EXERCISES [MAI 4.16-4.17]

HYPOTHESIS TEST FOR TWO MEANS

SOLUTIONS

A. Paper 1 questions (SHORT)

1. (a) We consider two samples from the two countries and compare the means.

We use a one-tailed t-test for the means.

We assume that the heights follow Normal distribution.

(b) Given that μ_1, μ_2 are the mean heights of men in Italy and Spain respectively,

Null hypothesis, $H_0: \mu_1 = \mu_2$

Alternative hypothesis, $H_1: \mu_1 > \mu_2$

2. (a) The probability the reject the null hypothesis H_0: $\mu_1 = \mu_2$ given that is true (that is to make a false-positive mistake) is 5%

(b) For a = 0.10 or 0.05 or 0.01

(i) if p-value = $0.78\times10^{-3} = 0.0078$,

in any case, since p-value < a, we reject the null hypothesis.

(ii) if p-value = 0.0392 the conclusion depends on the significance level

For a = 0.10 or 0.05, since p-value < a we reject the null hypothesis.

For a = 0.01, we do not have enough evidence to reject the null hypothesis

(iii) if p-value = 0.1109,

in any case, since p-value > a . we accept the null hypothesis

(in fact, we do not have enough evidence to reject the null hypothesis).

3. (a) $\bar{x}_1 = 18.25$, $\bar{x}_2 = 16$

(b) We use a two-tailed t-test for the means.

We assume that the values of the feature follow Normal distribution.

(c) Null hypothesis, $H_0: \mu_1 = \mu_2$

Alternative hypothesis, $H_1: \mu_1 \neq \mu_2$

(d) Remember to set **Data: List, Pooled: On**

(i) p -value = 0.0303,

(ii) t -value = 2.34

(e) $a = 0.05$, since p -value < a, we reject the null hypothesis (so there is a difference)

(f) $a = 0.01$, since p -value > a, we do not have enough evidence to reject the null hypothesis (so there is no significant difference).

4. (a) $\overline{x}_1 = 18.25$, $\overline{x}_2 = 16$

(b) We use a one-tailed t-test for the means.

We assume that the values of the feature follow Normal distribution.

(c) Null hypothesis, $H_0: \mu_1 = \mu_2$

Alternative hypothesis, $H_1: \mu_1 > \mu_2$

(d) Remember to set **Data: List, Pooled: On**

(i) p-value = 0.0151, (ii) t-value = 2.34

(e) $a = 0.05$, since p-value < a, we reject the null hypothesis (so population A has significantly greater values than population B)

(f) $a = 0.01$, since p-value > a, we do not have enough evidence to reject the null hypothesis (so there is no significant difference).

5. (a) (i) median 1 = 51, median 2 = 50

(ii) $\overline{x}_1 = 51.1$, $\overline{x}_2 = 58.6$

(b) According to the medians, Firstland has better results from Secondland.

According to the means, Secondland has better results from Firstland.

In general Secondland seems to perform better.

(c) We use a two-tailed t-test for the means.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Remember to set **Data: List, Pooled: On**

p-value = 0.185, $a = 0.10$

Since p-value > a, we do not have enough evidence to reject the null hypothesis

(so Peter's claim that the tow countries had almost the same performance is valid).

(d) We use a one-tailed t-test for the means.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 < \mu_2$

Remember to set **Data: List, Pooled: On**

(i) p-value = 0.092, $a = 0.10$

Since p-value < a, we reject the null hypothesis

(so Maria is right, Secondland had better results than Firstland).