

EXERCISES [MAI 4.1-4.3]
DESCRIPTIVE STATISTICS
SOLUTIONS

A. Paper 1 questions (SHORT)

1.

Number of children in a family	discrete
Height of Greek men	continuous
Time spent in a supermarket	continuous
Final grade in IB exams	discrete
100m sprint time	continuous
100m sprint time to the nearest second	discrete

2. 1-C, 2-D, 3-B, 4-A

3. mode = 6, median = 6, mean = 5,
range = 6, IQR = $6.5 - 2.5 = 4$, standard deviation = $2.0548 = 2.05$, variance = $2.0548^2 = 4.22$

4. mode = 6, median = 6, mean = 6.76
range = 4, IQR = $8.5 - 5 = 3.5$, standard deviation = $1.556 = 1.56$, variance = $1.556^2 = 2.45$

5. (a) 51, 53, 57, 60
(b) mode = 84, median = 78
(c) range = 48, IQR = $84 - 60 = 24$

6.

stem	leaf	Key 16 1 represents 161
16	1 1 5 6 7 9	
17	3 3 5 7 7	
18	0 3 4	
19	2 5 5 7	

7. mean = 25 standard deviation = $5\sqrt{5}$ variance = 125

	mean	standard deviation	variance
+ 2	27	$5\sqrt{5}$	125
$\times 2$	50	$10\sqrt{5}$	500
+ a	$25 + a$	$5\sqrt{5}$	125
$\times a$	$25a$	$5a\sqrt{5}$	$125a^2$

8. (a) mean = $\frac{3 \times 16 + 8 \times 28 + 13 \times 26 + 18 \times 14}{84} = 10.3$
(b) modal group: 6-10
(c) standard deviation = $4.90412... = 4.90$ variance = $(4.90412...)^2 = 24.1$

9. Median = middle value $\Rightarrow b = 11$
Mean = $\frac{a+b+c}{3} = \frac{a+11+c}{3} = 9 \Rightarrow a + 11 + c = 27 \Rightarrow a + c = 16$
Range = $c - a = 10$
Solving equations simultaneously gives $a = 3$
10. $d = 11; c = 11$
 $d - a = 8 \Rightarrow 11 - a = 8 \Rightarrow a = 3$
 $\frac{3+b+11+11}{4} = 8$
 $b = 7$
11. $b = 3, c = 3$
using mean $\left(\frac{a+b+c+d}{4} = 4 \right)$ and range ($d - a = 6$)
 $a = 2, d = 8$
12. (a) (i) $r = 10$
(ii) $s = 13$
(b) $\frac{\sum x}{12} = 10 \Rightarrow t = 18$
13. (a) $A = 18, B = 19, C = 23, D = 31, E = 36$
(b) IQR = 12
14. (a) 3
(b) 6
(c) 25% scored greater than 6,
 $0.25 \times 32 = 8$
15. (a) 18
(b) (i) 10
(ii) 44
16. (a) max = 27, min = 4, range = 23
(b) Graph 3
17. (a) mean = $\sum \frac{x}{n} \left(= \frac{2230}{45} \right) \bar{x} = 49.6$
(b) $\sum y = 2230 + 37 + 30$
 $y = \frac{2297}{47} = 48.9$
18. (a) $m = \frac{300}{25} = 12$
25
(b) $\sum_{i=1} x_i = 25 \times 10 = 250$
19. Mean = $\frac{(72 \times 1.79) + (28 \times 1.62)}{100} = 1.7424 (= 1.74 \text{ to 3 sf})$

20. Jan–Sept $\Sigma = 630 \times 9 = 5670$
 Oct–Dec $\Sigma = 810 \times 3 = 2430$

$$\bar{x} = \frac{5670 + 2430}{12} = 675$$

21. (a) $\Sigma f_i = 100 \Rightarrow k = 4$
 (b) (i) median = 3
 (ii) $Q_1 =$ and $Q_3 = 5$
 interquartile range = 4

22. (a) Median = 4
 (b) Mean = $\frac{5 + 18 + 48 + 72 + 100 + 42}{75} = \frac{285}{75} = 3.8$ (or directly mean = 3.8)

23. (a) $\sigma = 1.61$
 (b) median = 4.5
 (c) $Q_1 = 3, Q_3 = 5$
 IQR = 2

24.

x	f	Σf
4	2	2
5	5	7
6	4	11
7	3	14
8	4	18
10	2	20
12	1	21

- (a) $m = 6$
 (b) $Q_1 = 5$
 (c) $Q_3 = 8$
 IQR = $8 - 5 = 3$

25. $p < 10$ because 2 is the mode
 Possible values of p are 8 and 9

26. (a) (i) $p = 34 + 35 = 69$
 (ii) $q = 109 - 69 = 40$, **OR** $q = 120 - (9 + 25 + 35 + 11)$
 (b) mean = 3.16
 (c) 1.09

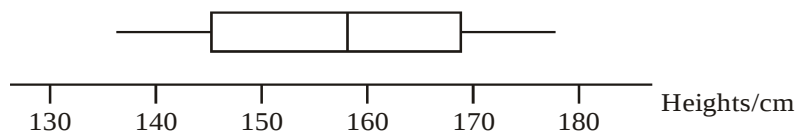
27. (a) (i) $p = 65$
 (ii) $q = 34$
 (b) median is 17 (sit-ups)
 (c) mean = 17.4

28. (a) $4.6 = \frac{144 + 4k}{k + 30}$
 $4.6k + 138 = 144 + 4k$
 $0.6k = 6$
 $k = 10$
 (b) Mode = 4

29.
$$\frac{(10 \times 1) + (20 \times 2) + (30 \times 5) + (40 \times k) + (50 \times 3)}{k + 11} = 34$$

$$\Rightarrow \frac{40k + 350}{k + 11} = 34 \Rightarrow k = 4$$

30. (a) (i) 145 (ii) 157 (iii) 167
 (b)

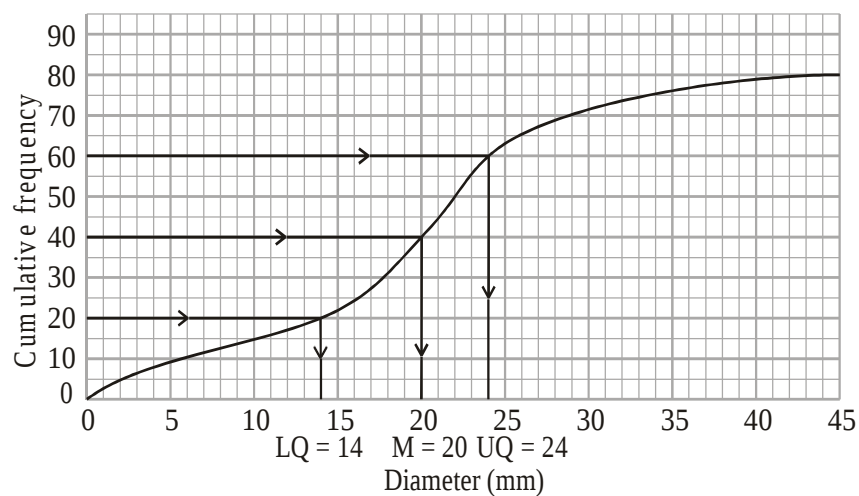


31. (a)

Age range	Frequency	Mid - interval value
$0 \leq \text{age} < 20$	40	10
$20 \leq \text{age} < 40$	70	30
$40 \leq \text{age} < 60$	100	50
$60 \leq \text{age} < 80$	50	70
$80 \leq \text{age} \leq 100$	10	90

- (b) Mean = 44.1

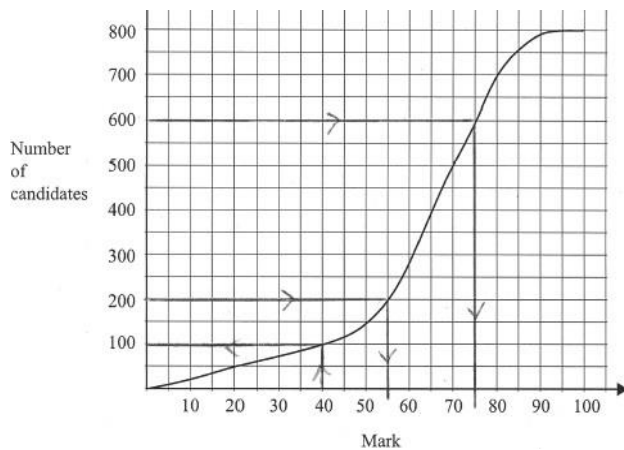
- 32.



- (a) (i) median = 20
 (ii) $Q_3 = 24$
 (b) $IQR = Q_3 - Q_1 = 10$

33. (a) median is 183
 (b) Lower quartile $Q_1 = 175$
 Upper quartile $Q_3 = 189$
 IQR is 14

34.



- (a) 100 students score 40 marks or fewer.
 (b) For 200 **and** 600, lines on graph
 $a = 55, b = 75$

35. (a) median $m = 32$
 (b) lower quartile $Q_1 = 22$, upper quartile $Q_3 = 40$
 interquartile range = 18

(c)

Time (minutes)	Number of students
$0 \leq t < 10$	5
$10 \leq t < 20$	11
$20 \leq t < 30$	20
$30 \leq t < 40$	24
$40 \leq t < 50$	14
$50 \leq t < 60$	6

36. (a) (i) $m = 165$
 (ii) $Q_1 = 160$ $Q_2 = 170$
 $IQR = 10$
 (b) 40th percentile, 48th student (a horizontal line through (0, 48))
 $a = 163$

37. (a)

Mark (x)	$0 \leq x < 20$	$20 \leq x < 40$	$40 \leq x < 60$	$60 \leq x < 80$	$80 \leq x < 100$
No of Students	22	50 (± 1)	66 (± 1)	42 (± 1)	20

- (b) 40th Percentile $\Rightarrow$ 80th student fails, (mark 42%)
 Pass mark 43% (Accept mark > 42 .)

38. (a) 76 (mice)
 (b) 11.2 (seconds)
 (c) (i) $p = 38$
 $q = 132 - 76 = 56$
 (ii) $x = 11.2$

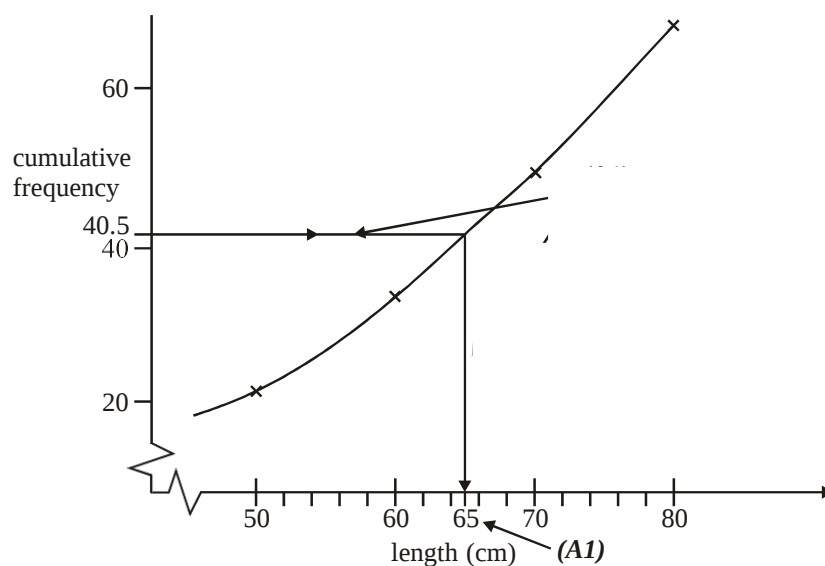
39. (a) D B C
 (b) B A C

B. Paper 2 questions (LONG)

40. (a) (i) 10
(ii) $14 + 10 = 24$
(b)

x_i	f_i
15	1
25	5
35	7
45	9
55	10
65	16
75	14
85	10
95	8
	80

- (i) $\mu = 63$
(ii) $\sigma = 20.5$ (3 sf)
(c) Assymmetric diagram/distribution
(d)



Median = 65

41. (a) (i) 50 (accept 49)
(ii) $90 - 50 = 40$
(iii) 75th person
median = 6.25 (min), 6 min 15 secs
(b) Top 40% of 150 = 60, so up to 90. $k = 7$

- (c) (i)

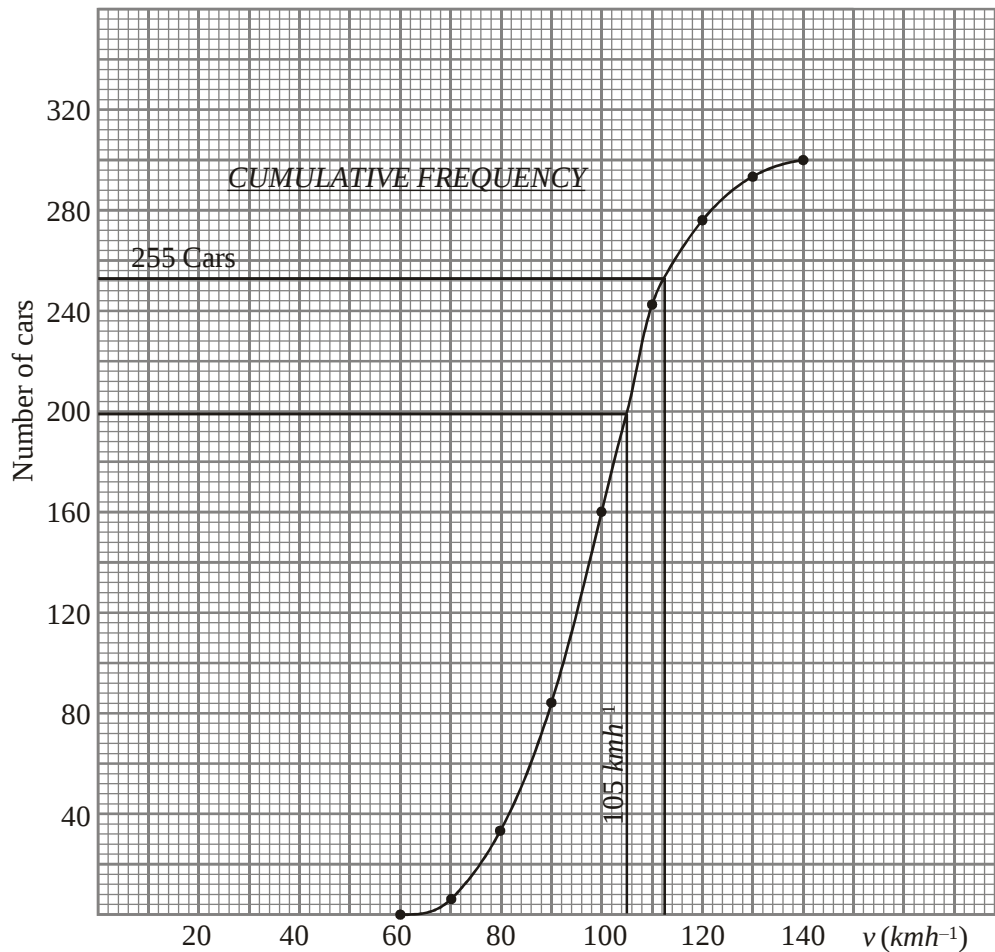
t (minutes)	$0 \leq t < 2$	$2 \leq t < 4$	$4 \leq t < 6$	$6 \leq t < 8$	$8 \leq t < 10$	$10 \leq t < 12$
Frequency	10	23	37	38	27	15

- (ii) mid-interval values (1, 3, 5, 7, 9, 11)
mean = $\left(\frac{1 \times 10 + 3 \times 23 + 5 \times 37 + 7 \times 38 + 9 \times 27 + 11 \times 15}{150} \right) = 6.25$ (min), 6 min 15 secs

OR directly by GSC: Statistics

42. (a) (Using mid-intervals) mean = 98.2 km h^{-1} $\left(\frac{29450}{300} \right)$

- (b) (i) $a = 165, b = 275$
(ii)



- (c) (i) Vertical line on graph at 105 km h^{-1}
percentage = $\frac{300 - 200}{300} \times 100\% = 33.3(\pm 1.3\%)$

- (ii) 15% of $300 = 45$ $300 - 45 = 255$
Horizontal line on graph at 255 cars
Speed = $114(\pm 2 \text{ km h}^{-1})$

43. (a)

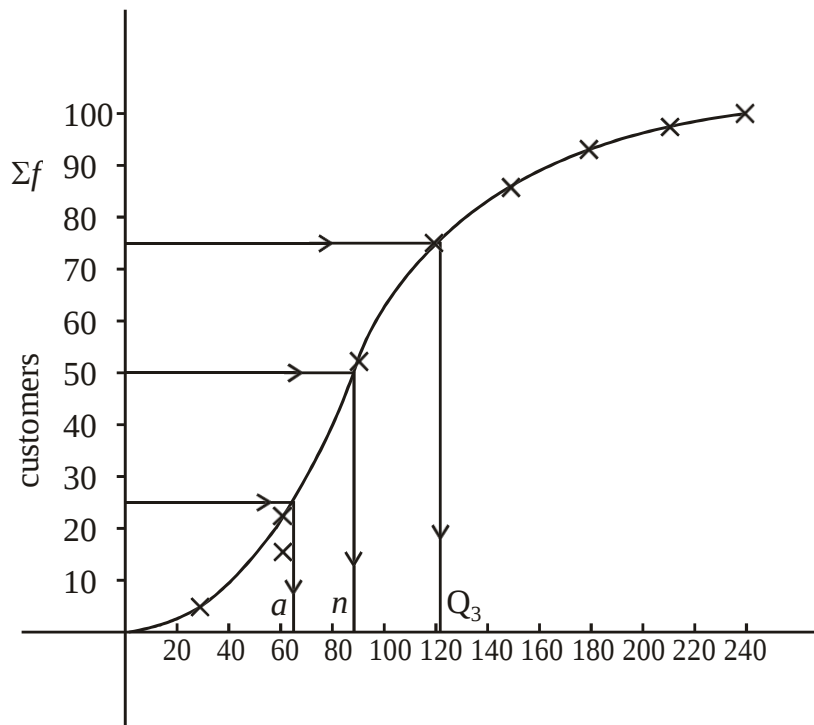
x	15	45	75	105	135	165	195	225
f	5	15	33	21	11	7	5	3

$$\bar{x} = 97.2 \text{ (exactly)}$$

(b)

x	30	60	90	120	150	180	210	240
Σf	5	20	53	74	85	92	97	100

(c)

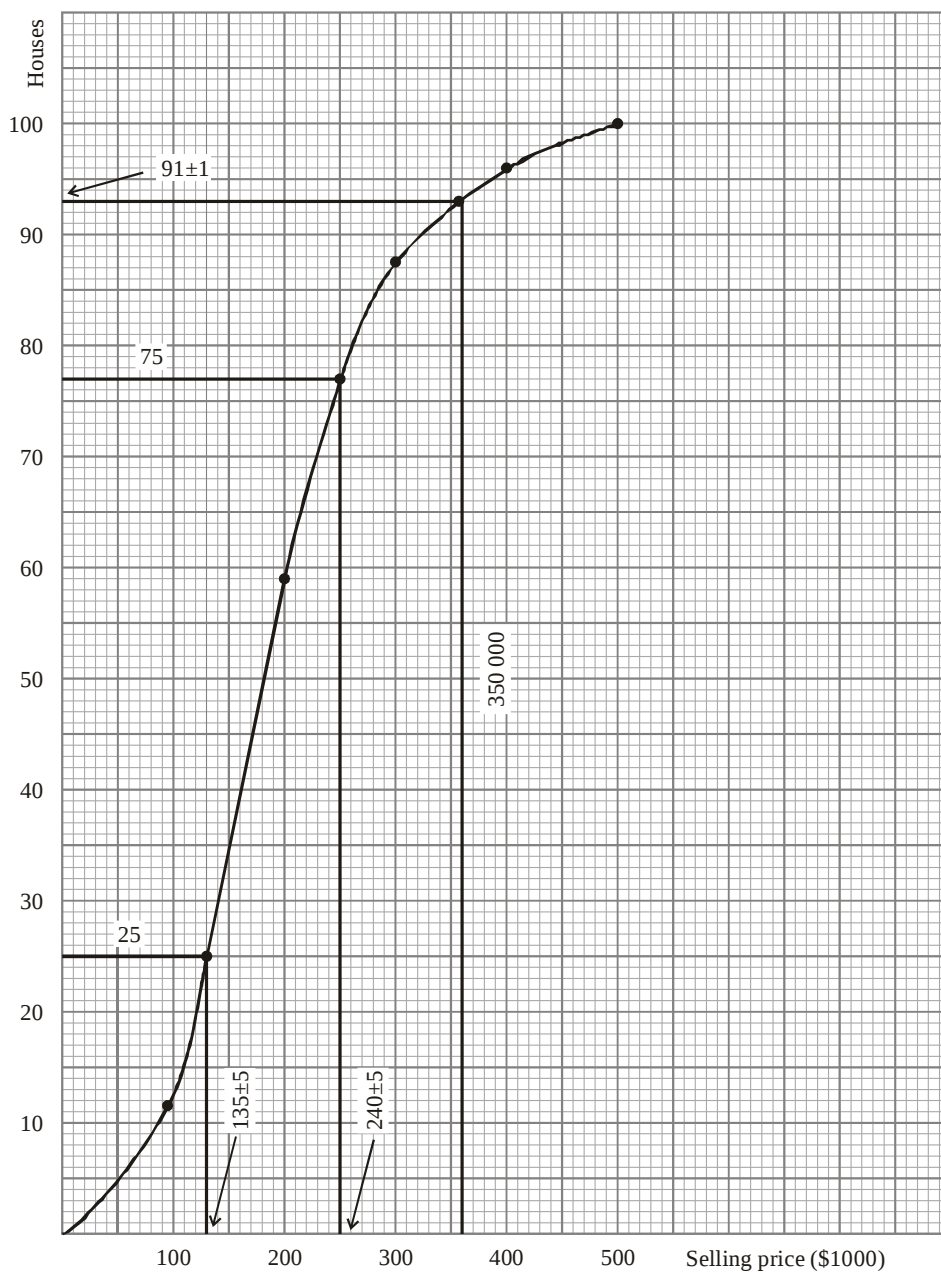


(d) Median = 87 ± 2

Lower quartile = 65 ± 2

Upper quartile = 123 ± 2

44. (a)



(b) $Q_1 = 135 \pm 5$ $Q_3 = 240 \pm 5$
Interquartile range = 105 ± 10 .

(c) $a = 94 - 87 = 7$, $b = 100 - 94 = 6$

(d) $\text{mean} = \frac{12(50) + 46(150) + 29(250) + 7(350) + 6(450)}{100} = 199 \text{ or } \199000

OR by using GDC: Statistics

(e) (i) $\$350000 \Rightarrow 91.5$

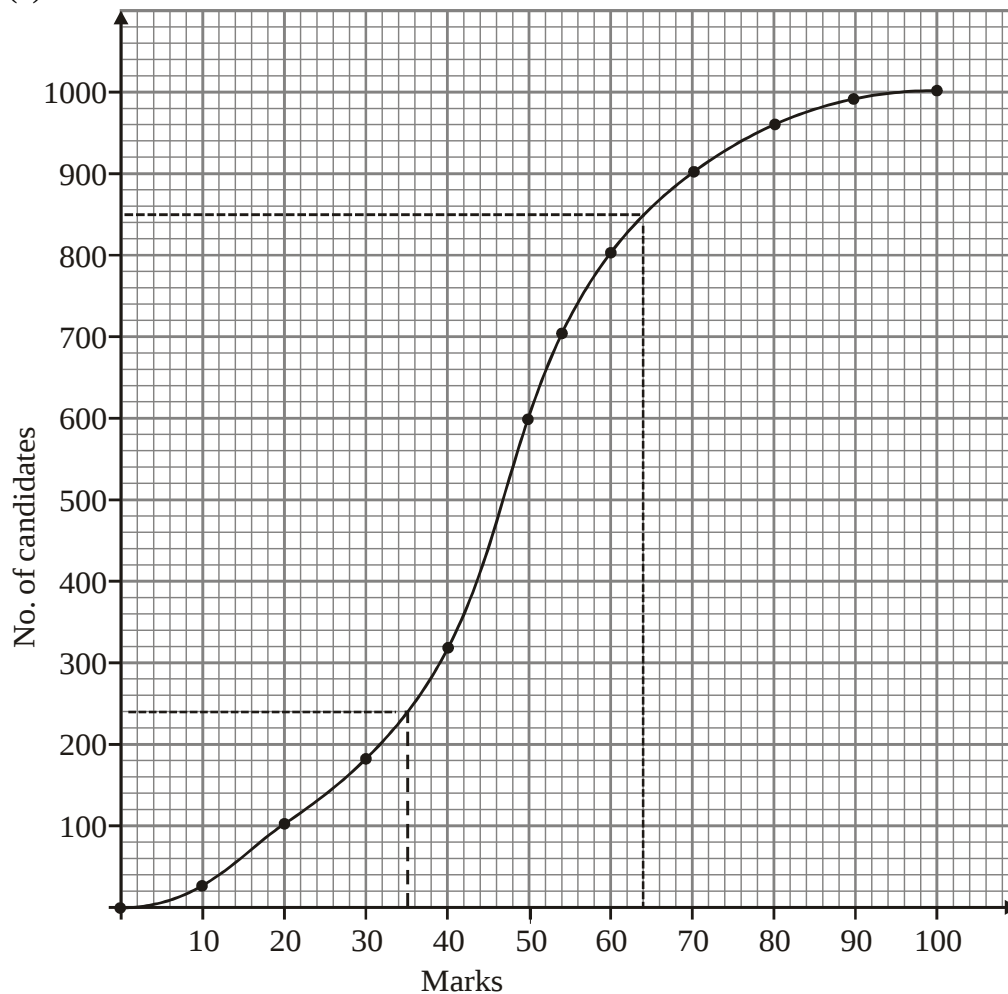
Number of *De luxe* houses $\simeq 100 - 91.5 = 9$ or 8

(ii) $P(\text{both} > 400000) = \frac{6(5)}{9(8)} = \frac{5}{12}$ or $\frac{6(5)}{8(7)} = \frac{15}{28}$

45. (a)

Mark	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70	≤ 80	≤ 90	≤ 100
No. of Candidates	15	65	165	335	595	815	905	950	980	1000

(b)



- (c) (i) Median = 46
(ii) Scores < 35: 240 candidates
(iii) Top 15% $\Rightarrow$ Mark $\geq$ 63

Note: Answers based on the student's graph are accepted.

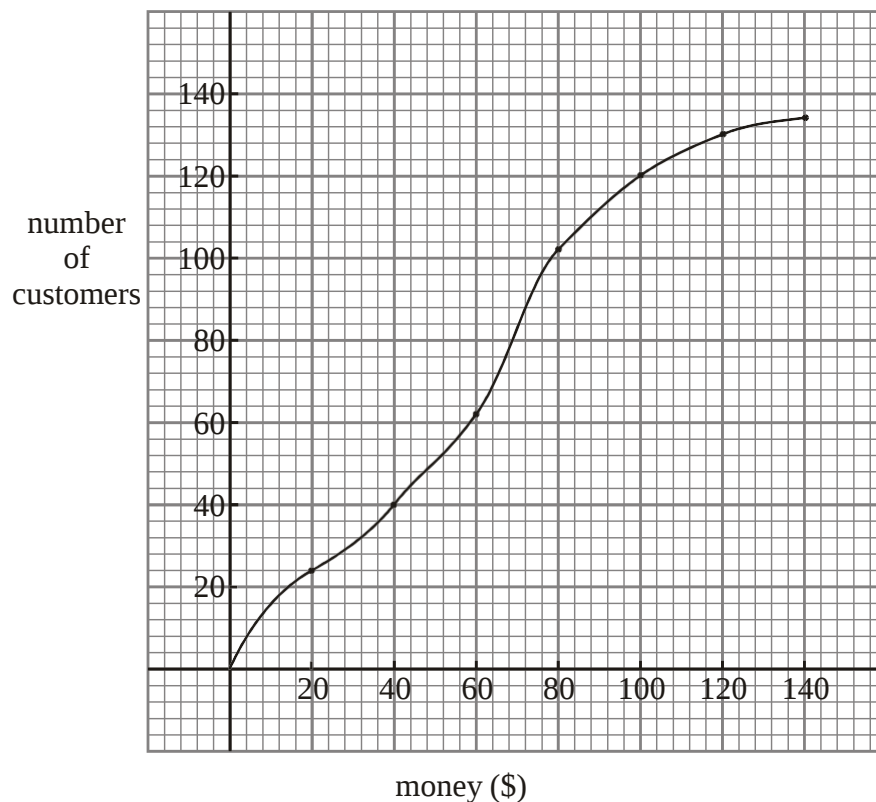
46. (a) $\bar{x} = \$59$ (by GDC : Statistics)

OR

$$\bar{x} = \frac{10 \times 24 + 30 \times 16 + \dots + 110 \times 10 + 130 \times 4}{24 + 16 + \dots + 10 + 4} = \frac{7860}{134} = \$59$$

(b)

Money (\$)	<20	<40	<60	<80	<100	<120	<140
Customers	24	40	62	102	120	130	134



- (c) (i) $t = 2d^{2/3} + 3$
Mean $d = 59$
Mean $t \approx 2 \times (59)^{2/3} + 3 \approx 33.3$ min. (3 sf)
- (ii) $t > 37 \Rightarrow 2d^{2/3} + 3 > 37$
By GDC $d > 70.1$
From the graph, when $d = 70.1$, $n = 82$
number of shoppers = $134 - 82 = 52$

47. (a) Mid interval values 14, 23, 32, 41, 50

$$\bar{w} = \frac{7(14) + 12(23) + 13(32) + 10(41) + 8(50)}{50} = \frac{1600}{50} = 32 \text{ (kg)}$$

(OR by GDC:Statistics)

- (b) **METHOD 1**

Total weight of other boxes = $1600 - 50x$

Total number of other boxes = $50 - x$

$$\frac{1600 - 50x}{50 - x} = 30 \Leftrightarrow x = 5$$

METHOD 2

Let z be the number of other boxes in Class E

Total weight of other boxes = $1200 + 50z$

Total number of other boxes = $42 + z$

$$\frac{1200 - 50z}{42 + z} = 30 \Leftrightarrow z = 3$$

Hence $x = 5$

- (c) $\frac{98 + 276 + 416 + 41(10 + y) + 400}{50 + y} < 33 \Leftrightarrow \frac{1600 - 41y}{50 + y} < 33 \Leftrightarrow y < 6.25$

The largest possible value of y is 6