (b) (i) $P(72 < X < 90) = P(-1 < Z < 1.25)$ (M1)
 $= 0.3413 + 0.3944$ (A1)
 $= 0.736$ (A1)
- OR**
 $P(72 < X < 90) = 0.736$ (G3)
- (ii)



(A1)(A1) 5

Note: Award (A1) for a normal curve and (A1) for the shaded area, which should not be symmetrical.

- (c) 4% fail in less than x months

$$\Rightarrow x = 80 - 8 \times \Phi^{-1}(0.96)$$

$$= 80 - 8 \times 1.751$$

$$= 66.0 \text{ months}$$

(M1)

(A1)

(A1)

OR

$$x = 66.0 \text{ months}$$

(G3) 3

[10]

15. (a) $P(M \geq 350) = 1 - P(M < 350) = 1 - P\left(Z < \frac{350 - 310}{30}\right)$

(M1)

$$= 1 - P(Z < 1.333) = 1 - 0.9088$$

$$= 0.0912 \text{ (accept 0.0910 to 0.0920)}$$

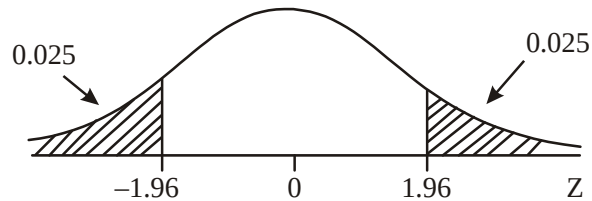
(A1)

OR

$$P(M \geq 350) = 0.0912$$

(G2)

- (b)



$$P(Z < 1.96) = 1 - 0.025 = 0.975$$

(A1)

$$1.96(30) = 58.8$$

(M1)

$$310 - 58.8 < M < 310 + 58.8 \Rightarrow a = 251, b = 369$$

(A1)

OR

$$251 < M < 369$$

(G3)

Note: Award (G1) if only one of the end points is correct.

[5]

16. (a) (i) $a = -1$

(A1)

$$b = 0.5$$

(A1)

(ii) (a) 0.841

(A2)

(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$)

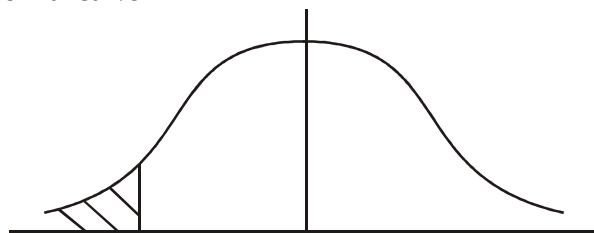
(M1)

$$= 0.533 \text{ (3 sf)}$$

(A1) (N2) 6

- (b) (i) Sketch of normal curve

(A1)(A1)



(ii) $c = 0.647$

(A2) 4

[10]