Chapter 13

Probability

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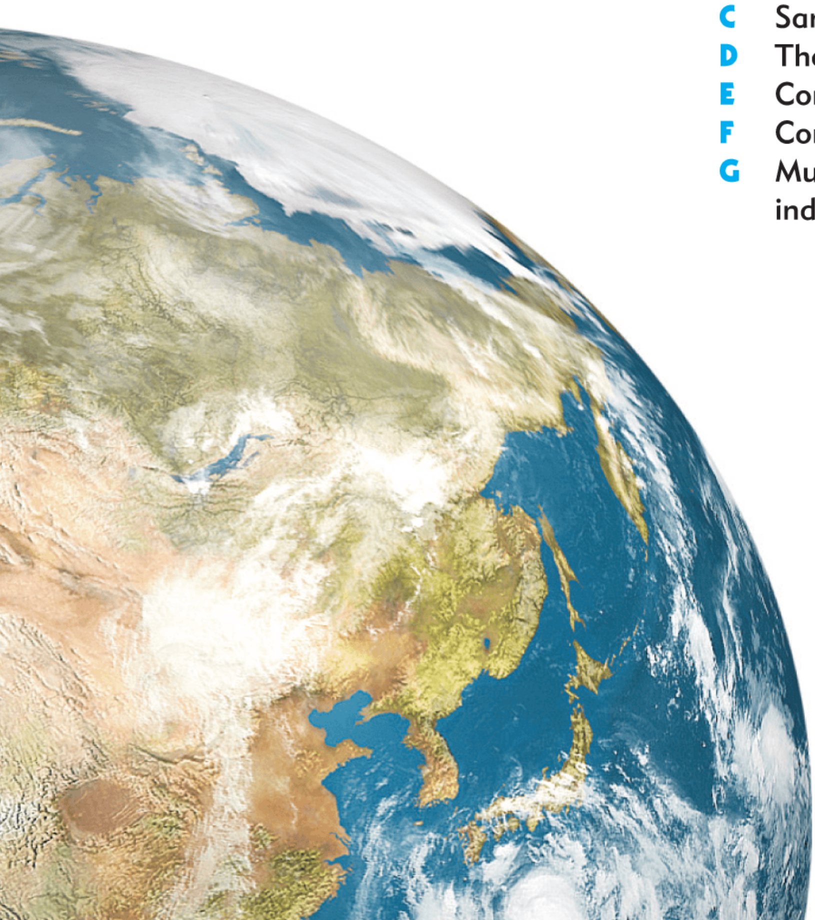
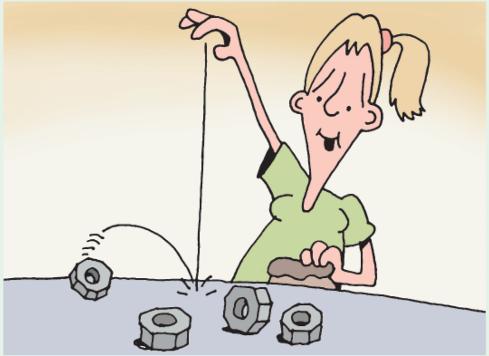


OPENING PROBLEM

When Karla dropped some metal nuts she noticed that they finished either on their ends or on their sides. She was interested to know how likely it was that a nut would finish on its end. So, she tossed a nut 200 times, and found that it finished on its end 137 times.



Her friend Sam repeated the experiment, and the nut finished on its end 145 times.

Things to think about:

- **a** What would Karla's best estimate be of the chance or probability that the nut will finish on its end?
- **b** What would Sam's estimate be?
- **c** How can we obtain a better estimate of the chance of an end occurring?
- **d** Hilda said that the best estimate would be obtained when the nut is tossed thousands of times. Do you agree with Hilda?

Consider these statements:

"The Wildcats will probably beat the Tigers on Saturday."
"It is unlikely that it will rain today."
"I will probably make the team."

Each of these statements indicates a **likelihood** or **chance** of a particular event happening.

We can indicate the likelihood of an event happening in the future by using a percentage.

0%	indicates the event **will not occur**.
100%	indicates the event **is certain to occur**.

All events can therefore be assigned a percentage between 0% and 100% inclusive.

A number close to 0% indicates the event is **unlikely** to occur, whereas a number close to 100% means that it is **highly likely** to occur.

In mathematics, we usually write probabilities as either decimals or fractions rather than percentages.

An **impossible** event has 0% chance of happening, and is assigned the probability 0.

A **certain** event has 100% chance of happening, and is assigned the probability 1.

All other events can be assigned a probability between 0 and 1.

Probabilities are usually determined by either:

- observing the results of an experiment (experimental probability), or
- using arguments of symmetry (theoretical probability).

HISTORICAL NOTE

- **Girolamo Cardano** (1501 to 1576) admitted in his autobiography that he gambled "not only every year, but .... every day, and with the loss at once of thought, of substance, and of time". He wrote a handbook on gambling, with tips on cheating and how to detect it. His book included discussion on equally likely events, frequency tables for dice probabilities, and expectations.
- **Pierre-Simon Laplace** (1749 - 1827) once described the theory of probability as "nothing but common sense reduced to calculation".
- **Blaise Pascal** (1623 - 1662) invented the first mechanical digital calculator. Pascal and his friend **Pierre de Fermat** (1601 - 1665) were the first to develop probability theory as we know it today. Pascal also developed the syringe and the hydraulic press, and he wrote a large number of articles on Christian beliefs and ethics.

Girolamo Cardano

A EXPERIMENTAL PROBABILITY

In experiments involving chance, we use the following terms to describe what we are doing and the results we are obtaining:

- The **number of trials** is the total number of times the experiment is repeated.
- The **outcomes** are the different results possible for one trial of the experiment.
- The **frequency** of a particular outcome is the number of times that this outcome is observed.
- The **relative frequency** of an outcome is the frequency of that outcome divided by the total number of trials. It can be expressed as a fraction, a decimal, or a percentage.

For example, suppose a tin can is tossed in the air 250 times, and it comes to rest on an end 29 times. We say:

- the number of trials is 250
- the outcomes are *ends* and *sides*
- the frequency of *ends* is 29, and of *sides* is 221
- the relative frequency of *ends* $= \frac{29}{250} = 0.116$
- the relative frequency of *sides* $= \frac{221}{250} = 0.884$.

EXPERIMENTAL PROBABILITY

Sometimes the only way of estimating the probability of a particular event occurring is by experimentation.

Tossing a tin can is one such example. The **relative frequencies** from an experiment can be used to estimate the probabilities of the corresponding events.

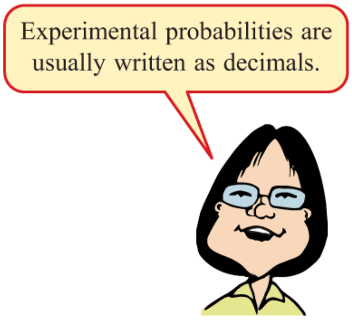
The larger the number of trials, the more accurate the estimate will be.

Example 1 **Self Tutor**

Larissa took 43 shots at a netball goal, and scored 29 times. Estimate the probability that Larissa will miss with her next shot.

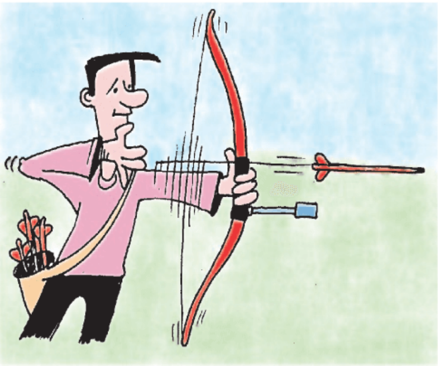
From 43 shots, Larissa scored 29 times and missed 14 times.

$\therefore$ P(misses next shot) $\approx \frac{14}{43}$
≈ 0.326



EXERCISE 13A

1 Clem fired 200 arrows at a target and hit the target 168 times. Estimate the probability that Clem will hit the target with his next shot.



2 Ivy has free-range hens. Out of the first 123 eggs that they laid, she found that 11 had double-yolks. Estimate the probability that the next egg laid will have a double-yolk.

3 Jackson leaves for work at the same time each day. Over a period of 227 working days, he had to wait for a train at the railway crossing on 58 days. Estimate the probability that Jackson will not have to wait for a train tomorrow.

4 Ravi has a circular spinner marked P, Q, and R. When the spinner was twirled 417 times, it finished on P 138 times, and on Q 107 times. Estimate the probability that the next spin will finish on R.

5 Answer the **Opening Problem** on page **278**.

B PROBABILITIES FROM TABLED DATA

If we are given a table of frequencies then we use **relative frequencies** to estimate the probabilities of the events.

$$\textbf{relative frequency} = \frac{\textbf{frequency}}{\textbf{number of trials}}$$

Example 2 **Self Tutor**

A marketing company surveys 80 people to discover what brand of shoe cleaner they use. The results are shown in the table alongside.

Estimate the probability that a randomly selected community member uses:

a Brite **b** Cleano or No scuff.

Brand	*Frequency*
Shine	27
Brite	22
Cleano	20
No scuff	11

a P(Brite) $\approx \frac{22}{80} \approx 0.275$

b P(Cleano or No scuff) $\approx \frac{20+11}{80} \approx 0.388$

PROBABILITIES FROM TWO-WAY TABLES

Two-way tables are tables comparing two variables. They usually result from a survey.

For example, the Year 10 students in a small school were asked whether they were good at mathematics. The results are summarised in the following two-way table:

	Boy	Girl
Good at mathematics	17	19
Not good at mathematics	8	9

← 9 girls were not good at mathematics.

In this case the variables are *ability in mathematics* and *gender*.

We can use two-way tables to estimate probabilities.

Example 3

Self Tutor

To investigate the breakfast habits of teenagers, a survey was conducted amongst some students from a high school. The results are shown alongside.

	Male	Female
Regularly eats breakfast	87	53
Does not regularly eat breakfast	68	92

Use this table to estimate the probability that a randomly selected student from the school:

a is male

b is male *and* regularly eats breakfast

c is female *or* regularly eats breakfast.

We extend the table to include totals:

	Male	Female	*Total*
Regularly eats breakfast	87	53	140
Does not regularly eat breakfast	68	92	160
Total	155	145	300

a There are 155 males amongst the 300 students surveyed.

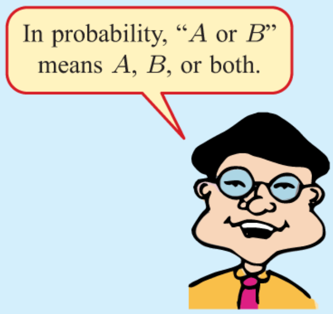
$\therefore$ $\text{P(male)} \approx \dfrac{155}{300} \approx 0.517$

b 87 of the 300 students are male and regularly eat breakfast.

$\therefore$ $\text{P(male } \textit{and} \text{ regularly eats breakfast)} \approx \dfrac{87}{300} \approx 0.29$

c $53 + 92 + 87 = 232$ out of the 300 are female or regularly eat breakfast.

$\therefore$ $\text{P(female } \textit{or} \text{ regularly eats breakfast)} \approx \dfrac{232}{300} \approx 0.773$



EXERCISE 13B

1 A marketing company surveyed people to discover which brand of soap they use. The results of the survey are given alongside.

Brand	*Frequency*
Silktouch	125
Super	107
Just Soap	93
Indulgence	82

a How many people were surveyed?

b Estimate the probability that a randomly selected person uses:

i Just Soap **ii** Indulgence or Silktouch.

2 This table shows the flavour of ice creams sold in a café one afternoon.

Flavour	*Frequency*
Chocolate	21
Strawberry	17
Lemon	4
Vanilla	15

- **a** How many ice creams were sold?
- **b** Estimate the probability that the next ice cream sold will be:
  - **i** strawberry
  - **ii** chocolate or vanilla.

3

Councillor	*Frequency*
Mr Tony Trimboli	216
Mrs Andrea Sims	72
Mrs Sara Chong	238
Mr John Henry	
Total	

Results from a poll for a local Council election are shown in the table. It is known that 600 people were surveyed in the poll.

- **a** Copy and complete the table.
- **b** Estimate the probability that a randomly selected person from this electorate will vote for:
  - **i** John Henry
  - **ii** a female councillor.

4 310 students at a high school in South Africa were surveyed on the question "Do you like watching rugby on TV?". The results are shown in the two-way table.

	Like	Dislike
Junior students	87	38
Senior students	129	56

- **a** Copy and complete the table to include 'totals'.
- **b** Estimate the probability that a randomly selected student:
  - **i** likes watching rugby on TV and is a junior student
  - **ii** likes watching rugby on TV and is a senior student
  - **iii** dislikes watching rugby on TV.
- **c** Find the total of the probabilities found in **b**. Explain your answer.

5 A random selection of students in a youth club were asked whether they wore glasses, contact lenses, or neither. The results were further categorised by gender.

	Glasses	Contact lenses	Neither
Male	15	6	26
Female	14	8	31

- **a** How many students were surveyed?
- **b** Estimate the probability that a randomly chosen student in the club:
  - **i** wears glasses
  - **ii** is female and wears contact lenses
  - **iii** is male and wears neither
  - **iv** is female or wears glasses.

6 The table alongside describes the types of cars advertised for sale in a newspaper.
Estimate the probability that a randomly selected car for sale:

	Manual	Automatic
Hatchback	26	27
Sedan	30	39
4WD	9	16

- **a** is a sedan
- **b** is a manual hatchback
- **c** is automatic, but not a sedan.

7 A random selection of hotels in Paris are given a gold star rating for quality, and a green star for environmental friendliness.
Estimate the probability that a randomly selected Paris hotel is given:

a 2 gold stars and 4 green stars
b 3 gold stars or higher
c the same number of gold stars as green stars
d more green stars than gold stars.

		Green star			
		★	★★	★★★	★★★★
Gold star	☆	5	2	1	1
	☆☆	4	10	4	3
	☆☆☆	2	8	13	8
	☆☆☆☆	1	7	9	4

8 The table alongside gives the age distribution of inmates in a prison on December 31, 2011.
A new prisoner entered the prison on January 1, 2012.
Estimate the probability that:

a the prisoner was male
b the prisoner was aged from 17 to 19
c the prisoner was 19 or under and was female
d the prisoner was aged from 30 to 49 and was male.

Age distribution of prison inmates			
Age	*Female*	*Male*	*Total*
15	0	6	6
16	5	23	28
17 - 19	26	422	448
20 - 24	41	1124	1165
25 - 29	36	1001	1037
30 - 34	32	751	783
35 - 39	31	520	551
40 - 49	24	593	617
50 - 59	16	234	250
60+	5	148	153
Total	216	4822	5038

Global context

click here

How much time do we have?

Statement of inquiry: Collecting and interpreting data can help us to understand our place in the world.
Global context: Identities and relationships
Key concept: Relationships
Related concepts: Change, Representation
Objectives: Knowing and understanding, Applying mathematics in real-life contexts
Approaches to learning: Communication, Self-management

C SAMPLE SPACE

A **sample space** is the set of all possible outcomes of an experiment.

Possible ways of representing sample spaces are:

- listing them
- using a 2-dimensional grid
- using a tree diagram
- using a Venn diagram.

Example 4 — Self Tutor

When two coins are tossed, the possible outcomes are:

two heads | head and tail | tail and head | two tails

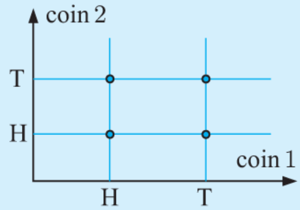
Represent the sample space for tossing two coins using:

a a list **b** a 2-D grid **c** a tree diagram.

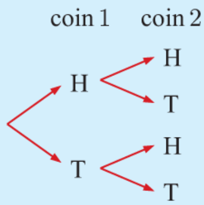
We let H represent a 'head' and T represent a 'tail'.

a {HH, HT, TH, TT}

b



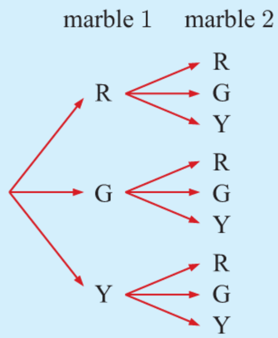
c

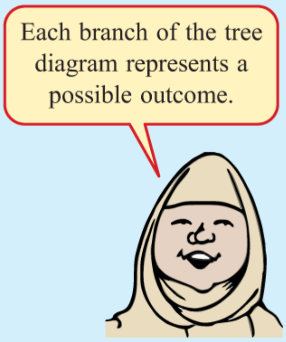


Example 5 — Self Tutor

Illustrate, using a tree diagram, the possible outcomes when drawing two marbles from a bag containing red, green, and yellow marbles.

Let R be the event of getting a red,
G be the event of getting a green, and
Y be the event of getting a yellow.





EXERCISE 13C

1 List the sample space for the following:

a twirling a square spinner labelled A, B, C, D

b the genders of a 2-child family

c the order in which 4 blocks A, B, C, and D can be lined up

d the 8 different 3-child families.

e tossing a coin **i** twice **ii** three times **iii** four times.

2 Use a 2-dimensional grid to illustrate the sample space for:

a rolling a die and tossing a coin simultaneously

b rolling two dice

c rolling a die and spinning a spinner with sides A, B, C, D

d twirling two square spinners, one labelled A, B, C, D, and the other 1, 2, 3, 4.

3 Illustrate on a tree diagram the sample space for:

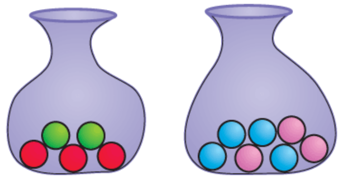
a tossing a 5-cent and 10-cent coin simultaneously

b tossing a coin and twirling an equilateral triangular spinner labelled A, B, and C

c twirling two equilateral triangular spinners labelled 1, 2, 3, and X, Y, Z respectively

d drawing two tickets from a hat containing pink, blue, and white tickets

e selecting bag A or bag B, then drawing a ball from that bag.



4 Draw a Venn diagram to show a class of 20 students in which 7 study History and Geography, 10 study History, and 15 study Geography.

D THEORETICAL PROBABILITY

Once we have represented the sample space of an experiment, we can use it to calculate probabilities.

If a sample space has n outcomes which are **equally likely** to occur when the experiment is performed once, then each outcome has probability $\frac{1}{n}$ of occurring.

An **event** occurs when we obtain an outcome with a particular property or feature.

When the outcomes of an experiment are equally likely, the probability of an event E occurring is given by:

$$\mathbf{P}(E) = \frac{\textbf{number of outcomes corresponding to } E}{\textbf{number of outcomes in the sample space}}$$

Example 6 Self Tutor

Suppose three coins are tossed simultaneously. Find the probability of getting:

a three heads

b at least one head.

The possible outcomes are:

{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

There are 8 possible outcomes.

a P(three heads) $= \frac{1}{8}$ ← three heads only occurs in the outcome HHH
← 8 possible outcomes

b P(at least one head) $= \frac{7}{8}$ ← all outcomes except TTT contain at least one head
← 8 possible outcomes

EXERCISE 13D

1 The three letters O, D, and G are placed at random in a row. Find the probability of:

- **a** spelling DOG
- **b** O appearing first
- **c** O not appearing first
- **d** spelling DOG or GOD.

2 Determine the probability that a randomly selected 3-child family consists of:

- **a** all boys
- **b** all girls
- **c** boy, then girl, then girl
- **d** two girls and a boy
- **e** a girl for the eldest
- **f** at least one boy.

3 Four friends Alex, Bodi, Claire, and Daniel sit randomly in a row. Determine the probability that:

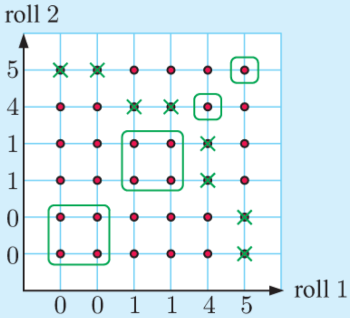
- **a** Alex is on one of the ends
- **b** Claire and Daniel are on the ends
- **c** Bodi is on an end, and Claire is seated next to him
- **d** Alex and Claire sit next to each other.

Example 7 — Self Tutor

A die has the numbers 0, 0, 1, 1, 4, and 5. It is rolled twice.

- **a** Illustrate the possible outcomes using a 2-dimensional grid.
- **b** Hence find the probability of getting:
  - **i** a total of 5
  - **ii** two numbers which are the same.

a



b There are $6 \times 6 = 36$ possible outcomes.

i P(total of 5)

$= \frac{8}{36}$ {those with a $\times$ }

$= \frac{2}{9}$

ii P(same numbers)

$= \frac{10}{36}$ {those circled}

$= \frac{5}{18}$

4 **a** Draw a grid to illustrate the sample space when a 10-cent and a 50-cent coin are tossed simultaneously.

b Hence, determine the probability of getting:

- **i** two heads
- **ii** two tails
- **iii** exactly one head
- **iv** at least one head.

5 A coin and a pentagonal spinner with sectors 1, 2, 3, 4, and 5 are tossed and spun respectively.

a Draw a grid to illustrate the sample space of possible outcomes.

b Use your grid to determine the chance of getting:

- **i** a head and a 4
- **ii** a tail and an odd number
- **iii** an even number
- **iv** a tail or a 3.

6 **a** Use a grid to display the possible outcomes when a pair of dice is rolled.

b Hence, determine the probability of:

- **i** one die showing a 4 and the other a 5
- **ii** both dice showing the same number
- **iii** at least one of the dice showing a 3
- **iv** either a 4 or 6 being displayed
- **v** both dice showing even numbers
- **vi** the sum of the numbers being 7.

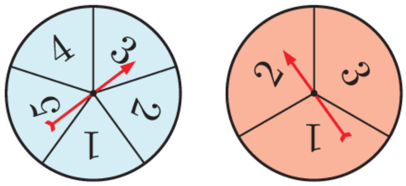
7 A die has the numbers 1, 2, 2, 2, 5, and 6. It is rolled twice.

a Draw a grid to illustrate the possible outcomes.

b Hence, find the probability of getting:

- **i** a 2 and a 5
- **ii** a total of 7
- **iii** numbers which are the same
- **iv** at least one 6
- **v** at least one prime number
- **vi** numbers whose product is at least 5.

8 The spinners shown are spun once, and the numbers spun are multiplied together.



a Find the probability that the result is:

- **i** 9
- **ii** 6
- **iii** greater than 5
- **iv** prime.

b Is the result more likely to be even or odd?

Example 8 **Self Tutor**

In a library group of 50 readers, 31 like science fiction, 20 like detective stories, and 12 dislike both.

a Draw a Venn diagram to represent this information.

b If a reader is randomly selected, find the probability that he or she:

- **i** likes science fiction and detective stories
- **ii** likes exactly one of science fiction and detective stories.

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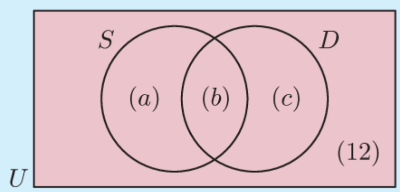
a Let S represent readers who like science fiction, and D represent readers who like detective stories.



We are given that $a + b = 31$

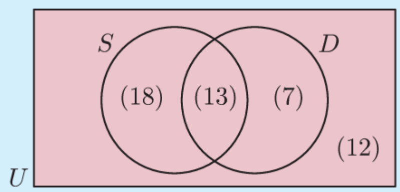
$b + c = 20$

$a + b + c = 50 - 12 = 38$

$\therefore\ c = 38 - 31 = 7$

$\therefore\ b = 13$ and $a = 18$

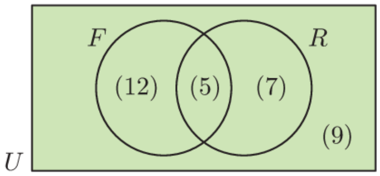
b **i** P(likes both) $= \frac{13}{50}$



ii P(likes exactly one) $= \dfrac{18 + 7}{50}$

$= \frac{1}{2}$

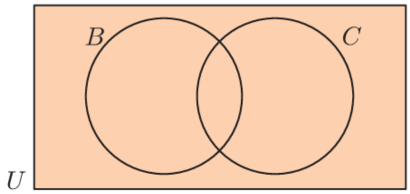
9 The Venn diagram shows the sports played by the students in a Year 10 class.



$F \equiv$ football
$R \equiv$ rugby

- **a** How many students are in the class?
- **b** A student is chosen at random.
  Find the probability that the student:
  - **i** plays football
  - **ii** plays both sports
  - **iii** plays football or rugby
  - **iv** plays exactly one of these sports.

10 In a class of 24 students, 10 study Biology, 12 study Chemistry, and 5 study neither Biology nor Chemistry.



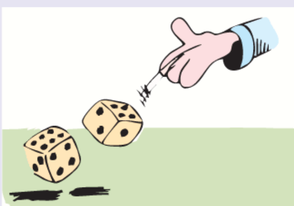
- **a** Copy and complete the Venn diagram.
- **b** Find the probability that a student picked at random from the class studies:
  - **i** Chemistry, but not Biology
  - **ii** both Chemistry and Biology.

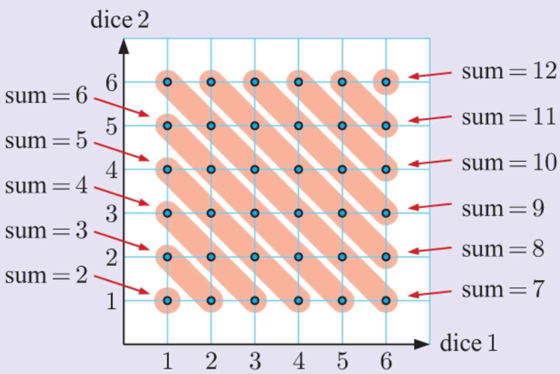
11 50 tourists went on a 'thrill seekers' holiday. 40 went white-water rafting, 21 went paragliding, and each tourist did at least one of these activities.
Find the probability that a randomly selected tourist:

- **a** participated in both activities
- **b** went white-water rafting but not paragliding.

PUZZLE — THE DICE PROBLEM

Suppose we roll a pair of standard dice, and find the sum of the numbers rolled.





We can use a 2-dimensional grid to find the probability of obtaining each sum:

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Can you find another way to label 2 dice with positive whole numbers, so that the sum probabilities are the same as those given above?

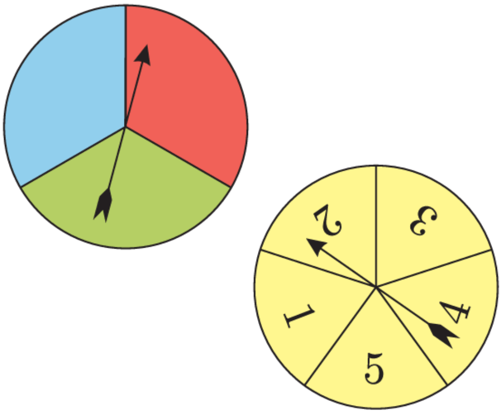
Hint: The same number can appear more than once on a die, and numbers greater than 6 can be used.

E COMPOUND EVENTS

We will now look at calculating probabilities for **combined events**, which are also called **compound events**. We will consider both **independent events** and **dependent events**.

INDEPENDENT EVENTS

Two events are **independent** if the occurrence of each event does not affect the occurrence of the other.



For example, suppose the spinners alongside are spun simultaneously.

Let A be the event *the first spinner lands on red*, and B be the event *the second spinner lands on an even number*.

Whether A occurs or not does not affect the probability of B occurring. Likewise, whether B occurs or not does not affect the probability of A occurring. Therefore, A and B are independent events.

Now, we know that $\text{P}(A) = \frac{1}{3}$, and $\text{P}(B) = \frac{2}{5}$.

By listing each of the possible outcomes, we can see that $\text{P}(A \text{ and } B) = \frac{2}{15}$.

{R1, **R2**, R3, **R4**, R5, G1, G2, G3, G4, G5, B1, B2, B3, B4, B5}

We notice that $\frac{2}{15} = \frac{1}{3} \times \frac{2}{5}$, so $\text{P}(A \text{ and } B) = \text{P}(A) \times \text{P}(B)$.

If two events A and B are **independent**, then $\text{P}(A \textbf{ and } B) = \text{P}(A) \times \text{P}(B)$.

Example 9 Self Tutor

A coin is tossed and a die rolled simultaneously. Find the probability that a tail and a '2' result.

'Getting a tail' and 'rolling a 2' are independent events.

$$\therefore \quad \text{P(a tail \textbf{and} a `2')} = \text{P(a tail)} \times \text{P(a `2')}$$
$$= \tfrac{1}{2} \times \tfrac{1}{6}$$
$$= \tfrac{1}{12}$$

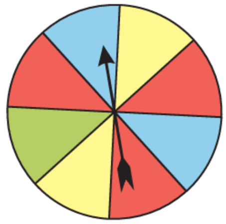
This rule can be extended to any number of independent events.

For example: If A, B, and C are all independent events, then

$$\text{P}(A \textbf{ and } B \textbf{ and } C) = \text{P}(A) \times \text{P}(B) \times \text{P}(C).$$

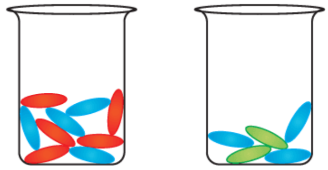
EXERCISE 13E.1

1 A die is rolled, and the spinner alongside is spun. Find the probability of obtaining:



- **a** a '5' and a green
- **b** an even number and a non-red.

2 A disc is selected from each of the containers alongside. Find the probability of selecting:



A **B**

- **a** a red disc from **A** and a green disc from **B**
- **b** a blue disc from both containers.

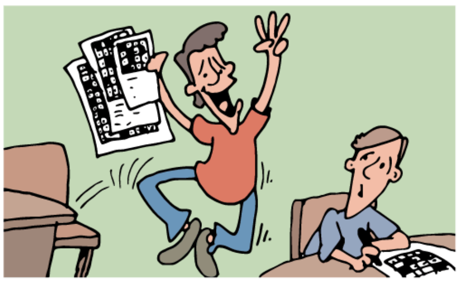
3 A school has two photocopiers. On any one day, machine A has an 8% chance of having a paper jam, and machine B has a 12% chance of having a paper jam.
Determine the probability that, on any one day, both machines will:

- **a** have paper jams
- **b** work uninterrupted.

4 A boy and a girl were each asked what day of the week they were born. Find the probability that:

- **a** the boy was born on a Monday and the girl was born on a Wednesday
- **b** the boy was born on a weekend, but the girl was not
- **c** both children were born on a weekday.

5 Each day, Steve attempts the easy, medium, and hard crosswords in the newspaper. He has probability 0.84 of completing the easy crossword, probability 0.59 of completing the medium crossword, and probability 0.11 of completing the hard crossword.
Find the probability that, on any given day, Steve will:



- **a** complete all 3 crosswords
- **b** leave all 3 crosswords incomplete
- **c** complete the easy and medium crosswords, but not the hard crossword
- **d** complete the medium crossword, but not the other two crosswords.

6 A drawing pin was tossed into the air 600 times. It landed on its back  243 times and on its side  for the remainder.

- **a** Estimate the probability that, when tossed once, this drawing pin will land on its:
  - **i** back
  - **ii** side.
- **b** Suppose the drawing pin is tossed twice. Estimate the probability that:
  - **i** it lands on its back both times
  - **ii** it lands on its side both times.

7 A biased coin is flipped 200 times. It lands on heads 143 times, and on tails for the remainder. If the coin is flipped 3 times, estimate the probability of getting:

- **a** all heads
- **b** all tails.

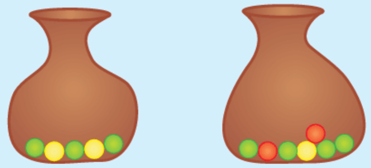
Example 10

Self Tutor

A marble is selected at random from each of the bags alongside.

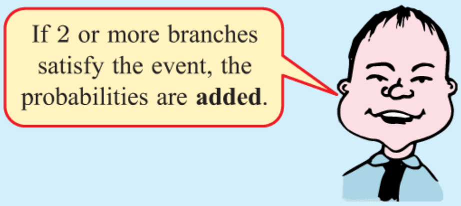
a Draw a tree diagram to display the possible outcomes.

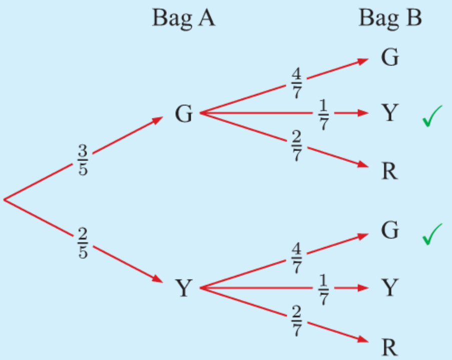
b Find the probability of obtaining a green marble and a yellow marble.



Bag A **Bag B**

a Let G represent a green marble,
Y represent a yellow marble, and
R represent a red marble.





b P(a green and a yellow)

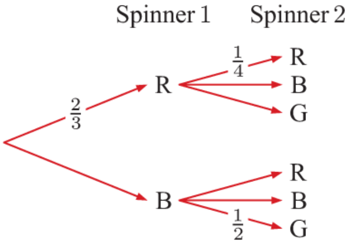
$= $ P(GY or YG) {branches marked ✓}

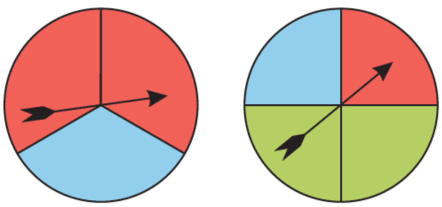
$= \frac{3}{5} \times \frac{1}{7} + \frac{2}{5} \times \frac{4}{7}$

$= \frac{11}{35}$

8 Each of the spinners alongside is spun once.

a Copy and complete this tree diagram for the possible outcomes:





Spinner 1 **Spinner 2**

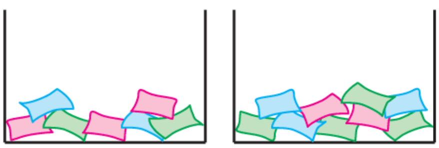
b Find the probability that:

i the spinners land on the same colour

ii the spinners land on different colours

iii exactly one of the spinners lands on blue.

9 A ticket is selected at random from each of the boxes alongside.



Box A **Box B**

a Draw a tree diagram to display the possible outcomes.

b Find the probability of obtaining:

i a blue ticket and a green ticket

ii two tickets of the same colour

iii exactly one pink ticket.

c Find the sum of the probabilities in **b**. Explain your result.

10 Sharon, Christine, and Keith have agreed to meet at a restaurant for lunch. Sharon has probability 0.8 of being on time, Christine has probability 0.7 of being on time, and Keith has probability 0.4 of being on time.

- **a** Draw a tree diagram to display the possible outcomes.
- **b** Determine the probability that at least 2 people will arrive on time.

DEPENDENT EVENTS

Dependent events are events for which the occurrence of one event *does affect* the occurrence of the other event.

Suppose a bag contains 5 blue discs and 3 yellow discs. One disc is selected at random, and put to one side. A second disc is then selected from the bag.

Let A be the event that the *first* disc is blue, and B be the event that the *second* disc is blue.

Now, if A occurs, then $P(B) = \frac{4}{7}$ ← 4 blue discs remaining, ← 7 discs left to choose from

However, if A does not occur, then $P(B) = \frac{5}{7}$ ← 5 blue discs remaining, ← 7 discs left to choose from

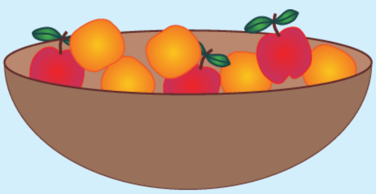
The occurrence of A *does* affect the probability of B occurring. Therefore A and B are **dependent events**.

The rule for finding compound event probabilities for dependent events is different from the rule for independent events.

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \times P(B \text{ given that } A \text{ has occurred})$.

Example 11 Self Tutor

A fruit bowl contains 3 apples and 5 oranges. Callan selects a piece of fruit at random, and eats it. His sister Michelle then selects a piece of fruit for herself.



Find the probability that:

- **a** both pieces of fruit selected are apples
- **b** Callan selects an orange and Michelle selects an apple.

a P(both are apples)

= P(Callan selects an apple *and* Michelle selects an apple)

= P(Callan selects an apple)

$\times$ P(Michelle selects an apple *given* that Callan selects an apple)

$= \frac{3}{8} \times \frac{2}{7}$ ← 2 apples remaining, ← 7 pieces of fruit to choose from

$= \frac{6}{56}$

$= \frac{3}{28}$

b P(Callan selects an orange *and* Michelle selects an apple)

$= $ P(Callan selects an orange)

$\times$ P(Michelle selects an apple *given* that Callan selects an orange)

$= \frac{5}{8} \times \frac{3}{7}$ ← 3 apples remaining ← 7 pieces of fruit to choose from

$= \frac{15}{56}$

EXERCISE 13E.2

1 A bucket contains 2 orange, 5 white, and 3 yellow ping pong balls. Two balls are selected at random from the bucket, the second being selected without replacing the first. Find the probability that:

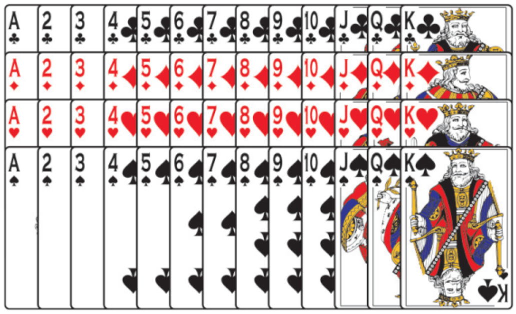
a both balls are orange **b** the first ball is yellow, and the second ball is white.

2 Two cards are selected, without replacement, from a standard deck of 52 cards. Find the probability that:

a both of the cards are red

b the first card is a club, and the second card is a diamond

c both of the cards are aces.



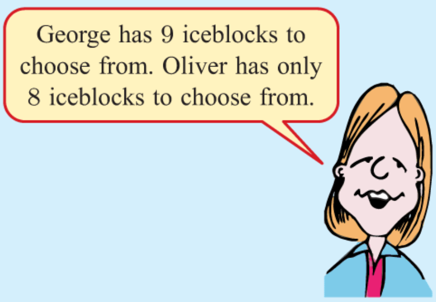
Example 12 **Self Tutor**

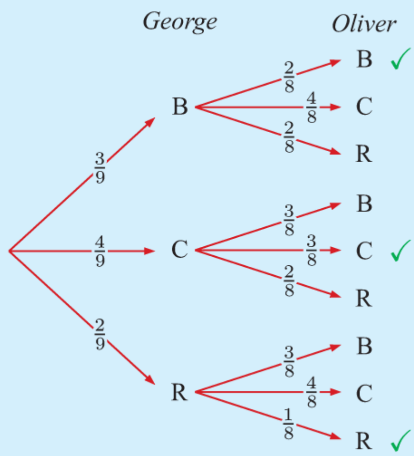
A container holds 3 banana iceblocks, 4 chocolate iceblocks, and 2 raspberry iceblocks. George randomly selects an iceblock from the container and eats it. His brother Oliver then randomly selects an iceblock from the container.

a Draw a tree diagram to display the possible outcomes.

b Find the probability that the brothers selected the same type of iceblock.

a Let B represent a banana iceblock,
C represent a chocolate iceblock, and
R represent a raspberry iceblock.





b P(brothers selected the same type of iceblock)

$= $ P(BB or CC or RR) {those marked ✓}

$= \frac{3}{9} \times \frac{2}{8} + \frac{4}{9} \times \frac{3}{8} + \frac{2}{9} \times \frac{1}{8}$

$= \frac{20}{72}$

$= \frac{5}{18}$

3 A drawer contains 5 blue pens, 3 red pens, and 2 green pens. A pen is selected at random from the drawer and put to one side. A second pen is then selected at random from the drawer.

a Draw a tree diagram to display the possible outcomes.

b Find the probability that:

i the selected pens are the same colour **ii** exactly one of the pens is green.

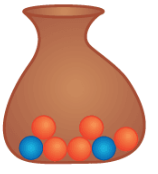
4 Matt is the best player in his lacrosse team. He has an injured knee, and has only a 60% chance of playing the next game. The team has a 70% chance of winning the next game if Matt plays, but only a 45% chance of winning if he does not play.

a Represent this information on a tree diagram.

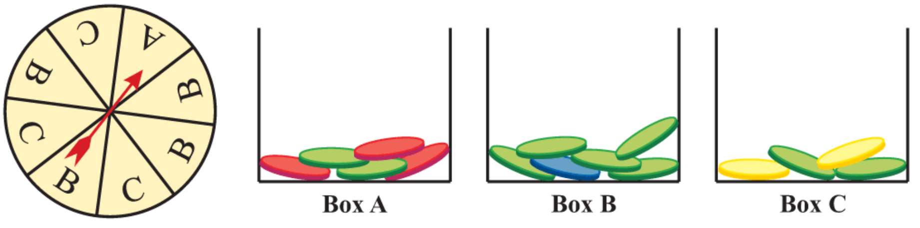
b Find the probability that the team will win the next game.

5 In a netball team, the Goal Shooter takes 65% of the team's shots, and scores 70% of the time. The Goal Attack takes the remainder of the shots, and scores 60% of the time.
Find the probability that the team will score with their next shot.

6 A fair coin is tossed. If the result is heads, one marble is selected from the bag alongside. If the result is tails, two marbles are selected, without replacement. Find the probability that at least one red marble will be selected.

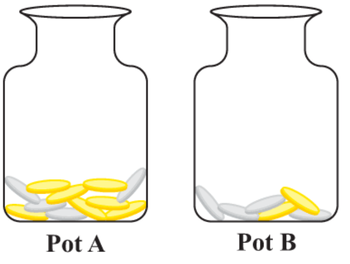


7 The spinner below is used to select box A, B, or C. Two discs are then randomly selected without replacement from that box.



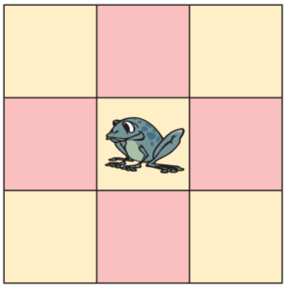
Find the probability that exactly one green disc is selected.

8 A coin is selected at random from pot A and placed in pot B. Then, a coin is selected at random from pot B and placed in pot A. Finally, a coin is selected at random from pot A. Find the probability that this coin is gold.



9 Jill and Mandy are considering whether to go to a party. If their friend Donna does *not* go, the probability that Jill will attend is 0.4, and the probability that Mandy will attend is 0.6. If Donna *does* go to the party, Jill and Mandy's probabilities will increase to 0.5 and 0.7 respectively.
The probability that Donna will go to the party is 0.8. Find the probability that exactly two of the three friends will attend.

10 A frog is in the middle of a 3×3 arrangement of squares. Each time the frog jumps, it lands with equal probability on one of the adjacent squares (either horizontally, vertically, or diagonally). Find the probability that, after 3 jumps, the frog is on:



a the middle square **b** a corner square

c an edge square that is not a corner square.

F CONDITIONAL PROBABILITY

We are often interested in the probability of an event occurring *given* that another event occurs.

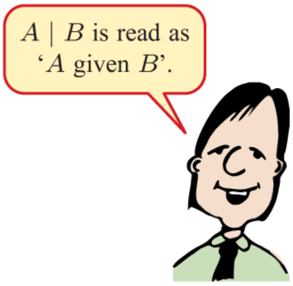
For example:

"Given that it is raining, what is the probability of Newcastle winning?"

"Given that George likes sausages, what is the probability that he also likes steak?"

"Given that Mae has to pick up her son on the way, what is the probability that she will be late?"

If A and B are events, then $\mathrm{P}(A \mid B)$ means "the probability of A occurring given that B has occurred".



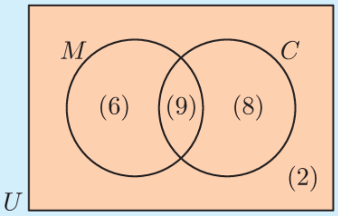
Venn diagrams, two-way tables, and 2-dimensional grids are useful for answering questions involving conditional probability.

Example 13 **Self Tutor**

In a group of 25 students, 15 like milk (M), and 17 like iced coffee (C). Two students like neither, and 9 students like both.

One student is randomly selected from the class. Find the probability that the student:

a likes milk **b** likes milk given that he or she likes iced coffee.



a 15 of the 25 students like milk.

$\therefore \quad \mathrm{P}(M) = \frac{15}{25} = \frac{3}{5}$

b Of the 17 who like iced coffee, 9 also like milk.

$\therefore \quad \mathrm{P}(M \mid C) = \frac{9}{17}$

EXERCISE 13F

1 In a class of 25 students, 19 have fair hair, 15 have blue eyes, and 22 have fair hair, blue eyes, or both. A child is selected at random. Find the probability that the child has:

- **a** fair hair and blue eyes
- **b** blue eyes, given that the child has fair hair.

2 A French examination has an aural part and a written part. When 30 students sit for the examination, 25 pass aural, 26 pass written, and 3 fail both parts. Determine the probability that a randomly selected student:

- **a** passed written, given that they passed aural
- **b** failed written, given that they passed aural.

3 

28 members of a youth group went bushwalking. 23 got sunburn, 8 got blisters, and 5 got both sunburn and blisters. Determine the probability that a randomly selected person:

- **a** got blisters or sunburn
- **b** got blisters, given that the person was sunburnt
- **c** was sunburnt, given that the person did not get blisters.

4 In a small town there are 3 supermarkets: P, Q, and R. 60% of the population shop at P, 36% shop at Q, and 34% shop at R. 18% shop at P and Q, 15% shop at P and R, 4% shop at Q and R, and 2% shop at all 3 supermarkets. A person is selected at random.
Determine the probability that the person shops at:

- **a** none of the supermarkets
- **b** P, given that the person shops at at least one supermarket
- **c** R, given that the person shops at P or Q or both
- **d** Q, given that the person shops at exactly one supermarket.

Example 14 **Self Tutor**

A restaurant owner collected the following data from his customers one night:

		Dine in	Takeaway	Delivery
Amount spent	Less than \$20	12	17	9
	At least \$20	31	15	13

Find the probability that a randomly selected customer:

- **a** spent at least \$20
- **b** spent less than \$20, given that they had their meal delivered
- **c** dined in, given that they spent less than \$20.

		Dine in	Takeaway	Delivery	*Total*
Amount spent	Less than \$20	12	17	9	38
	At least \$20	31	15	13	59
	Total	43	32	22	97

a P(spent at least \$20) $= \frac{59}{97}$

b Of the 22 customers who had their meal delivered, 9 spent less than \$20.

$\therefore$ P(spent less than \$20 | delivered) $= \frac{9}{22}$

c Of the 38 customers who spent less than \$20, 12 dined in.

$\therefore$ P(dined in | spent less than \$20) $= \frac{12}{38} = \frac{6}{19}$

5 Lee's soccer team played 37 games this season. Their results are shown in the two-way table alongside.
Find the probability that a randomly selected game was:

a won **b** won, given that it was at home

c at home, given that it was won.

	Win	Draw	Lose
Home	11	5	3
Away	6	8	4

6 The Year 10 students at a school were asked how many people and pets lived in their house. The results are shown alongside.
Find the probability that a randomly selected student's house has:

a 2 pets, given that it has 4 people

b 3 people, given that it has at least 2 pets

c at most 1 pet, given that it has at least 4 people.

		Number of people			
		2	3	4	5+
Number of pets	0	4	6	7	3
	1	4	11	12	6
	2	2	9	11	7
	3+	1	5	7	5

Example 15 **Self Tutor**

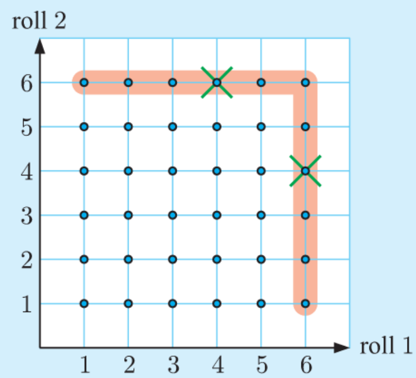
A pair of dice is rolled. Given that at least one of the dice shows a 6, find the probability that the sum of the rolls is 10.

Let A be the event "the sum of the rolls is 10".

Let B be the event "at least one of the dice shows a 6".

Of the 11 outcomes in B {those highlighted}, 2 of the outcomes are also in A {those with a $\times$}.

$\therefore$ $\text{P}(A \mid B) = \frac{2}{11}$

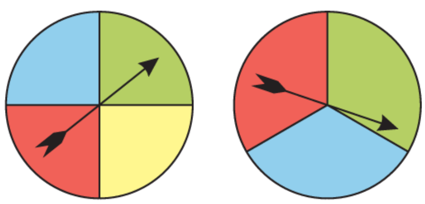


7 A pair of dice is rolled.

a Given that both of the dice show less than 5, find the probability that the sum of the rolls is 6.

b Given that the sum of the rolls is 8, find the probability that at least one of the dice shows a 3.

8 Suppose each spinner alongside is spun once.
Given that exactly one of the spins is green, find the probability that the result of spinner 2 is green.

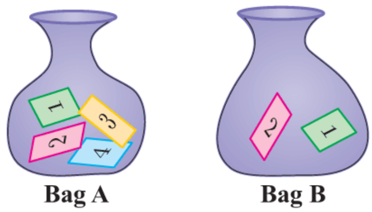


Spinner 1 **Spinner 2**

9 A card is selected at random from each of the bags shown.

a Find the probability that the card drawn from bag A is a 4.

b You are told that the sum of the selected cards is 5.



i Do you think this increases or decreases the probability that the card drawn from bag A is a 4? Explain your answer.

ii Find the probability that the card drawn from bag A is a 4, given that the sum of the selected cards is 5.

G MUTUALLY EXCLUSIVE AND INDEPENDENT EVENTS

MUTUALLY EXCLUSIVE EVENTS

Two events are **mutually exclusive** or **disjoint** if they have no common outcomes.

If A and B are mutually exclusive events then P(A and B) $= 0$.

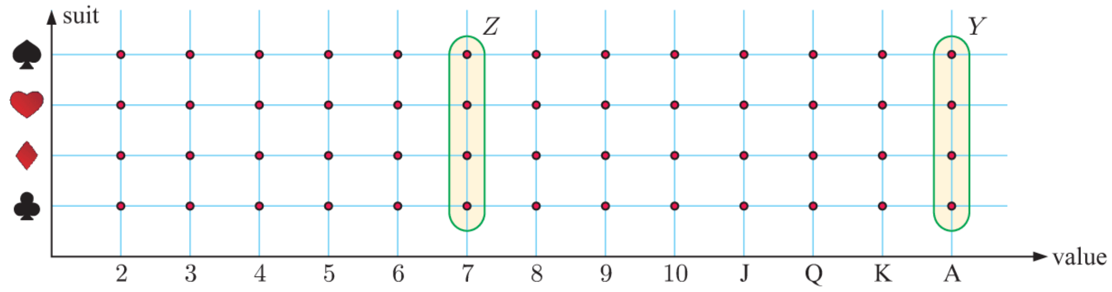
For example, suppose we select a card at random from a normal pack of 52 playing cards. Consider these events:

Event X: the card is a heart *Event Y*: the card is an ace *Event Z*: the card is a 7

Notice that:

- X and Y have a common outcome, the Ace of hearts
- X and Z have a common outcome, the 7 of hearts
- Y and Z do not have a common outcome, so they are mutually exclusive.

The events Y and Z are shown on the 2-dimensional grid below:

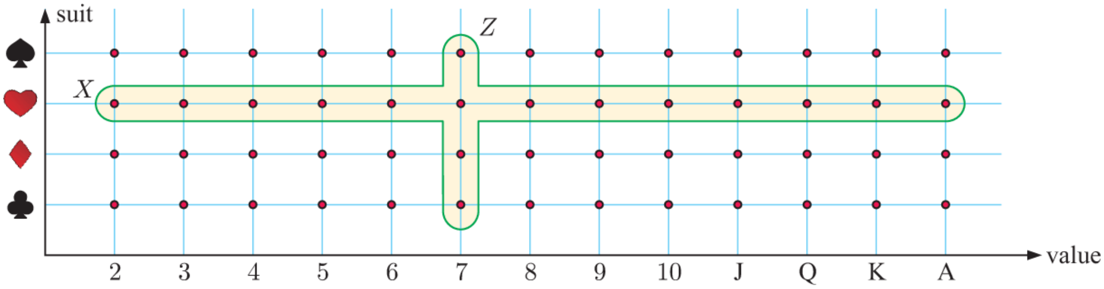


Notice that $\text{P}(Y \textbf{ or } Z) = \frac{8}{52}$ and $\text{P}(Y) + \text{P}(Z) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$.

If two events A and B are **mutually exclusive**, then

$$\text{P}(A \textbf{ or } B) = \text{P}(A) + \text{P}(B)$$

Now consider the events X and Z, which are *not* mutually exclusive since they have a common outcome: the 7 of hearts.



Notice that $\text{P}(X \textbf{ or } Z) = \frac{16}{52}$ and $\text{P}(X) + \text{P}(Z) = \frac{13}{52} + \frac{4}{52} = \frac{17}{52}$.

These values are not the same, since when we find $\text{P}(X) + \text{P}(Z)$ the common outcome is counted *twice*.

Actually, $\text{P}(X \textbf{ or } Z) = \text{P}(X) + \text{P}(Z) - \text{P}(X \textbf{ and } Z)$.

If two events A and B are **not mutually exclusive** then

$$\text{P}(A \textbf{ or } B) = \text{P}(A) + \text{P}(B) - \text{P}(A \textbf{ and } B).$$

Notice the similarity to $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Example 16 **Self Tutor**

Suppose $\text{P}(A) = 0.2$, $\text{P}(B) = 0.6$, and $\text{P}(A \text{ and } B) = 0.1$.

a Are A and B mutually exclusive events? Explain your answer.

b Find $\text{P}(A \text{ or } B)$.

a A and B are *not* mutually exclusive, since $\text{P}(A \text{ and } B) \neq 0$.

b $$\begin{aligned}\text{P}(A \text{ or } B) &= \text{P}(A) + \text{P}(B) - \text{P}(A \text{ and } B)\\ &= 0.2 + 0.6 - 0.1\\ &= 0.7\end{aligned}$$

INDEPENDENT EVENTS

In **Section E**, we saw that two events are **independent** if the occurrence of each event does not affect the occurrence of the other.

If A and B are independent events, then $\text{P}(A \text{ and } B) = \text{P}(A) \times \text{P}(B)$.

Example 17 **Self Tutor**

Suppose $\text{P}(X) = 0.6$ and $\text{P}(Y) = 0.2$. Find $\text{P}(X \text{ and } Y)$ given that X and Y are:

a mutually exclusive **b** independent.

a If X and Y are mutually exclusive, then $\text{P}(X \text{ and } Y) = 0$.

b If X and Y are independent, then $$\begin{aligned}\text{P}(X \text{ and } Y) &= \text{P}(X) \times \text{P}(Y)\\ &= 0.6 \times 0.2\\ &= 0.12\end{aligned}$$

EXERCISE 13G

1 An ordinary die with faces 1, 2, 3, 4, 5, and 6 is rolled once. Consider these events:

A: rolling a 1 — B: rolling a 3
C: rolling an odd number — D: rolling an even number
E: rolling a prime number — F: rolling a result greater than 3.

List the pairs of events which are mutually exclusive.

2 A coin and an ordinary die are tossed and rolled simultaneously.

- **a** Draw a grid showing the 12 possible outcomes.
- **b** Let A be the event 'tossing a head' and B be the event 'rolling a 5'.
  - **i** Are A and B mutually exclusive? Explain your answer.
  - **ii** Find $\text{P}(A \text{ or } B)$ and $\text{P}(A \text{ and } B)$.
  - **iii** Check that $\text{P}(A \text{ or } B) = \text{P}(A) + \text{P}(B) - \text{P}(A \text{ and } B)$.

3 Suppose $\text{P}(A) = 0.7$, $\text{P}(B) = 0.2$, and $\text{P}(A \text{ and } B) = 0.15$.

- **a** Are A and B mutually exclusive events? Explain your answer.
- **b** Find $\text{P}(A \text{ or } B)$.

4 Suppose $\text{P}(X) = 0.3$ and $\text{P}(X \text{ or } Y) = 0.7$. Given that X and Y are mutually exclusive, find $\text{P}(Y)$.

5 Suppose $\text{P}(C) = 0.6$ and $\text{P}(D) = 0.7$. Explain why C and D are not mutually exclusive.

6 Let $\text{P}(A) = 0.4$ and $\text{P}(B) = 0.25$. Find $\text{P}(A \text{ and } B)$ given that A and B are:

- **a** mutually exclusive
- **b** independent.

7 X and Y are independent events such that $\text{P}(X) = 0.5$ and $\text{P}(Y) = 0.3$. Find $\text{P}(X \text{ or } Y)$.

8 A and B are independent events such that $\text{P}(A \text{ or } B) = 0.9$ and $\text{P}(A \text{ and } B) = 0.4$. Find $\text{P}(A)$ and $\text{P}(B)$ given that $\text{P}(A) > \text{P}(B)$.

REVIEW SET 13A

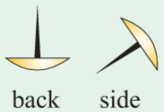
1 Donna kept records of the number of clients she interviewed over consecutive days.

- **a** For how many days did Donna keep records?
- **b** Estimate the probability that tomorrow Donna will interview:
  - **i** no clients
  - **ii** four or more clients
  - **iii** less than three clients.

Number of clients	*Frequency*
0	1
1	6
2	12
3	8
4	6
5	3
6	0
7	2

2 A coin and a pentagonal spinner with sides labelled A, B, C, D, and E are tossed and spun simultaneously. Illustrate the possible outcomes using a 2-dimensional grid.

3 When a box of drawing pins was dropped onto the floor, 49 pins landed on their backs, and 32 landed on their sides. Estimate, to 2 decimal places, the probability of a drawing pin landing:



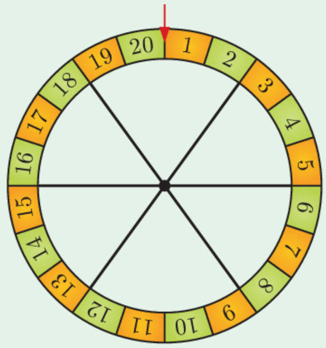
- **a** on its back
- **b** on its side.

4 A wheel numbered 1 to 20 is spun.
Find the probability that the result is:

a 13

b a multiple of 3

c greater than 11.



5 On a particular day, 500 people visited a carnival. 300 people rode the Ferris wheel, and 350 people rode the roller coaster. Each person rode at least one of these attractions.

a Display this information on a Venn diagram.

b Find the probability that a randomly chosen person:

i rode the Ferris wheel, but not the roller coaster

ii rode the roller coaster, given that they rode the Ferris wheel.

6 A bag contains 4 green and 3 red marbles. Two marbles are randomly selected from the bag, the first being put to one side before the second is drawn. Determine the probability that:

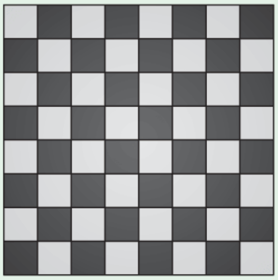
a both are green

b they are different in colour.

7 A chess piece is placed on a random square of an 8×8 chess board. A second piece is then placed at random on one of the unoccupied squares.
Find the probability that:

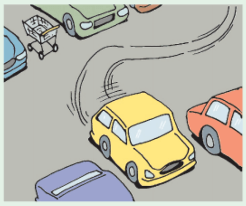
a the two pieces lie on the same row

b the pieces lie on the same row or column.



8 Brothers Paul, Cameron, and Bruce play in the same rugby team. Their probabilities of getting injured during the season are 0.4, 0.3, and 0.2 respectively. What is the most likely number of injured brothers during the season?

9 A married couple own a large car and a small car. Glen uses the small car 30% of the time. When he goes to the shops, the probability that he can park in the car park is 80% if he has the small car, and 60% if he has the large car.
Find the probability that, on a given day, Glen is able to park in the shop's car park.



10 The two-way table alongside describes the books on Elizabeth's bookshelf.

	Biography	Novel	Reference
Softcover	3	22	6
Hardcover	5	7	4

a How many books are on Elizabeth's bookshelf?

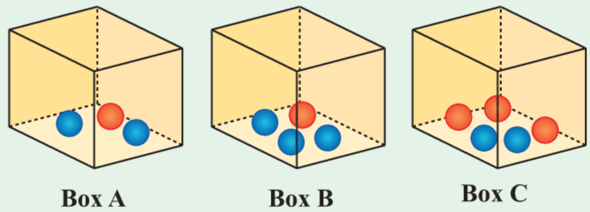
b Find the probability that a randomly selected book is:

i a softcover book

ii a hardcover reference book

iii a novel, given that it has a hard cover.

11 A ball is selected from box A and placed in box B. A ball is then selected from box B, and placed in box C. A ball is then selected from box C.
Find the probability that the ball is blue.



12 Tyson has one of each type of Australian coin in his pocket. He randomly selects two coins from his pocket, replacing the first coin before selecting the second.

- **a** Draw a 2-dimensional grid to display the possible outcomes.
- **b** Find the probability that:
  - **i** both coins are silver
  - **ii** at least one of the coins is the 50 cent coin
  - **iii** the value of the coins is at least 65 cents.
- **c** Given that the value of the coins is at least 65 cents, find the probability that at least one of the coins is gold.

13 Let $\mathrm{P}(A) = 0.2$ and $\mathrm{P}(B) = 0.7$. Find $\mathrm{P}(A \text{ and } B)$ given that A and B are:

- **a** mutually exclusive
- **b** independent.

REVIEW SET 13B

1 Pierre conducted a survey to determine the ages of people walking through a shopping mall. The results are shown in the table alongside. Estimate the probability that the next person Pierre meets in the shopping mall will be:

Age	*Frequency*
0 - 19	22
20 - 39	43
40 - 59	39
60+	14

- **a** between 20 and 39 years of age
- **b** less than 40 years of age
- **c** at least 20 years of age.

2 Use a tree diagram to illustrate the sample spaces for the following:

- **a** Bags A, B, and C contain green and yellow tickets. A bag is selected and then a ticket taken from it.
- **b** Martina and Justine play tennis. The first to win three sets wins the match.

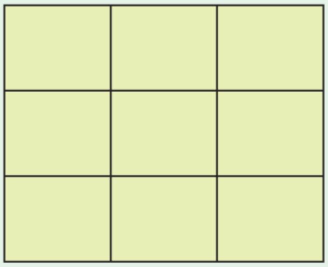
3 The two-way table alongside shows the results from asking the question "Do you like the school uniform?".
If a student is randomly selected from these year groups, estimate the probability that the student:

	Likes	Dislikes
Year 8	129	21
Year 9	108	42
Year 10	81	69

- **a** likes the school uniform
- **b** dislikes the school uniform
- **c** is in Year 8 and dislikes the uniform.

4 The digits 1, 6, and 9 are placed in random order to create a 3 digit number. Find the probability that this number will be a perfect square.

5 A farmer fences his rectangular property into 9 rectangular paddocks as shown.
A paddock is selected at random. Find the probability that it has:



a no fences on the boundary of the property
b one fence on the boundary of the property
c two fences on the boundary of the property.

6 A paper plate is tossed in the air 50 times. It lands face up 37 times, and lands face down 13 times.

a Estimate the probability that the paper plate will land face up next time it is thrown.
b If the plate is tossed in the air three times, estimate the probability that the plate will land face down on all three occasions.

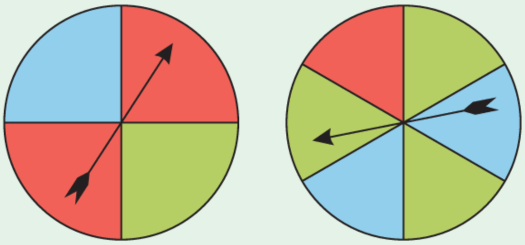
7 Bag X contains three white and two red marbles. Bag Y contains one white and three red marbles. A bag is randomly chosen, and two marbles are drawn from it.

a Illustrate the given information on a tree diagram.
b Determine the probability of drawing two marbles of the same colour.

8 A knife block contains 6 knives of different sizes. Each knife fits exactly into its specific hole, and will also fit into the holes of larger knives.
Suppose a knife is chosen at random, and a hole in the knife block is chosen at random. Find the probability that the knife will fit in the hole.

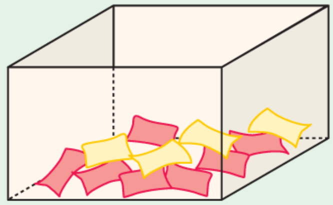
9 Matthew is taking a mathematics test. There is a 2% chance that his calculator will not work. If his calculator works, Matthew has a 70% probability of passing the test. If his calculator does not work, Matthew has a 55% probability of passing the test.
Find the probability that Matthew passes the test.

10 The spinners alongside are each spun once. Given that the spins are the same colour, find the probability that they are both red.



11 Events A, B, and C are all mutually exclusive with each other.
$\text{P}(A \text{ or } B) = 0.5$, $\text{P}(B \text{ or } C) = 0.75$, and $\text{P}(A \text{ or } C) = 0.55$. Find the probability of each event occurring.

12 Shelley draws 3 cards without replacement from the container alongside. She will win a prize if all 3 cards are the same colour.



a Find the probability that Shelley will win a prize.

b Suppose the rules change so that Shelley now draws her cards *with* replacement.

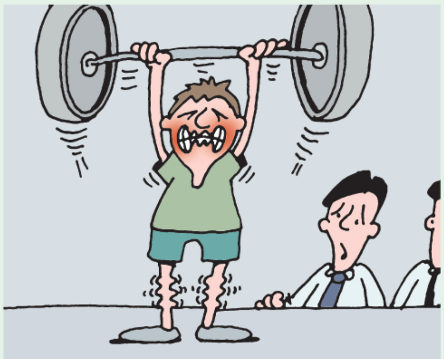
i Do you think this increases or decreases her probability of winning? Explain your answer.

ii Find the probability of Shelley winning.

13 In a weightlifting competition, 3 competitors are given one chance to lift a weight of their choosing. The table alongside shows the weights chosen by each competitor, and their probability of lifting it.

Competitor	*Weight*	*Probability*
Ihor	210 kg	0.5
Ruslan	225 kg	0.4
Behdad	240 kg	0.3

The winner is the competitor who successfully lifts the greatest weight. If nobody lifts their weight, the competition is a tie.



a Find the probability that:

i Ihor **ii** Ruslan **iii** Behdad

will win the competition.

b Ihor is considering increasing his weight to 230 kg, so he is lifting more than Ruslan.
However, the probability that he can lift this weight is only 0.34.
Would this strategy increase or decrease Ihor's probability of winning the competition?