

EXERCISES [MAI 4.10]
BINOMIAL DISTRIBUTION
SOLUTIONS

A. Paper 1 questions (SHORT)

1. (a)

exactly 4 heads	0.273
exactly 3 heads	0.219
3, 4 or 5 heads	0.711
no heads	0.00391
always heads	0.00391
at most 2 heads	0.145
at least 3 heads	0.855

(b)

$E(X)$	4	$\text{Var}(X)$	2
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2. $np = 10$ and $np(1-p) = 6$. Hence $10(1-p) = 6 \Leftrightarrow p = 0.4$ and $n = 25$

3. $B(n, p)$ with $n = 5$ and $p = \frac{1}{2}$

(a) $P(X = 3) = 0.3125 \dots = 0.313$

(b) $P(X \geq 1) = 0.969$

4. $B(n, p)$ with $n = 7$ and $p = 0.18$

(a) $P(X = 2) = 0.252$

(b) $P(X \geq 2) = 0.368$

5. $B(n, p)$ with $n = 100$ and $p = 0.04$

(a) mean $= np = 100 \times 0.04 = 4$

(b) $P(X = 6) = 0.105$

(c) $P(X \geq 1) = 0.983$

6. $X \sim B(100, 0.02)$

(a) $E(X) = 100 \times 0.02 = 2$

(b) (i) $P(X = 3) = 0.182$ (ii) $P(X > 1) = 0.597$

7. $p(\text{Red}) = \frac{35}{40} = \frac{7}{8}$ $p(\text{Black}) = \frac{5}{40} = \frac{1}{8}$

(a) $B(n, p)$ with $n = 8$, $p = \frac{1}{8}$

(i) $p(\text{one black}) = P(X = 1) = 0.393$ to 3 s.f. (ii) $p(\text{at least one black}) = P(X \geq 1) = 0.656$

(b) 400 draws: expected number of blacks $= \frac{400}{8} = 50$

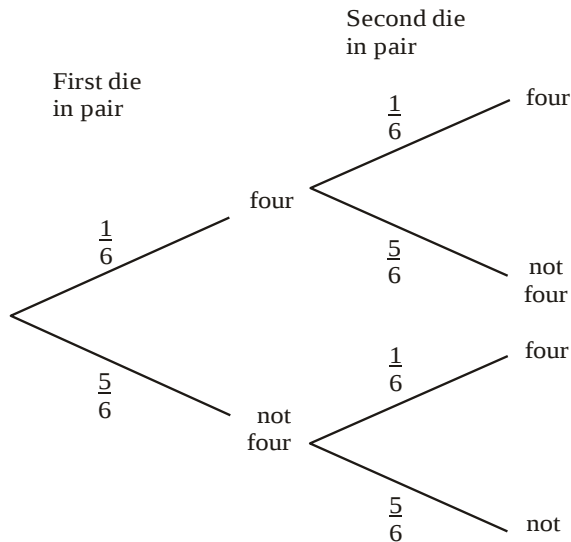
8. $X \sim B(n, p)$ with $n = 5$ and $p = \frac{1}{3}$

Therefore $P(X = 3) = 0.165$

9. (a) Probability = 0.138
 (b) Probability = $(0.6)^2 \times 0.4 = 0.144$ (or $\frac{18}{125}$)
10. (a) X is $B(10, 0.25)$
 $E(X) = 10 \times 0.25 = 2.5$
 (b) $P(X \leq 2) = 0.526$
11. X is Binomial $n = 5$ $p = 0.4$
 $P(X \leq 3) = 0.913$ to 3 s.f.
12. (a) $B(n, p)$ with $n = 3$, $p = \frac{1}{3}$
 (i) $P(X = 3) = 0.0370$ or $P(3H) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$
 (ii) $P(X = 3) = 0.222$ or $P(2H, 1T) = 3 \left(\frac{1}{3}\right)^2 \frac{2}{3} = \frac{2}{9}$
 (b) (i) expected number of heads = $np = \frac{1}{3} \times 12 = 4$
 (ii) 4 heads, so 8 tails
 $E(\text{winnings}) = 4 \times 10 - 8 \times 6 (= 40 - 48) = -\$ 8$
13. $B(n, p)$ with $n = 7$, $p = \frac{1}{5}$
 $P(X = 4) = 0.0287$
 $P(X \geq 4) = 0.0333$
14. $B(n, p)$ with $n = 20$, $p = \frac{1}{4}$
 (a) $E(X) = 20 \times \frac{1}{4} = 5$
 (b) $P(X = 5) = 0.202$ to 3 s.f.
 (c) $P(X < 5) = 0.415$ to 3 s.f. [less than five means $P(X \leq 4)$]
15. (a) $P(\text{all ten cells fail}) = 0.107$ (or 0.8^{10})
 (b) (satellite is still operating at the end of one year if $X \geq 1$)
 $P(X \geq 1) = 0.893$ (or $1 - 0.107 = 0.893$)
16. (i) mean = $10 \times 0.4 = 4$
 (ii) check $P(X = 3) = 0.214$, $P(X = 4) = 0.251$, $P(X = 5) = 0.201$ so mode = 4
 (iii) variance = $10 \times 0.4 \times 0.6 = 2.4$
 (iv) st. dev = $\sqrt{2.4} = 1.55$
17. (i) mean = $10 \times \frac{1}{4} = 2.5$
 (ii) check $P(X = 2) = 0.281$, $P(X = 3) = 0.250$ so mode = 2
 (iii) variance = $10 \times \frac{1}{4} \times \frac{3}{4} = \frac{15}{8} = 1.875$
 (iv) st. dev = $\sqrt{1.875} = 1.37$

B. Paper 2 questions (LONG)

18. (a)



$$(b) \quad P(E) = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{36} + \frac{5}{36} \right) = \frac{10}{36} = \frac{5}{18} \quad \left(= \frac{5}{18} \text{ or } 0.278 \right)$$

$$(c) \quad X \sim B\left(5, \frac{5}{18}\right)$$

$$P(X=3) = 0.112 \quad \left[\text{in fact } \binom{5}{3} \left(\frac{5}{18}\right)^3 \left(\frac{13}{18}\right)^2 = 0.112 \right]$$

$$(d) \quad P(X \geq 3) = 0.135$$

19. $B(n, p)$ with $n = 10$, $p = \frac{1}{4}$

$$(a) \quad E(X) = 10 \times \frac{1}{4} = 2.5$$

$$(b) \quad P(X=6) = 0.0162$$

$$(c) \quad P(X \geq 2) = 0.756$$

(d) Since $E(X) = 2.5$ the mode is 2 or 3
Using GDC

x	$P(X=x)$
1	0.188
2	0.282
3	0.250

From these values the most likely number of yellow ribbons is 2.

(e) The probability that a ribbon is yellow remains constant ($= \frac{1}{4}$)

20. $B(n, p)$ with $n = 20$, $p = 0.3$

$$(a) \quad \text{Mean} = 20 \times 0.3 = 6 \quad \text{Variance} = 20 \times 0.3 \times 0.7 = 4.2$$

$$(b) \quad (i) \quad P(X=5) = 0.179 \quad (ii) \quad P(4 \leq X \leq 8) = 0.780$$

$$(c) \quad 0.3$$

$$(d) \quad 0.7 \times 0.7 \times 0.3 = 0.147$$