

EXERCISES [MAI 4.26]
CRITICAL REGION – TYPE I-II ERRORS
SOLUTIONS
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A. Paper 1 questions (SHORT)

CRITICAL REGION

1. We use InvN with $\mu = 205$ and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}}$

(a) InvN, tail: central $1-a$

	Critical region	Non-critical region
If $a = 0.10$	$(-\infty, 202.21) \cup (207.79, +\infty)$	$[202.21, 207.79]$
If $a = 0.05$	$(-\infty, 201.67) \cup (208.33, +\infty)$	$[201.67, 208.33]$
If $a = 0.01$	$(-\infty, 200.63) \cup (209.37, +\infty)$	$[200.63, 209.37]$

(b) InvN, tail: left a

	Critical region	Non-critical region
If $a = 0.10$	$(-\infty, 202.83)$	$[202.83, +\infty)$
If $a = 0.05$	$(-\infty, 202.21)$	$[202.21, +\infty)$
If $a = 0.01$	$(-\infty, 201.05)$	$[201.05, +\infty)$

(c) InvN, tail: right a

	Critical region	Non-critical region
If $a = 0.10$	$(207.17, +\infty)$	$(-\infty, 207.17]$
If $a = 0.05$	$(207.79, +\infty)$	$(-\infty, 207.79]$
If $a = 0.01$	$(208.95, +\infty)$	$(-\infty, 208.95]$

2. We use Poisson with $\lambda = 63$.

(a) Looking for r such that $P(X \leq r) < a$

	Critical region for X	Non-critical region for X
If $a = 0.10$	$X \leq 52$	$X \geq 53$
	Probability of critical region: 0.0900	
If $a = 0.05$	$X \leq 49$	$X \geq 50$
	Probability of critical region: 0.0404	
If $a = 0.01$	$X \leq 44$	$X \geq 45$
	Probability of critical region: 0.00736	

(b) Looking for r such that $P(X \geq r) < a$

	Critical region for X	Non-critical region for X
If $a = 0.10$	$X \geq 74$	$X \leq 73$
	Probability of critical region: 0.0953	
If $a = 0.05$	$X \geq 77$	$X \leq 76$
	Probability of critical region: 0.0479	
If $a = 0.01$	$X \geq 83$	$X \leq 82$
	Probability of critical region: 0.00904	

3. We use Binomial with $n = 150$ and $p = 0.18$

(a) Looking for r such that $P(X \leq r) < \alpha$

	Critical region for X	Non-critical region for X
If $\alpha = 0.10$	$X \leq 20$	$X \geq 21$
	Probability of critical region: 0.0799	
If $\alpha = 0.05$	$X \leq 18$	$X \geq 19$
	Probability of critical region: 0.0308	
If $\alpha = 0.01$	$X \leq 16$	$X \geq 17$
	Probability of critical region: 0.00952	

(b) Looking for r such that $P(X \geq r) < \alpha$

	Critical region for X	Non-critical region for X
If $\alpha = 0.10$	$X \geq 34$	$X \leq 33$
	Probability of critical region: 0.0863	
If $\alpha = 0.05$	$X \geq 36$	$X \leq 35$
	Probability of critical region: 0.0390	
If $\alpha = 0.01$	$X \geq 39$	$X \leq 38$
	Probability of critical region: 0.00936	

TYPE I AND TYPE II ERRORS

4. (a) (i) It is just the significance level α

(ii) We use Normal with $\mu = 200$ and $\sigma = \frac{12}{\sqrt{50}}$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $\alpha = 0.10$	0.10	0.0964
If $\alpha = 0.05$	0.05	0.163
If $\alpha = 0.01$	0.01	0.355

(b) (i) It is just the significance level α

(ii) We use Normal with $\mu = 200$ and $\sigma = \frac{12}{\sqrt{50}}$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $\alpha = 0.10$	0.10	0.0477
If $\alpha = 0.05$	0.05	0.0964
If $\alpha = 0.01$	0.01	0.268

(c) (i) It is just the significance level α

(ii) We use Normal with $\mu = 210$ and $\sigma = \frac{12}{\sqrt{50}}$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $\alpha = 0.10$	0.10	0.0477
If $\alpha = 0.05$	0.05	0.0964
If $\alpha = 0.01$	0.01	0.268

5. (a) (i) It is $P(\text{critical region})$.

In fact, we find the closest value $a' (< a)$ such that $P(X \leq r) = a'$

- (ii) We use Poisson with $\lambda = 42$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $a = 0.10$	0.0900	0.0566
If $a = 0.05$	0.0404	0.125
If $a = 0.01$	0.00736	0.342

- (b) (i) It is $P(\text{critical region})$.

In fact, we find the closest value $a' (< a)$ such that $P(X \geq r) = a'$

- (ii) We use Poisson with $\lambda = 85$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $a = 0.10$	0.0953	0.104
If $a = 0.05$	0.0479	0.178
If $a = 0.01$	0.00904	0.400

6. (a) (i) It is $P(\text{critical region})$.

In fact, we find the closest value $a' (< a)$ such that $P(X \leq r) = a'$

- (ii) We use Binomial with $n = 150$ and $p = 0.14$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $a = 0.10$	0.0799	0.536
If $a = 0.05$	0.0308	0.715
If $a = 0.01$	0.00952	0.856

- (b) (i) It is $P(\text{critical region})$.

In fact, we find the closest value $a' (< a)$ such that $P(X \geq r) = a'$

- (ii) We use Binomial with $n = 150$ and $p = 0.23$ to find $P(\text{non-critical region})$

	Type I error	Type II error
If $a = 0.10$	0.0863	0.430
If $a = 0.05$	0.0390	0.583
If $a = 0.01$	0.00936	0.783