

INTERNATIONAL BACCALAUREATE
Mathematics: applications and interpretation

MAI

**EXERCISES [MAI 5.6-5.7]
RULES OF DIFFERENTIATION**

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

BASIC RULES OF DIFFERENTIATION

1. [Maximum mark: 3 per function]

Differentiate the following functions:

$y = 7x^3 - 2 - 5e^x - 3\sin x$	
$y = \sqrt{x} + \ln x$	
$y = \sqrt{x} \ln x$	
$y = \frac{\ln x}{\sqrt{x}}$	
$y = \frac{2x+1}{3x-5}$	
$y = x + ex + \ln \pi$	
$y = x^2 + \ln x + x^2 \ln x$	
$y = x \sin x \ln x$	
$y = x^2 e^x \ln x$	

5. [Maximum mark: 4]

Let $f(x) = 6\sqrt[3]{x^2}$. Find $f'(x)$.

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6. [Maximum mark: 6]

Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$

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7. [Maximum mark: 5]

Let $g(x) = 2x \sin x$.

(a) Find $g'(x)$. [3]

(b) Find the **exact** value of the gradient of the graph of g at $x = \pi$. [2]

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8. [Maximum mark: 4]

Consider the function $f(x) = k \sin x + 3x$, where k is a constant.

(a) Find $f'(x)$. [2]

(b) When $x = \frac{\pi}{3}$, the gradient of the curve of $f(x)$ is 8. Find the value of k . [2]

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9. [Maximum mark: 5]

Let $f(x) = \frac{3x^2}{5x-1}$.

(a) Write down the **equation** of the vertical asymptote of $y = f(x)$.

(b) Find $f'(x)$. Give your answer in the form $\frac{ax^2 + bx}{(5x-1)^2}$ where a and $b \in \mathbb{Z}$.

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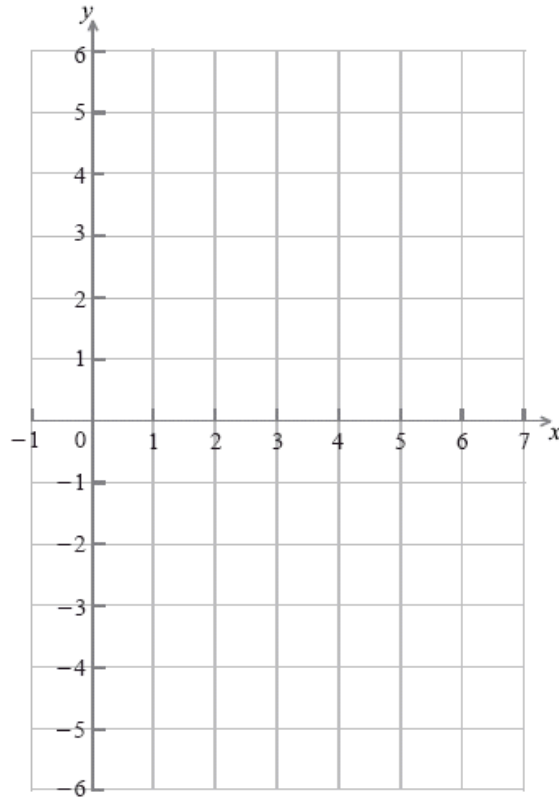
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10. [Maximum mark: 8]

Let $f(x) = x \cos x$, for $0 \leq x \leq 6$.

- (a) Find $f'(x)$. [3]
- (b) On the grid below, sketch the graph of $y = f'(x)$. [3]
- (c) Write down the range of the function $y = f'(x)$, for $0 \leq x \leq 6$ [2]



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11. [Maximum mark: 7]

Let $f(x) = e^x \cos x$.

- (a) Find $f'(x)$. [3]
- (b) Find the gradient of the normal to the curve of f at $x = \pi$. [2]
- (c) Find the gradient of the tangent to the curve of f at $x = \frac{\pi}{4}$. [2]

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12. [Maximum mark: 7]

Let $f(x) = xe^x$.

- (a) Find the equation of the tangent line at $x = 0$. [3]
- (b) Find the equation of the normal line at $x = 0$. [2]
- (c) Solve the equation $f'(x) = 0$. [2]

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13. [Maximum mark: 3 per function]

Find the derivative of each function below. [cover 3rd column; then compare with your answer]

Function $f(x)$	Derivative $f'(x)$	Compare with correct answer
$f(x) = e^{x-5}$		$f'(x) = e^{x-5}$
$f(x) = e^{2x}$		$f'(x) = 2e^{2x}$
$f(x) = e^{2x-5}$		$f'(x) = 2e^{2x-5}$
$f(x) = e^{5-2x}$		$f'(x) = -2e^{5-2x}$
$f(x) = e^{2x^2-5}$		$f'(x) = 4xe^{2x^2-5}$
$f(x) = e^{2x^7-5}$		$f'(x) = 14x^6e^{2x^7-5}$
$f(x) = \sin(x-5)$		$f'(x) = \cos(x-5)$
$f(x) = \sin 2x$		$f'(x) = 2 \cos 2x$
$f(x) = \sin(2x-5)$		$f'(x) = 2 \cos(2x-5)$
$f(x) = \sin(5-2x)$		$f'(x) = -2 \cos(5-2x)$
$f(x) = \sin(2x^2-5)$		$f'(x) = 4x \cos(2x^2-5)$
$f(x) = \sin(2x^7-5)$		$f'(x) = 14x^6 \cos(2x^7-5)$
$f(x) = \cos(x-5)$		$f'(x) = -\sin(x-5)$
$f(x) = \cos 2x$		$f'(x) = -2 \sin 2x$
$f(x) = \cos(2x-5)$		$f'(x) = -2 \sin(2x-5)$
$f(x) = \cos(5-2x)$		$f'(x) = 2 \sin(5-2x)$
$f(x) = \cos(2x^2-5)$		$f'(x) = -4x \sin(2x^2-5)$
$f(x) = \cos(2x^7-5)$		$f'(x) = -14x^6 \sin(2x^7-5)$

Function $f(x)$	Derivative $f'(x)$	Compare with correct answer
$f(x) = \tan(x-5)$		$f'(x) = \frac{1}{\cos^2(x-5)}$
$f(x) = \tan 2x$		$f'(x) = \frac{2}{\cos^2(2x)}$
$f(x) = \tan(2x-5)$		$f'(x) = \frac{2}{\cos^2(2x-5)}$
$f(x) = \tan(5-2x)$		$f'(x) = \frac{-2}{\cos^2(5-2x)}$
$f(x) = \tan(2x^2-5)$		$f'(x) = \frac{4x}{\cos^2(2x^2-5)}$
$f(x) = \tan(2x^7-5)$		$f'(x) = \frac{14x^6}{\cos^2(2x^7-5)}$
$f(x) = \ln(x-5)$		$f'(x) = \frac{1}{x-5}$
$f(x) = \ln 2x$		$f'(x) = \frac{2}{2x} = \frac{1}{x}$
$f(x) = \ln(2x-5)$		$f'(x) = \frac{2}{2x-5}$
$f(x) = \ln(5-2x)$		$f'(x) = \frac{-2}{5-2x} = \frac{2}{2x-5}$
$f(x) = \ln(2x^2-5)$		$f'(x) = \frac{4x}{2x^2-5}$
$f(x) = \ln(2x^7-5)$		$f'(x) = \frac{14x^6}{2x^7-5}$
$f(x) = \sqrt{x-5}$		$f'(x) = \frac{1}{2\sqrt{x-5}}$
$f(x) = \sqrt{2x-5}$		$f'(x) = \frac{1}{\sqrt{2x-5}}$
$f(x) = \sqrt{5-2x}$		$f'(x) = \frac{-1}{\sqrt{5-2x}}$
$f(x) = \sqrt{2x^2-5}$		$f'(x) = \frac{2x}{\sqrt{2x^2-5}}$
$f(x) = \sqrt{2x^7-5}$		$f'(x) = \frac{7x^6}{\sqrt{2x^7-5}}$

Function $f(x)$	Derivative $f'(x)$	Compare with correct answer
$f(x) = (x-5)^3$		$f'(x) = 3(x-5)^2$
$f(x) = (2x-5)^3$		$f'(x) = 6(2x-5)^2$
$f(x) = (5-2x)^3$		$f'(x) = -6(5-2x)^2$
$f(x) = (2x^2-5)^3$		$f'(x) = 12x(2x^2-5)^2$
$f(x) = (2x^7-5)^3$		$f'(x) = 42x^6(2x^7-5)^2$
$f(x) = (x-5)^{-3}$		$f'(x) = -3(x-5)^{-4}$
$f(x) = (2x-5)^{-3}$		$f'(x) = -6(2x-5)^{-4}$
$f(x) = (5-2x)^{-3}$		$f'(x) = 6(5-2x)^{-4}$
$f(x) = (2x^2-5)^{-3}$		$f'(x) = -12x(2x^2-5)^{-4}$
$f(x) = (2x^7-5)^{-3}$		$f'(x) = -42x^6(2x^7-5)^{-4}$
$f(x) = \frac{1}{x-5}$ $= (x-5)^{-1}$		$f'(x) = \frac{-1}{(x-5)^2}$
$f(x) = \frac{1}{2x-5}$		$f'(x) = \frac{-2}{(2x-5)^2}$
$f(x) = \frac{1}{5-2x}$		$f'(x) = \frac{2}{(5-2x)^2}$
$f(x) = \frac{1}{2x^2-5}$		$f'(x) = \frac{-4x}{(2x^2-5)^2}$
$f(x) = \frac{1}{2x^7-5}$		$f'(x) = \frac{-14x^6}{(2x^7-5)^2}$

Function $f(x)$	Derivative $f'(x)$	Compare with correct answer
$f(x) = e^{\sin x}$		$f'(x) = e^{\sin x} \cos x$
$f(x) = \sin(\ln x)$		$f'(x) = \frac{\cos(\ln x)}{x}$
$f(x) = \sin^4 x$		$f'(x) = 4 \sin^3 x \cos x$
$f(x) = \cos(e^x)$		$f'(x) = -e^x \sin(e^x)$
$f(x) = 3 \cos^2 x$		$f'(x) = -6 \cos x \sin x$
$f(x) = \ln \cos x$		$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$
$f(x) = (1 + e^x)^3$		$f'(x) = 3e^x (1 + e^x)^2$
$f(x) = \sqrt{x + e^x}$		$f'(x) = \frac{1 + e^x}{2\sqrt{x + e^x}}$
$f(x) = \sqrt{\sin x}$		$f'(x) = \frac{\cos x}{2\sqrt{\sin x}}$
$f(x) = \frac{1}{\cos x}$		$f'(x) = \frac{\sin x}{\cos^2 x}$
$f(x) = \frac{3}{\sin^2 x}$		$f'(x) = -\frac{6 \cos x}{\sin^3 x}$
$f(x) = e^{x \sin x}$		$f'(x) = e^{x \sin x} (\sin x + x \cos x)$
$f(x) = x^2 e^{3x}$		$f'(x) = 2x e^{3x} + 3x^2 e^{3x}$

14. [Maximum mark: 3 per function]

Differentiate the following functions:

Function $f(x)$	Derivative $f'(x)$
$f(x) = 2(x^2 + 5)^3$	
$f(x) = 2e^{x^2+1}$	
$f(x) = 7e^{-x} + 8e^{\frac{x}{2}}$	
$f(x) = 3\cos 3x + x$	
$f(x) = \ln(x^2 + 1) + 2\cos(\frac{\pi}{2}x)$	
$f(x) = \sqrt{x^2 + 5} + \sqrt[3]{x}$	
$f(x) = (2x + 5)^3 + (2x + 5)^2 + 2x + 5$	
$f(x) = \sin^2 x + \cos^2 x$	
$y = e^{2x} + \ln(2x + 3)$	
$y = \sqrt{x^5 + 2x + 1}$	
$y = \frac{2x + 1}{3x - 5}$	
$y = \frac{2x + 1}{(3x - 5)^2}$	
$y = \ln \frac{2x + 1}{(3x - 5)^2}$	

18. [Maximum mark: 4]

Differentiate each of the following with respect to x .

(a) $y = x \sin 3x$ [2]

(b) $y = \frac{\ln x}{x}$ [2]

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19. [Maximum mark: 4]

The point $P\left(\frac{1}{2}, 0\right)$ lies on the graph of the curve of $y = \sin(2x - 1)$.

Find the gradient of the tangent to the curve at P.

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20. [Maximum mark: 4]

Differentiate with respect to x :

(i) $y = (x^2 + 1)^2$.

(ii) $y = \ln(3x - 1)$

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21. [Maximum mark: 4]

Differentiate with respect to x

(i) $\sqrt{3 - 4x}$

(ii) $e^{\sin x}$

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22. [Maximum mark: 5]

Let $f(x) = e^{\frac{x}{3}} + 5 \cos^2 x$. Find $f'(x)$.

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23. [Maximum mark: 6]

Let $f(x) = \cos 2x$ and $g(x) = \ln(3x - 5)$.

- (a) Find $f'(x)$. [2]
 (b) Find $g'(x)$. [2]
 (c) Let $h(x) = f(x) \times g(x)$. Find $h'(x)$. [2]

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24. [Maximum mark: 6]

- (a) Let $f(x) = e^{5x}$. Write down $f'(x)$. [2]
 (b) Let $g(x) = \sin 2x$. Write down $g'(x)$. [2]
 (c) Let $h(x) = e^{5x} \sin 2x$. Find $h'(x)$. [2]

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25. [Maximum mark: 6]

Let $f(x) = e^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

(a) Write down (i) $f'(x)$; (ii) $g'(x)$. [2]

(b) Let $h(x) = e^{-3x} \sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. [4]

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26. [Maximum mark: 6]

Let $f(x) = (2x + 7)^3$ and $g(x) = \cos^2(4x)$. Find (i) $f'(x)$; (ii) $g'(x)$

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27. [Maximum mark: 6]

The population p of bacteria at time t is given by $p = 100e^{0.05t}$. Calculate

- (a) the value of p when $t = 0$; [2]
- (b) the rate of increase of the population when $t = 10$. [4]

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28. [Maximum mark: 8]

The number of bacteria, n , in a dish, after t minutes is given by $n = 800e^{0.13t}$.

- (a) Find the value of n when $t = 0$. [2]
- (b) Find the rate at which n is increasing when $t = 15$. [2]
- (c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k , where $k \in \mathbb{Z}$. [4]

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29. [Maximum mark: 8]

Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

- (a) Write down the gradient of the curve at P. [2]
- (b) Find the equation of the tangent to the curve at P. [2]
- (c) The normal to the curve at P cuts the x -axis at R. Find the coordinates of R. [4]

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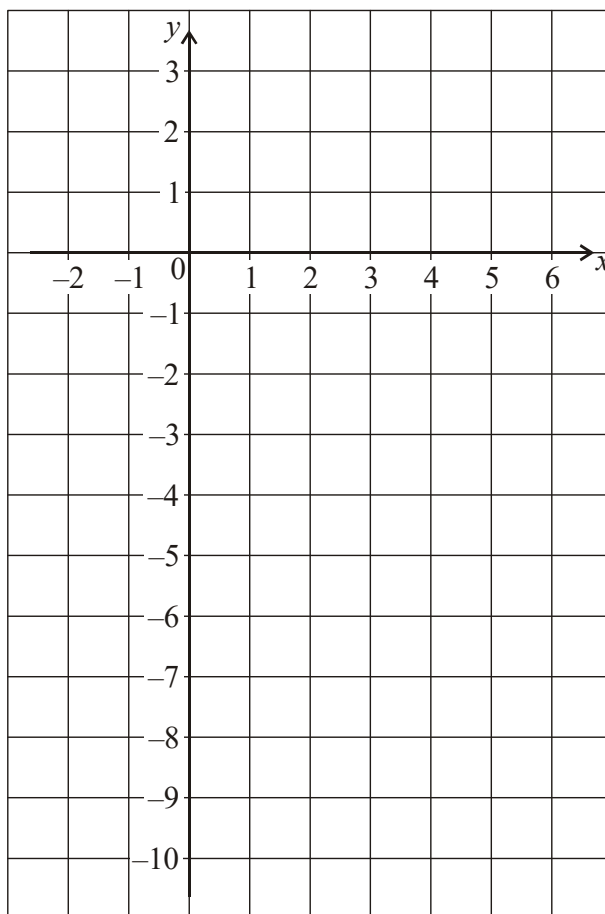
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30. [Maximum mark: 7]

Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \leq x \leq 5$.

- (a) Find the x -intercepts of the graph of f . [3]
- (b) On the grid below, sketch the graph of f . [3]
- (c) Write down the gradient of the graph of f at $x = 2$. [1]



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31. [Maximum mark: 6]

If $y = \ln(2x - 1)$ find $\frac{d^2 y}{dx^2}$.

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32. [Maximum mark: 6]

Consider the function $y = \tan x - 8 \sin x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the value of $\cos x$ for which $\frac{dy}{dx} = 0$. [2]

(c) Solve the equation $\frac{dy}{dx} = 0$ for $-\pi \leq x \leq 2\pi$. [2]

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33. [Maximum mark: 6]

Let $y = e^{3x} \sin(\pi x)$. Find

(a) $\frac{dy}{dx}$. [4]

(b) the smallest positive value of x for which $\frac{dy}{dx} = 0$. [2]

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34. [Maximum mark: 6]

Let $f(x) = \cos^3(4x + 1)$, $0 \leq x \leq 1$. Find

(a) $f'(x)$ [3]

(b) the **exact** values of the three roots of $f'(x) = 0$. [3]

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35. [Maximum mark: 6]

Let f be a cubic polynomial function. Given that $f(0) = 2$, $f'(0) = -3$, $f(1) = f'(1)$ and $f''(-1) = 6$, find $f(x)$.

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36. [Maximum mark: 3 per function]

The following table shows the values of two functions f and g and their derivatives when $x = 1$ and $x = 0$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	1	-4	5
1	-2	3	-1	2

Find the derivatives of the following functions when $x = 1$.

$3f(x) + 5g(x)$	
$f(x)g(x)$	
$\frac{f(x)}{g(x)}$	
$f(x)^3$	
$\ln f(x)$	
$f(\ln x)$	
$e^{f(x)}$	
$f(e^{x-1})$	
$f(x-1)$	
$f(2x-2)$	
$f(-g(x))$	
$g(f(x)+2)$	

Let $f(x) = 1 + 3\cos(2x)$ for $0 \leq x \leq \pi$, and x is in radians.

- (a) (i) Find $f'(x)$.
- (ii) Find the values for x for which $f'(x) = 0$; Give your answers in terms of π . [6]

The function $g(x)$ is defined as $g(x) = f(2x) - 1$, $0 \leq x \leq \frac{\pi}{2}$.

- (b) (i) The graph of f may be transformed to the graph of g by a stretch in the x -direction with scale factor $\frac{1}{2}$ followed by another transformation.

Describe fully this other transformation.

- (ii) Find the solution to the equation $g(x) = f(x)$. [4]

[illegible]

39. [Maximum mark: 14]

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counterclockwise) direction.

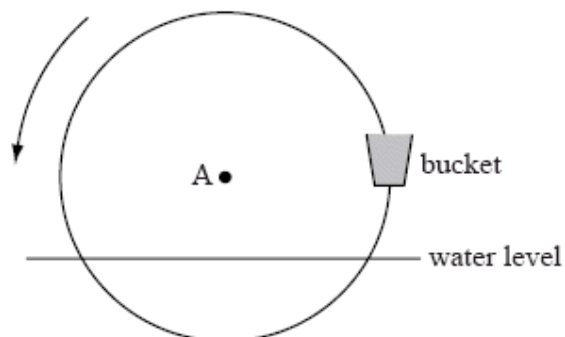


diagram not to scale

The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a \sin bt + 2$.

(a) Show that $a = 4$. [2]

The wheel turns at a rate of one rotation every 30 seconds.

(b) Show that $b = \frac{\pi}{15}$. [2]

In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 m s^{-1} .

(c) Find these values of t . [6]

(d) Determine whether the bucket is underwater at the second value of t . [4]

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(a) Sketch the graph of f . [3]

(i) the amplitude; (ii) the period; (iii) the x -intercept between $-\frac{\pi}{2}$ and 0. [3]

(d) Write down one value of x such that $f'(x) = 0$. [2]

(f) Let $g(x) = \ln(x + 1)$, for $0 \leq x \leq \pi$. There is a value of x , between 0 and 1, for which the gradient of f is equal to the gradient of g . Find this value of x . [5]

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