INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 5.17]

KINEMATICS

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A. Paper 1 questions (SHORT)

1. [Maximum mark: 10]

The displacement s metres of a car, t seconds after leaving a fixed point A, is given by

$$s = 12t - t^3, \quad t \geq 0$$

(a) Calculate the velocity of the car (i) in terms of t; (ii) after 1 sec. [2]

(b) Calculate the acceleration of the car (i) in terms of t; (ii) after 1 sec. [2]

(c) Find the value of the time t in the following instances

(i) when the car has velocity -15ms^{-1}

(ii) when the car is stationary;

(iii) when the car passes again from point A. [6]

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2. [Maximum mark: 12]

The acceleration a of a car, t seconds after leaving a fixed point A, is given by

$$a = -6t$$

Calculate

(a) the velocity of the car in terms of t, given that the initial velocity is 12ms^{-1}. [3]

(b) the displacement of the car in terms of t, given that the initial displacement is 0*m.* [3]

(c) the maximum positive displacement. [2]

(d) the time intervals in which the velocity is positive. [2]

(e) the time intervals in which the displacement is positive. [2]

3. [Maximum mark: 12]

The velocity v of a car, t seconds after leaving a fixed point A, is given by

$$v = 12 - 3t^2$$

(a) Calculate the acceleration of the car in terms of t. [1]

(b) Calculate the displacement of the car in terms of t, given that the initial displacement is 0. [3]

(c) The car is stationary at time $t = a$. Find the value of a. [2]

(d) Find, in the first 3 seconds (i) the displacement (ii) the distance travelled. [4]

(e) Sketch the diagram of v in terms of time t, and shade the area that indicates the distance travelled in the first 3 seconds. [2]

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4. [Maximum mark: 6]

The displacement s metres of a car, t seconds after leaving a fixed point A, is given by

$$s = 10t - 0.5t^2 .$$

(a) Calculate the velocity when $t = 0$. [3]

(b) Calculate the value of t when the velocity is zero. [2]

(c) Calculate the displacement of the car from A when the velocity is zero. [1]

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5. [Maximum mark: 6]

The displacement s metres at time t sec is given by $s = 5\cos 3t + t^2 + 10$, for $t \geq 0$.

(a) Write down the minimum value of s. [1]

(b) Find the acceleration, a, at time t. [3]

(c) Find the value of t when the **maximum** value of a first occurs. [2]

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6. [Maximum mark: 6]

The velocity, v, in m s^{-1} of a particle moving in a straight line is given by $v = e^{3t-2}$, where t is the time in seconds.

(a) Find the acceleration of the particle at $t = 1$. [2]

(b) At what value of t does the particle have a velocity of 22.3 m s^{-1}? [2]

(c) Find the distance travelled in the first second. [2]

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7. [Maximum mark: 6]

The velocity, $v \text{ m s}^{-1}$, of a moving object at time t seconds is given by $v = 4t^3 - 2t$.

When $t = 2$, the displacement, s, of the object is 8 metres.

(a) Find an expression for s in terms of t. [4]

(b) Find the acceleration of the object after 1 second. [2]

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8. [Maximum mark: 6]

The velocity v in m s^{-1} of a moving body at time t seconds is given by $v = e^{2t-1}$. When $t = 0.5$. the displacement of the body is 10 m. Find the displacement when $t = 1$.

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9. [Maximum mark: 6]

The velocity v of a particle at time t is given by $v = e^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

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10. [Maximum mark: 6]

A particle moves along a straight line so that its velocity, v ms^{-1} at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s, of the particle is 7 metres. Find an expression for s in terms of *t*.

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11. [Maximum mark: 6]

The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}. [2]

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t. [4]

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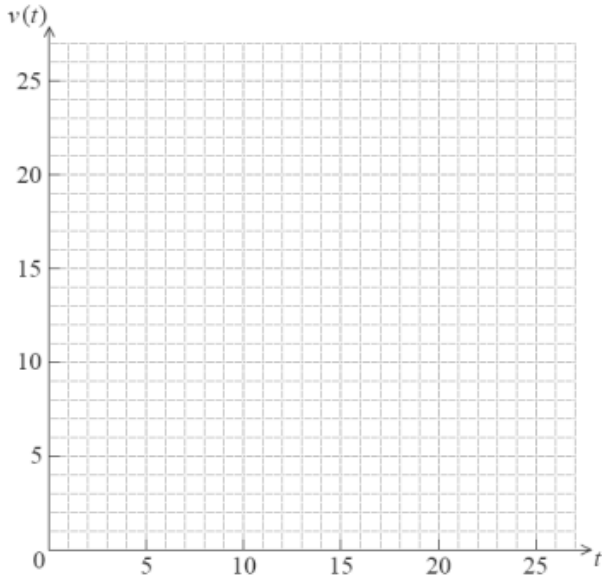
12. [Maximum mark: 7]

The velocity v ms^{-1} of an object after t sec is given by $v(t) = 15\sqrt{t} - 3t$, for $0 \le t \le 25$.

(a) On the grid below, sketch the graph of v, clearly indicating the maximum point.



[3]

Let d be the distance travelled in the first nine seconds.

(b) (i) Write down an expression for d. (ii) Hence, write down the value of d. [4]

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13. [Maximum mark: 7]

The acceleration, a ms^{-2}, of a particle at time t seconds is given by $a = \frac{1}{t} + 3\sin 2t$, for $t \geq 1$. The particle is at rest when $t = 1$. Find the velocity of the particle when $t = 5$.

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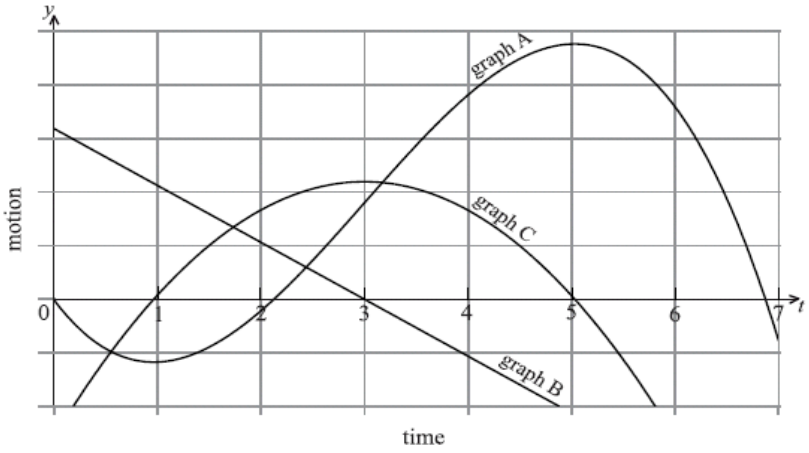
14. [Maximum mark: 6]

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t.



(a) Complete the following table by noting which graph A, B or C corresponds to each function.

Function	Graph
displacement	
acceleration	

[4]

(b) Write down the value of t when the velocity is greatest. [2]

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15. [Maximum mark: 9]

A particle moves with a velocity v m s^{-1} given by $v = 25 - 4t^2$, where $t \geq 0$.

(a) The displacement, s is 10m when t is 3. Find an expression for s in terms of t. [4]

(b) Find t when s reaches its maximum value. [2]

(c) The particle has a positive displacement for $m \leq t \leq n$.

Find the values of m and n [3]

16. [Maximum mark: 6]

The displacement s metres of a moving body B from a fixed-point O at time t seconds is given by $s = 50t - 10t^2 + 1000$.

(a) Find the velocity of B in ms^{-1}. [2]

(b) Find its maximum displacement from O. [3]

(c) Find the acceleration in ms^{-2}. [1]

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17. [Maximum mark: 4]

An astronaut on the moon throws a ball vertically upwards. The height, s metres, of the ball, after t seconds, is given by the equation $s = 40t + 0.5at^2$, where a is a constant. If the ball reaches its maximum height when $t = 25$, find the value of a.

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18. [Maximum mark: 6]

A particle moves in a straight line. At time t seconds, its displacement from a fixed point O is s metres, and its velocity, v metres per second, is given by $v = 3t^2 - 4t + 2$, $t \geq 0$. When $t = 0$, $s = -3$. Find the value of t when the particle is at O.

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19. [Maximum mark: 6]

The acceleration, $a(t)\ ms^{-2}$, of a fast train during the first 80 seconds of motion is given by $a(t) = -\frac{1}{20}t + 2$, where t is the time in seconds. If the train starts from rest at $t = 0$, find the distance travelled by the train in the first minute.

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20. [Maximum mark: 6]

A particle moves in a straight line with velocity, in metres per sec, at time t seconds, given by $v(t) = 6t^2 - 6t,\ t \geq 0$

(a) Find the total distance travelled by the particle in the first two seconds of motion. [2]

(b) Find the displacement in the first two seconds of motion. [2]

(c) Explain why the two results in (a) and (b) are different. [2]

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21. [Maximum mark: 4]

The velocity, v, of an object, at a time t, is given by $v = ke^{-\frac{t}{2}}$, where t is in seconds and v is in m s^{-1}. Find the distance travelled between $t = 0$ and $t = a$.

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22. [Maximum mark: 5]

A particle is projected along a straight line path. After t seconds, its velocity v metres per second is given by $v = \frac{1}{2+t^2}$.

(a) Find the distance travelled in the first second. [2]

(b) Find an expression for the acceleration at time t. [3]

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23. [Maximum mark: 6]

A particle moves in a straight line. Its velocity v ms^{-1} after t seconds is given by $v = e^{-\sqrt{t}} \sin t$.

(a) Find the first time it changes direction. [1]

(b) In the time interval [0, 2π], write down an expression for and find the value of

(i) the displacement; (ii) the total distance travelled. [5]

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24. [Maximum mark: 7]

A particle is moving along a straight line so that t seconds after passing through a fixed point O on the line, its velocity v ms^{-1} is given by $v(t)=t\sin\left(\frac{\pi}{3}t\right)$.

(a) Find the values of t for which $v(t)=0$, given that $0\le t\le 6$. [3]

(b) (i) Write down a mathematical expression for the **total** distance travelled by the particle in the first six seconds after passing through O.

(ii) Find this distance. [4]

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25. [Maximum mark: 5]

(a) A particle moves along a straight line and its velocity at time t is given by $v=10t$. Find its acceleration. [2]

(b) A particle moves along a straight line. When it is in distance s from a fixed point, the velocity v is given by $v=10s$. Find the acceleration in terms of s. [3]

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26. [Maximum mark: 5]

Find the acceleration

(a) if $v = t^2 + 10e^t$ [2]

(b) if $v = s^2 + 10e^s$ [3]

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27. [Maximum mark: 6]

A particle moves along a straight line. When it is in distance s from a fixed point, where $s > 1$, the velocity is given by $v = \frac{(3s+2)}{(2s-1)}$. Find the acceleration when $s = 2$.

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B. Paper 2 questions (LONG)

28. [Maximum mark: 13]

The velocity *v* of a car, *t* seconds after leaving a fixed point A, is given by

$$v = 16 - 4e^{2t}$$

(a) Calculate the acceleration of the car after ln3 seconds. [3]

(b) Calculate the displacement of the car in terms of t, given that the initial displacement is 0. [3]

(c) The car is stationary at time t. Find the **exact** value of t. [3]

(d) Find the displacement and the distance travelled after ln4 seconds. [4]

29. [Maximum mark: 12]

The displacement s of an oscillating body from a fixed point O in a straight line, is given by $s = \sin\left(\frac{t}{4}\right), \quad t \geq 0$

(a) Find expressions for the velocity and the acceleration in terms of time t. [2]

(b) Find the maximum displacement. [2]

(c) Find the times it passes the fixed point O. [2]

(d) Find the two values of the velocity at these times (found in (c)) [2]

(e) Find the times when the body changes direction. [2]

(f) Find the two values of the acceleration at these times (found in (e)) [2]

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30. [Maximum mark: 15]

A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t},\ t \geq 0$$

where *v* is in metres per second and *t* is in seconds.

(a) Find the value of v when (i) $t = 0$; (ii) $t = 10$. [2]

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$? [3]

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii). [3]

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** s.f. [7]

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31. [Maximum mark: 15]

In this question, s represents displacement in metres, and t represents time in sec.

(a) The velocity v of a moving body is given by $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, a is a constant.

Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t. [5]

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station. [5]

(c) Train 2 slows down so that it stops at the station. Its velocity is given by

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a. [5]

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32. [Maximum mark: 15]

A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by $h = 2 + 20t - 5t^2,\ t \geq 0$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released). [2]

(b) Show that the height of the ball after one second is 17 metres. [2]

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that t must satisfy when the ball is at $\mathrm{h} = 17\ \mathrm{m}$.

(ii) Solve the equation **algebraically**. [4]

(d) (i) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$.

(ii) Find the **initial** velocity of the ball (i.e. when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball. [7]

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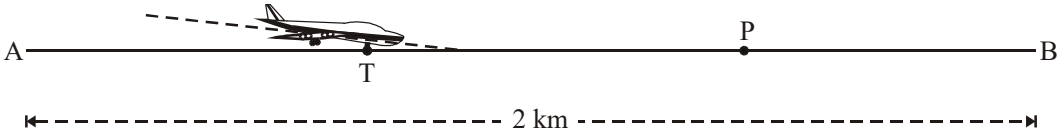
33. [Maximum mark: 15]

The main runway at *Concordville* airport is 2 km long. An airplane, landing at *Concordville,* touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



As the airplane slows down, its distance, s, from A, is given by $s = c + 100t - 4t^2$, where t is the time in sec after touchdown, and c metres is the distance of T from A.

(a) The airplane touches down 800 m from A, (*ie* $c = 800$).

(i) Find the distance travelled by the airplane in the first 5 sec after touchdown.

(ii) Write down an expression for the velocity of the airplane at time t seconds after touchdown, and hence find the velocity after 5 seconds. [5]

(b) The airplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(i) how many seconds after touchdown it passes the marker;

(ii) the distance from P to A. [5]

(c) Show that if the airplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway. [5]

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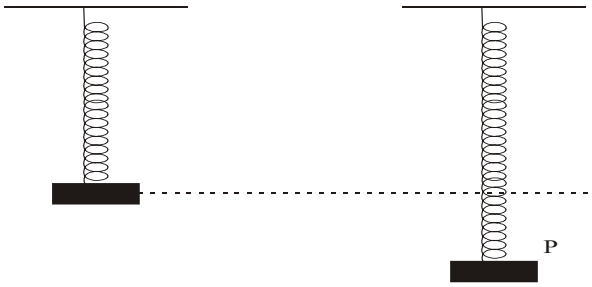
34. [Maximum mark: 16]

Note: Radians are used throughout this question.

A mass is suspended from the ceiling on a spring. It is pulled down to point P and then released. It oscillates up and down.



Its distance, s cm, from the ceiling, is modelled by the function $s = 48 + 10\cos 2\pi t$, where t is the time in seconds from release.

(a) (i) What is the distance of the point P from the ceiling?

(ii) How long is it until the mass is next at P? [5]

(b) (i) Find $\frac{\mathrm{d}s}{\mathrm{d}t}$.

(ii) Where is the mass when the velocity is zero? [7]

A second mass is suspended on another spring. Its distance r cm from the ceiling is modelled by the function $r = 60 + 15\cos 4\pi t$. The two masses are released at the same instant.

(c) Find the value of t when they are first at the same distance below the ceiling. [2]

(d) In the first three sec, how many times are the two masses at the same height? [2]

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35. [Maximum mark: 15]

A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height *h* metres of the rock-climber after t seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \le t \le 2$$
$$h = 90 - 40t + 5t^2, \qquad 2 \le t \le 5$$

(a) Find the height of the rock-climber when $t = 2$. [1]

(b) Sketch a graph of *h* against *t* for $0 \le t \le 5$. [4]

(c) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ for: (i) $0 \le t \le 2$ (ii) $2 \le t \le 5$ [2]

(d) Find the velocity of the rock-climber when $t = 2$. [2]

(e) Find the times when the velocity of the rock-climber is zero. [3]

(f) Find the minimum height of the rock-climber for $0 \le t \le 5$. [3]

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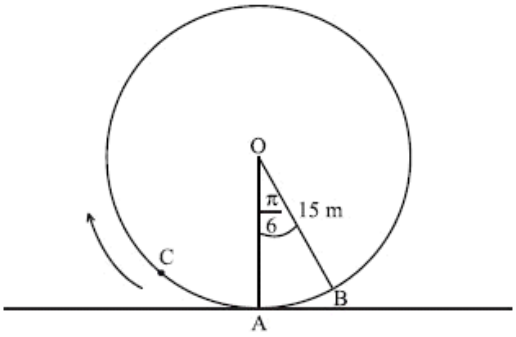
36. [Maximum mark: 22]

A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $\text{A}\hat{\text{O}}\text{B} = \frac{\pi}{6}$.



(a) Find the length of the arc AB. [2]

(b) Find the area of the sector AOB [2]

(c) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$.

Find the height of A above the ground. [3]

The height, *h* metres, of seat C above the ground after t minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right)$$

(d) (i) Find the height of seat C when $t = \frac{\pi}{4}$.

(ii) Find the initial height of seat C.

(iii) Find the time at which seat C first reaches its highest point. [8]

(e) Find $h'(t)$. [2]

(f) For $0 \le t \le \pi$

(i) sketch the graph of h';

(ii) find the time at which the height is changing most rapidly. [5]

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37. [Maximum mark: 16]

The velocity $v\ \text{ms}^{-1}$ of a particle at time t sec, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

(a) Write down the velocity of the particle when $t = 0$. [1]

When $t = k$, the acceleration is zero.

(b) (i) Show that $k = \frac{\pi}{4}$. (ii) Find the exact velocity when $t = \frac{\pi}{4}$. [8]

(c) When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. Sketch a graph of v against t. [4]

(d) Let d be the distance travelled by the particle for $0 \le t \le 1$.

(i) Write down an expression for d. (ii) Represent d on your sketch. [3]

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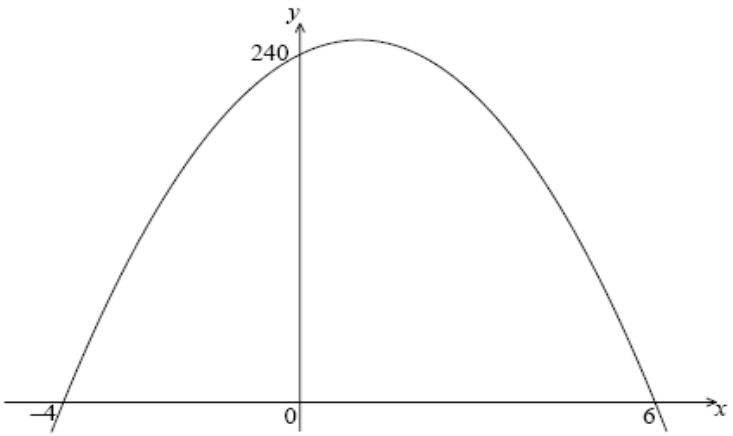
38. [Maximum mark: 15]

The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at (–4, 0) and (6, 0) and the y-intercept is at (0, 240).

(a) Write down $f(x)$ in the form $f(x) = -10(x-p)(x-q)$. [2]

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. [4]

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [2]

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero. [7]

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39. [Maximum mark: 17]

In this question s represents displacement in metres and t represents time in seconds.

The velocity $v \text{ m s}^{-1}$ of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t. [6]

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop; give answer in terms of a.

(ii) Hence show that $a = \frac{8}{5}$. [6]

(c) For a different train N, the value of a is 4.

Show that this train will stop **before** it reaches the station. [5]

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40. [Maximum mark: 11]

The acceleration in ms^{-2} of a particle moving in a straight line at time t seconds, $t > 0$, is given by the formula $a = -\frac{1}{(1+t)^2}$. When $t = 1$ second, the velocity is $v = 8\ \text{ms}^{-1}$.

(a) Find the velocity when $t = 3$. [6]

(b) Find the limit of the velocity as $t \to \infty$. [1]

(c) Find the exact distance travelled between $t = 1$ and $t = 3$. [4]

41. [Maximum mark: 23]

Particle A moves in a straight line, starting from O_A, such that its velocity in metres per second for $0 \le t \le 9$ is given by

$$v_A = -\frac{1}{2}t^2 + 3t + \frac{3}{2}.$$

Particle B moves in a straight line, starting from O_B, such that its velocity in metres per second for $0 \le t \le 9$ is given by

$$v_B = \mathrm{e}^{0.2t}.$$

(a) Find the maximum value of v_A, justifying that it is a maximum. [5]

(b) Find the acceleration of B when $t = 4$. [3]

The displacements of A and B from O_A and O_B respectively, at time t are s_A metres and s_B metres. When $t = 0$, $s_A = 0$, and $s_B = 5$.

(b) Find an expression for s_A and for s_B, giving your answers in terms of t. [7]

(c) (i) Sketch the curves of s_A and s_B on the same diagram.

(ii) Find the values of t at which $s_A = s_B$. [8]

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