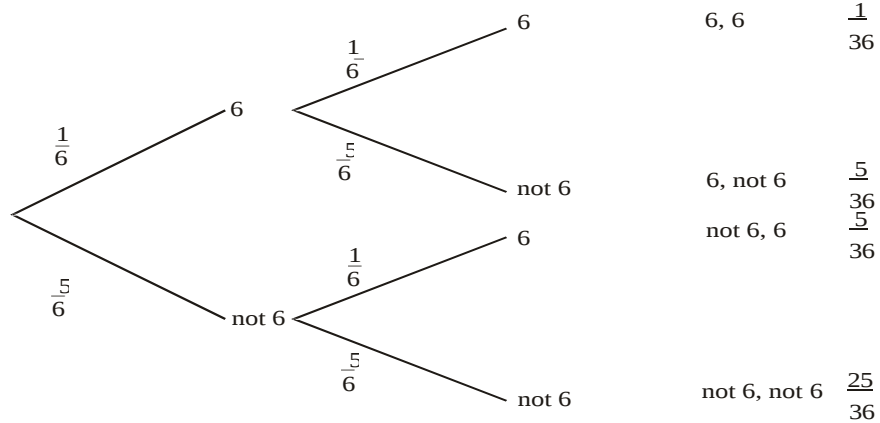


EXERCISES [MAI 4.8]
PROBABILITY II (TREE DIAGRAMS)
SOLUTIONS

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A. Paper 1 questions (SHORT)

1. (a)



(b) $P(\text{two sixes}) = \frac{1}{36}$

(c) $P(\text{one six only}) = \frac{5}{36} + \frac{5}{36} = \frac{10}{36}$

(d) $P(\text{one or more sixes}) = \frac{1}{36} + \frac{10}{36} = \frac{11}{36}$ or $1 - \frac{25}{36} = \frac{11}{36}$

2. The four scenarios have probabilities 0.18, 0.12, 0.14, 0.56

(a)

$P(A)$	0.3	$P(A')$	0.7	$P(A \cap B)$	0.18
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$P(A \cap B')$	0.12	$P(A' \cap B)$	0.14	$P(A' \cap B')$	0.56
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(b)

$P(B)$	0.32	$P(B')$	0.68	$P(A \cup B)$	0.44
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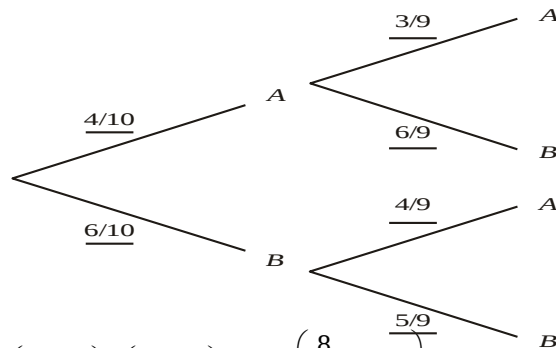
(c)

$P(B A)$	0.6	$P(B' A)$	0.4	$P(B A')$	0.2	$P(B' A')$	0.8
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(d)

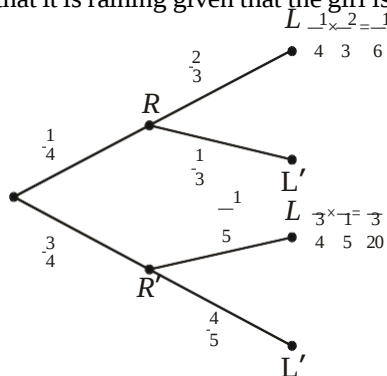
$P(A B)$	$0.18/0.32 = 9/16$
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3. (a)



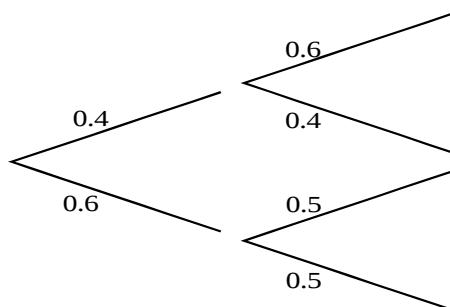
(b) $\left(\frac{4}{10} \times \frac{6}{9} \right) + \left(\frac{6}{10} \times \frac{4}{9} \right) = \frac{48}{90} = \frac{8}{15} \approx 0.533$

4. Let $P(R|L)$ be the probability that it is raining given that the girl is late.



$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{1/6}{1/6 + 3/20} = \frac{10}{19}$$

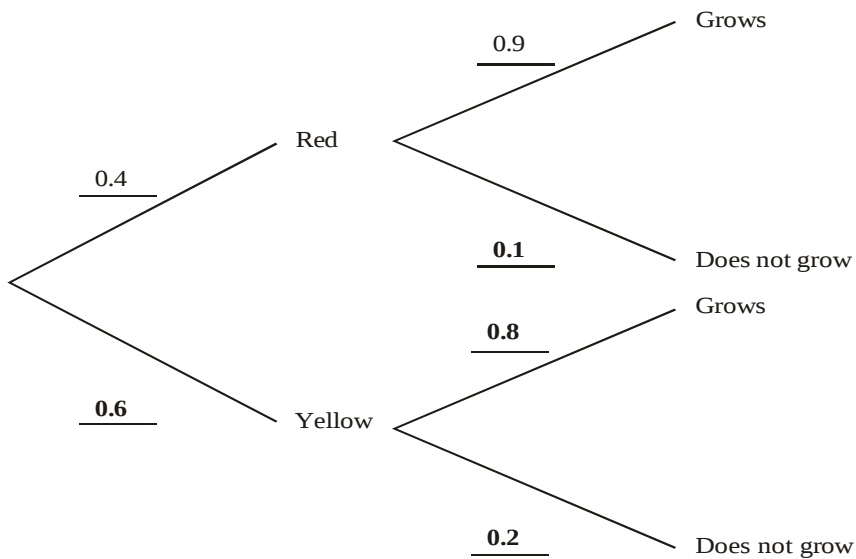
5. (a)



(b) $P(B) = 0.4(0.6) + 0.6(0.5) = 0.24 + 0.30 = 0.54$

(c) $P(C|B) = \frac{P(B \cap C)}{P(B)} = \frac{0.24}{0.54} = \frac{4}{9}$ ($= 0.444$, 3 sf)

6. (a)

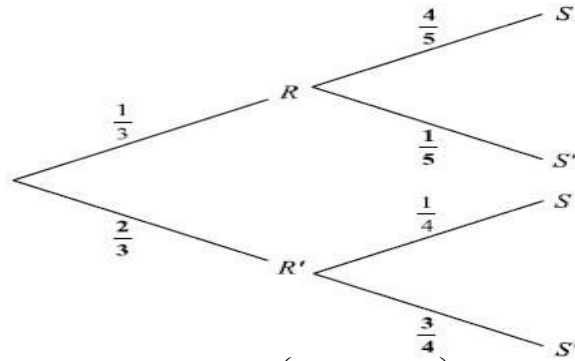


(b) (i) $0.4 \times 0.9 = 0.36$

(ii) $0.36 + 0.6 \times 0.8$ ($= 0.36 + 0.48$) $= 0.84$

(iii) $\frac{0.36}{0.84} = 0.429$ ($\frac{3}{7}$)

7. (a)



$$\begin{aligned} \text{(b) (i)} \quad & P(R \cap S) = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15} \left(= 0.267 \right) \\ \text{(ii)} \quad & P(S) = \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} = \frac{13}{30} \quad (= 0.433) \\ \text{(iii)} \quad & P(R|S) = (4/15) / (13/30) = \frac{8}{13} \quad (= 0.615) \end{aligned}$$

8. (a) $p = \frac{4}{5}$

$$\text{(b)} \quad P(B) = \frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8} = \frac{14}{40} \left(= \frac{7}{20} \right)$$

$$\text{(c)} \quad P(A' | B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}} = \frac{12}{14} \left(= \frac{6}{7} \right)$$

9. (a) $P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9 = 0.84$

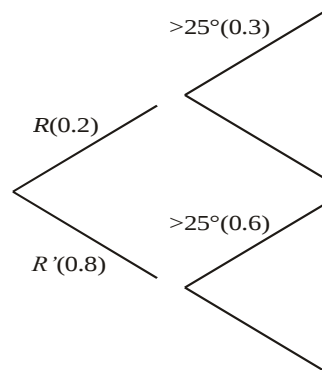
(b) $P(B) = x, P(A) = 1 - x$

$$0.8(1 - x) + 0.9x = 0.87 \Leftrightarrow x = 0.7 \quad 70 \% \text{ from B}$$

10. (a) $P(\text{win}) = (0.65)(0.83) + (0.35)(0.26) = 0.6305 \text{ (or } 0.631)$

$$\text{(b)} \quad P(H | W) = \frac{(0.65)(0.17)}{0.3695} \left(= \frac{0.1105}{0.3695} \right) = 0.299$$

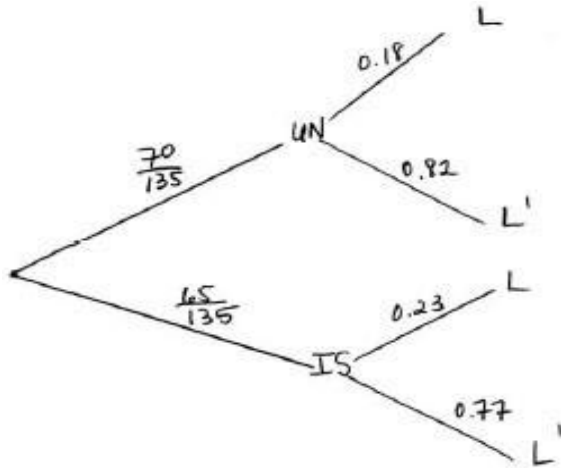
11.



$$P(> 25^\circ) = 0.2 \times 0.3 + 0.8 \times 0.6 = 0.54$$

$$P(R | > 25^\circ) = \frac{0.06}{0.54} = \frac{1}{9} \text{ (or } 0.111)$$

12. METHOD 1



$$P(I|L) = \frac{0.23 \times \frac{65}{135}}{0.18 \times \frac{70}{135} + 0.23 \times \frac{65}{135}} = \frac{299}{551} (=0.543, \text{ accept } 0.542)$$

METHOD 2

Expected number of suitcases lost by UN Air is $0.18 \times 70 = 12.6$

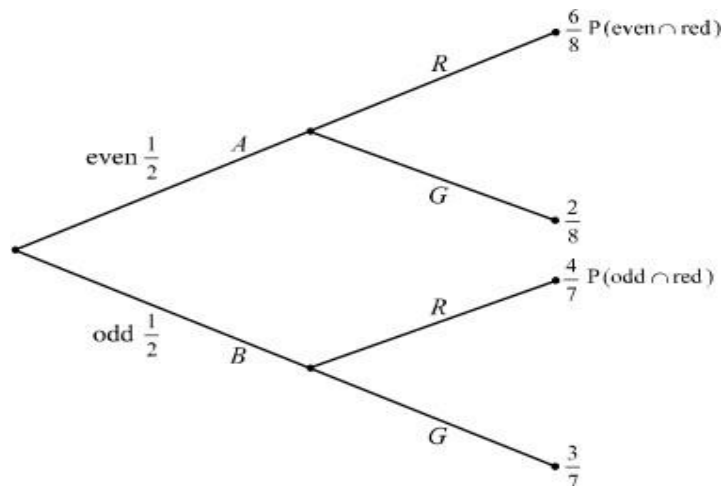
Expected number of suitcases lost by IS Air is $0.23 \times 65 = 14.95$

$$P(I|L) = \frac{14.95}{12.6 + 14.95} = 0.543$$

13. (a) Probability = $0.2 \times 0.66 + 0.8 \times 0.75 = 0.732$
 (b) Probability = $\frac{P(\text{Mon} \cap \text{catches train})}{P(\text{catches train})} = \frac{0.2 \times 0.66}{0.732} = 0.180 \left(= \frac{11}{61} \right)$

14. Required prob = $\frac{3}{9} \times \frac{2}{5} + \frac{6}{9} \times \frac{3}{4} = \frac{19}{30}$

15.



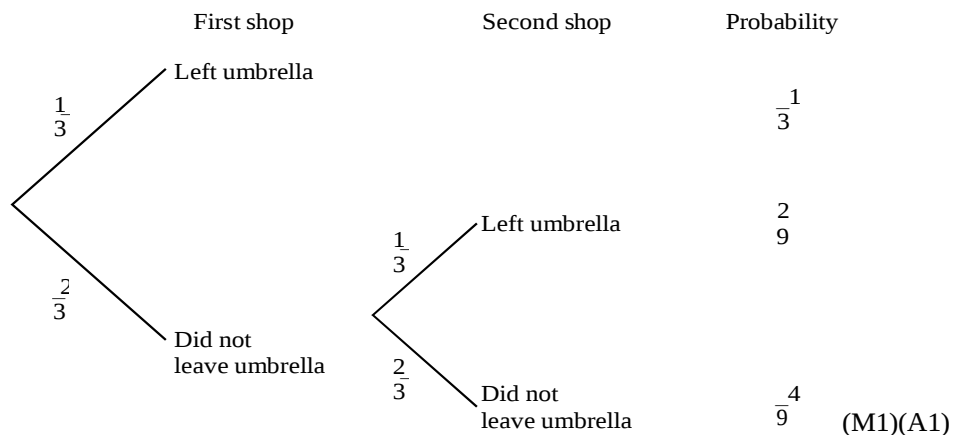
(i) $P(R) = \frac{1}{2} \times \frac{6}{8} + \frac{1}{2} \times \frac{4}{7} = \frac{3}{8} + \frac{2}{7} = \frac{37}{56} (0.661)$

(ii) $P(B|R) = \frac{P(B \cap R)}{P(R)} = \frac{\frac{2}{7}}{\frac{37}{56}} = \frac{16}{37} (0.432)$

16. (a) $P(\text{same colour}) = \left(\frac{4}{9}\right)\left(\frac{7}{9}\right) + \left(\frac{5}{9}\right)\left(\frac{2}{9}\right) = \left(\frac{28}{81}\right) + \left(\frac{10}{81}\right) = \frac{38}{81} (=0.469)$

(b) $P(\text{first red}|\text{different}) = \frac{\left(\frac{4}{9}\right)\left(\frac{2}{9}\right)}{\left(\frac{4}{9}\right)\left(\frac{2}{9}\right) + \left(\frac{5}{9}\right)\left(\frac{7}{9}\right)} = \frac{8}{43} (=0.186)$

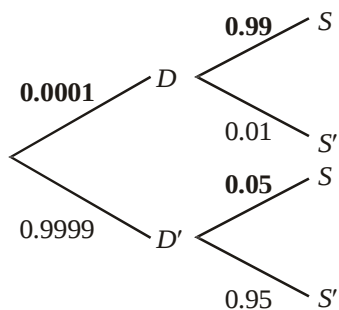
17.



(a) (i) $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ (ii) $1 - \frac{4}{9} = \frac{5}{9}$ or from tree diagram $\frac{2}{9} + \frac{1}{3} = \frac{5}{9}$

(b) Required probability = $\frac{\frac{2}{9}}{\frac{2}{9} + \frac{1}{3}} = \frac{2}{5}$.

18. Let D be the event that the patient has the disease and S be the event that the new blood test shows that the patient has the disease. Let D' be the complement of D , i.e. the patient does not have the disease.



Therefore $p(S) = 0.0001 \times 0.99 + 0.9999 \times 0.05 = 0.0500939$

$$p(D|S) = \frac{0.0001 \times 0.99}{0.0500939} = 0.00198 \text{ (3 s.f.)}$$

19. $P(4 \text{ girls}) = \left(\frac{13}{24} \times \frac{12}{23} \times \frac{11}{22} \times \frac{10}{21} \right) = \frac{17160}{255024} \left(= \frac{65}{966} = 0.0673 \right)$

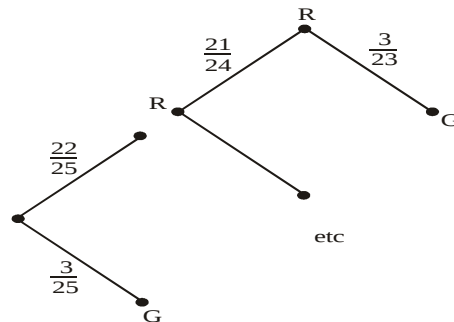
$$20. \quad P(RR) = \frac{7}{12} \times \frac{6}{11} \left(= \frac{7}{22} \right)$$

$$P(YY) = \frac{5}{12} \times \frac{4}{11} \left(= \frac{5}{33} \right)$$

$$P(\text{same colour}) = P(RR) + P(YY) = \frac{31}{66} (= 0.470, 3 \text{ sf})$$

$$21. \quad (a) \quad P = \frac{22}{23} (= 0.957 (3 \text{ sf}))$$

(b)



$$P = P(RRG) + P(RGR) + P(GRR)$$

$$\frac{22}{25} \times \frac{21}{24} \times \frac{3}{23} + \frac{22}{25} \times \frac{3}{24} \times \frac{21}{23} + \frac{3}{25} \times \frac{22}{24} \times \frac{21}{23} = \frac{693}{2300} (= 0.301 (3 \text{ sf}))$$

$$22. \quad P(\text{different colours}) = 1 - [P(GG) + P(RR) + P(WW)]$$

$$= 1 - \left(\frac{10}{26} \times \frac{9}{25} + \frac{10}{26} \times \frac{9}{25} + \frac{6}{26} \times \frac{5}{25} \right) = 1 - \left(\frac{210}{650} \right) = \frac{44}{65} (= 0.677, \text{ to } 3 \text{ sf})$$

OR

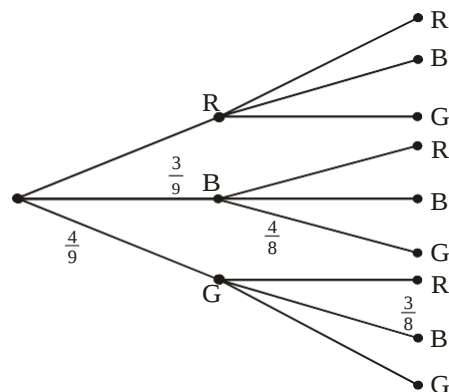
$$P(\text{different colours}) = P(GR) + P(RG) + P(GW) + P(WG) + P(RW) + P(WR)$$

$$= 4 \left(\frac{10}{26} \times \frac{6}{25} \right) + 2 \left(\frac{10}{26} \times \frac{10}{25} \right) = \frac{44}{65} (= 0.677, \text{ to } 3 \text{ sf})$$

$$23. \quad (a) \quad p(GG) = \frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$$

$$(b) \quad p(RR) + P(BB) + P(GG) = \frac{2}{9} \times \frac{1}{8} + \frac{3}{9} \times \frac{2}{8} + \frac{4}{9} \times \frac{3}{8} = \frac{20}{72} = \frac{5}{18}$$

(c) Using a tree diagram,



$$p(\text{BG or GB}) = \binom{3}{9} \times \binom{4}{8} + \binom{4}{9} \times \binom{3}{8} = \frac{1}{6} + \frac{1}{6} \quad \text{OR} \quad p(\text{BG or GB}) = 2 \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{3}$$

24.

Let there be n black disks and $25 - n$ white disks.

EITHER

Since the two probabilities are equal,

$$\begin{aligned} \frac{1}{2} &= \frac{\binom{n}{2} + \binom{25-n}{2}}{\binom{25}{2}} \\ \frac{1}{2} \binom{25}{2} &= \frac{n!}{(n-2)!2!} + \frac{(25-n)!}{(23-n)!2!} \\ 300 &= n(n-1) + (25-n)(24-n) \\ 0 &= 2n^2 - 50n + 300 \\ 0 &= n^2 - 25n + 150 \\ 0 &= (n-15)(n-10) \\ n &= 15, 10 \end{aligned}$$

OR

$$P(\text{same color}) = \frac{n(n-1)}{25 \times 24} + \frac{(25-n)(24-n)}{25 \times 24}$$

$$P(\text{different color}) = \frac{n(25-n)}{25 \times 24} + \frac{(25-n)n}{25 \times 24}$$

probabilities the same so,

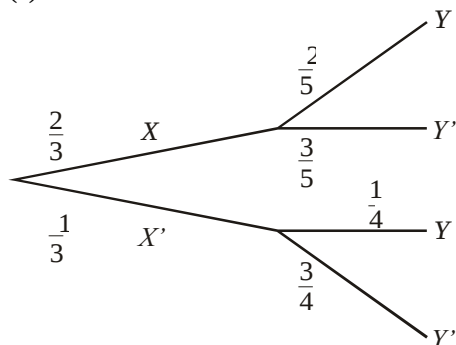
$$\begin{aligned} \frac{n(n-1)}{25 \times 24} + \frac{(25-n)(24-n)}{25 \times 24} &= \frac{n(25-n)}{25 \times 24} + \frac{(25-n)n}{25 \times 24} \\ n^2 - n + 600 - 49n + n^2 &= 25n - n^2 + 25n - n^2 \\ 4n^2 - 100n + 600 &= 0 \\ n^2 - 25n + 150 &= 0 \\ (n-10)(n-15) &= 0 \\ n &= 10, 15 \end{aligned}$$

25. (a) (i) $P(\text{Ann wins on her first throw}) = \frac{1}{6}$
- (ii) $P(\text{Bridget wins on her first throw}) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$
- (iii) $P(\text{Ann wins on her second throw}) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{25}{216}$
- (b) $P(\text{Ann wins}) = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \dots \times$
- This is an infinite GS with common ratio $r = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$
- The sum is $S_{\infty} = \frac{6}{11}$
- (c) $P(\text{Bridget wins}) = 1 - \frac{6}{11} = \frac{5}{11}$

26. (a) Probability that Jack wins on his first throw = $\frac{2}{3}$ (or 0.667).
- (b) Probability that Jill wins on her first throw: $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ (or 0.222).
- (c) **EITHER** Probability that Jack wins the game:
- $$\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) + \dots = \frac{2}{3} \times \frac{1}{1 - \frac{4}{9}} = \frac{3}{1 - \frac{4}{9}} = \frac{3}{\frac{5}{9}} = \frac{27}{5}$$

B. Paper 2 questions (LONG)

27. (a)

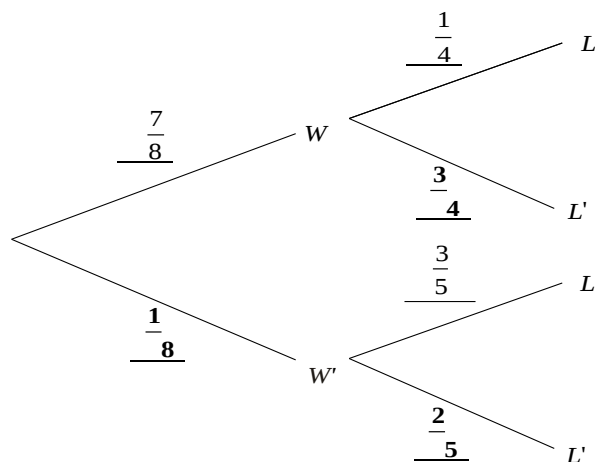


$$(b) \quad P(Y') = \frac{2}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{3}{4} = \frac{13}{20}$$

$$(c) \quad P(X' \cup Y') = 1 - P(X' \cap Y') = 1 - \frac{4}{15} = \frac{11}{15}$$

$$(d) \quad P(X' | Y') = \frac{\frac{1}{3} \times \frac{3}{4}}{\frac{2}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{3}{4}} = \frac{\frac{1}{4}}{\frac{13}{20}} = \frac{5}{13}$$

28. (a)



$$(b) \quad \text{Probability that he will be late is } \frac{7}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{3}{5} = \frac{47}{160} (0.294)$$

$$(c) \quad P(W|L) = \frac{P(W \cap L)}{P(L)} \quad P(W \cap L) = \frac{7}{8} \times \frac{1}{4} \quad P(L) = \frac{47}{160}$$

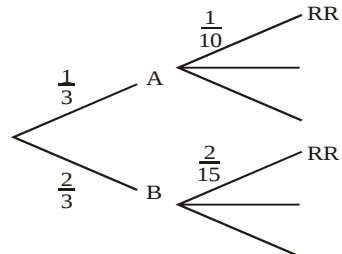
$$P(W|L) = \frac{\frac{7}{32}}{\frac{47}{160}} = \frac{35}{47} (= 0.745)$$

29. (a) $P(RR) = \frac{\binom{2}{5} \binom{1}{4}}{\binom{1}{10}} = \frac{1}{10}$

(b) $P(RR) = \frac{4}{4+n} \times \frac{3}{3+n} = \frac{2}{15}$

$$\Leftrightarrow 12 \times 5 = 2(4+n)(3+n) \Leftrightarrow 12 + 7n + n^2 = 90 \Rightarrow n^2 + 7n - 78 = 0 \Rightarrow n = 6$$

(c)



$$P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{15} = \frac{11}{90}$$

(d) $P(1 \text{ or } 6) = P(A)$

$$P(A|RR) = \frac{P(A \cap RR)}{P(RR)} = \frac{\left[\left(\frac{1}{3} \right) \left(\frac{1}{10} \right) \right]}{\frac{11}{90}} = \frac{3}{11}$$