EXERCISES [MAI 4.20-4.22]

CONFIDENCE INTERVAL – HYPOTHESIS TEST FOR μ

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

STANDARD DEVIATION – UNBIASED ESTIMATES FOR μ AND σ²

1. (a)

the mean of the population	μ	29.5
the standard deviation of the population	σ	4.42
The variance of the population	σ^2	19.5

(b)

the mean of the sample	$\bar{x}$	29.5
the standard deviation of the sample	s_n	4.42
The variance of the sample	${s_n}^2$	19.5
the value of s_{n-1}	s_{n-1}	4.72

(c)

an unbiased estimate for the mean of the population	$\bar{x} = 29.5$
an unbiased estimate for the variance of the population	${s_{n-1}}^2 = 4.72077...^2 \cong 22.3$

(d) $${s_{n-1}}^2 = \frac{n}{n-1}{s_n}^2 \Rightarrow {s_{n-1}}^2 = \frac{8}{7}{s_n}^2$$

CONFIDENCE INTERVAL FOR μ

2. (a) [Statistics – INTR – Z – 1 sample – Data: **List** – enter C-Level and σ]

90% confidence interval	$[26.6 - 32.4]$
95% confidence interval	$[26.0 - 33.0]$
99% confidence interval	$[24.9 - 34.1]$

(b) [Statistics – INTR – t – 1 sample – Data: **List** – enter C-Level]

90% confidence interval	$[26.3 - 32.7]$
95% confidence interval	$[25.6 - 33.4]$
99% confidence interval	$[23.7 - 35.3]$

3. (a) [Statistics – INTR – Z – [1 sample] – Data: **Var** – enter C-Level and the statistics]

$[76.8 - 83.2]$

(b) [Statistics – INTR – t – [1 sample] – Data: **Var** – enter C-Level and the statistics]

$[76.4 - 83.6]$

(c) [Statistics – INTR – t – [1 sample] – Data: **Var** – enter C-Level and the statistics]

$${s_{n-1}}^2 = \frac{n}{n-1}{s_n}^2 \Rightarrow {s_{n-1}}^2 = \frac{10}{9}5^2 \Rightarrow s_{n-1} = \frac{\sqrt{10}}{3}5 = 5.27$$

$[76.2 - 83.8]$

HYPOTHESIS TEST FOR μ

4. (a) H$_0$: $\mu = 32$,

H$_1$: $\mu \neq 32$

[Statistics – TEST – Z – [1 sample] – Data: **List** – enter $\mu \neq 32$ and σ]

$p-$value = 0.157 (if needed, the statistic is $z_{stat} = -1.41$)

Since $p-$value > a, (0.157 > 0.10), we do not have enough evidence to reject H$_0$

[we accept $\mu = 32$]

(b) H$_0$: $\mu = 32$,

H$_1$: $\mu < 32$

[Statistics – TEST – Z – [1 sample] – Data: **List** – enter $\mu < 32$ and σ]

$p-$value = 0.0786 (if needed, the statistic is $z_{stat} = -1.41$)

Since $p-$value < a, (0.0786 < 0.10), we reject H$_0$

[we reject $\mu = 32$]

(c) H$_0$: $\mu = 28$,

H$_1$: $\mu > 28$

[Statistics – TEST – Z – [1 sample] – Data: **List** – enter $\mu > 28$ and σ]

$p-$value = 0.198 (if needed, the statistic is $z_{stat} = 0.849$)

Since $p-$value > a, (0.198 > 0.10), we do not have enough evidence to reject H$_0$

[we accept $\mu = 28$]

5. (a) H$_0$: $\mu = 32$,

H$_1$: $\mu \neq 32$

[Statistics – TEST – t – [1 sample] – Data: **List** – enter $\mu \neq 32$]

$p-$value = 0.178 (if needed, the statistic is $t_{stat} = -1.50$)

Since $p-$value > a, (0.178 > 0.10), we do not have enough evidence to reject H$_0$

[we accept $\mu = 32$]

(b) H$_0$: $\mu = 32$,

H$_1$: $\mu < 32$

[Statistics – TEST – t – [1 sample] – Data: **List** – enter $\mu < 32$]

$p-$value = 0.0889 (if needed, the statistic is $t_{stat} = -1.50$)

Since $p-$value < a, (0.0889 < 0.10), we reject H$_0$

[we reject $\mu = 32$]

(c) H$_0$: $\mu = 28$,

H$_1$: $\mu > 28$

[Statistics – TEST – t – [1 sample] – Data: **List** – enter $\mu > 28$]

$p-$value = 0.199 (if needed, the statistic is $t_{stat} = 0.899$)

Since $p-$value > a, (0.199 > 0.10), we do not have enough evidence to reject H$_0$

[we accept $\mu = 28$]

6. (a) H$_0$: $\mu = 83$,

H$_1$: $\mu \neq 83$

[Statistics – TEST – Z – [1 sample] – Data: **Var** – enter $\mu \neq 83$ and σ]

$p-$value = 0.0681 (if needed, the statistic is $z_{stat} = -1.82$)

Since $p-$value > a, (0.0680 > 0.05), we do not have enough evidence to reject H$_0$

[we accept $\mu = 83$]

(b) H$_0$: $\mu = 83$,

H$_1$: $\mu < 83$

[Statistics – TEST – Z – [1 sample] – Data: **Var** – enter $\mu < 83$ and σ]

$p-$value = 0.0340 (if needed, the statistic is $z_{stat} = -1.82$)

Since $p-$value < a, (0.0340 < 0.05), we reject H$_0$

[we reject $\mu = 83$]

(c) H$_0$: $\mu = 78$,

H$_1$: $\mu > 78$

[Statistics – TEST – Z – [1 sample] – Data: **Var** – enter $\mu > 78$ and σ]

$p-$value = 0.112 (if needed, the statistic is $z_{stat} = 1.22$)

Since $p-$value > a, (0.112 > 0.05), we do not have enough evidence to reject H$_0$

[we accept $\mu = 78$]

7. (a) H$_0$: $\mu = 83$,

H$_1$: $\mu \neq 83$

[Statistics – TEST – t – [1 sample] – Data: **Var** – enter $\mu \neq 83$ and the statistics]

$p-$value = 0.0903 (if needed, the statistic is $t_{stat} = -1.89$)

Since $p-$value > a, (0.0903 > 0.05), we do not have enough evidence to reject H$_0$

[we accept $\mu = 83$]

(b) H$_0$: $\mu = 83$,

H$_1$: $\mu < 83$

[Statistics – TEST – t – [1 sample] – Data: **Var** – enter $\mu < 83$ and the statistics]

$p-$value = 0.0451 (if needed, the statistic is $t_{stat} = -1.89$)

Since $p-$value < a, (0.0340 < 0.05), we reject H$_0$

[we reject $\mu = 83$]

(c) H$_0$: $\mu = 78$,

H$_1$: $\mu > 78$

[Statistics – TEST – t – [1 sample] – Data: **Var** – enter $\mu > 78$ and the statistics]

$p-$value = 0.119 (if needed, the statistic is $t_{stat} = 1.26$)

Since $p-$value > a, (0.119 > 0.05), we do not have enough evidence to reject H$_0$

[we accept $\mu = 78$]

8. As in 7, but now $s_{n-1}{}^2 = \dfrac{n}{n-1} s_n{}^2 \Rightarrow s_{n-1}{}^2 = \dfrac{10}{9} 5^2 \Rightarrow s_{n-1} = \dfrac{\sqrt{10}}{3} 5 = 5.27$

In all cases $p-$value > 0.05

(a) $p-$value = 0.105, we do not have enough evidence to reject H$_0$: $\mu = 83$

(b) $p-$value = 0.0527, we do not have enough evidence to reject H$_0$: $\mu = 83$

(c) $p-$value = 0.130, we do not have enough evidence to reject H$_0$: $\mu = 78$

9. We find the differences of paired data

Year 1	23	24	27	29	31	31	35	36
Year 2	25	24	32	28	35	29	35	39
d	**2**	**0**	**5**	**– 1**	**4**	**– 2**	**0**	**3**

(a) H_0: $d = 0$,
H_1: $d \neq 0$
[Statistics – TEST – t – [1 sample] – Data: **List** – enter $\mu \neq 0$]
$p -$value = 0.164 (if needed, the statistic is $t_{stat} = 1.55$)
Since $p -$value > a, (0.164 > 0.10), we do not have enough evidence to reject H_0
[we accept $d = 0$]

(b) H_0: $d = 0$,
H_1: $d > 0$
[Statistics – TEST – t – [1 sample] – Data: **List** – enter $\mu > 0$]
$p -$value = 0.0821 (if needed, the statistic is $t_{stat} = 1.55$)
Since $p -$value < a, (0.0821 < 0.10), we reject H_0
[we accept $d > 0$, that is the values have increased.]