

EXERCISES [MAI 3.8]
TRIGONOMETRIC FUNCTIONS
SOLUTIONS
 Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1.

Function	Amplitude	Period	Range
$f(x) = \sin x$	1	2π	$-1 \leq y \leq 1$
$f(x) = \cos x$	1	2π	$-1 \leq y \leq 1$
$f(x) = \sin x + 1$	1	2π	$0 \leq y \leq 2$
$f(x) = \sin x - 1$	1	2π	$-2 \leq y \leq 0$
$f(x) = 5 \sin x$	5	2π	$-5 \leq y \leq 5$
$f(x) = -7 \sin x$	7	2π	$-7 \leq y \leq 7$
$f(x) = \sin 4x$	1	$\pi/2$	$-1 \leq y \leq 1$
$f(x) = -\cos 4x$	1	$\pi/2$	$-1 \leq y \leq 1$
$f(x) = 3 \sin 4x$	3	$\pi/2$	$-3 \leq y \leq 3$
$f(x) = 3 \sin 4x + 10$	3	$\pi/2$	$7 \leq y \leq 13$
$f(x) = 3 \sin 4x - 2$	3	$\pi/2$	$-5 \leq y \leq 1$
$f(x) = -5 \sin 3x$	5	$2\pi/3$	$-5 \leq y \leq 5$
$f(x) = -5 \sin x + 10$	5	2π	$5 \leq y \leq 15$
$f(x) = -5 \sin x - 10$	5	2π	$-15 \leq y \leq -5$
$f(x) = -5 \sin x - 10$	5	2π	$-15 \leq y \leq -5$

2. (a) amplitude = 80, central value = 100, period = $\pi/2$

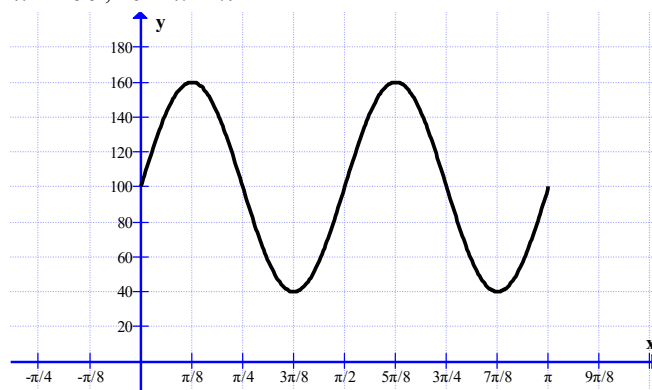
(b) $f(x) = 80 \sin 4x + 100$, since $B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{\pi/2} = 4$

(c) (i) $f(x) = -80 \sin 4(x - \frac{\pi}{4}) + 100$, ($D = \frac{\pi}{4}$ is the position of the 2nd ($\downarrow$) root)

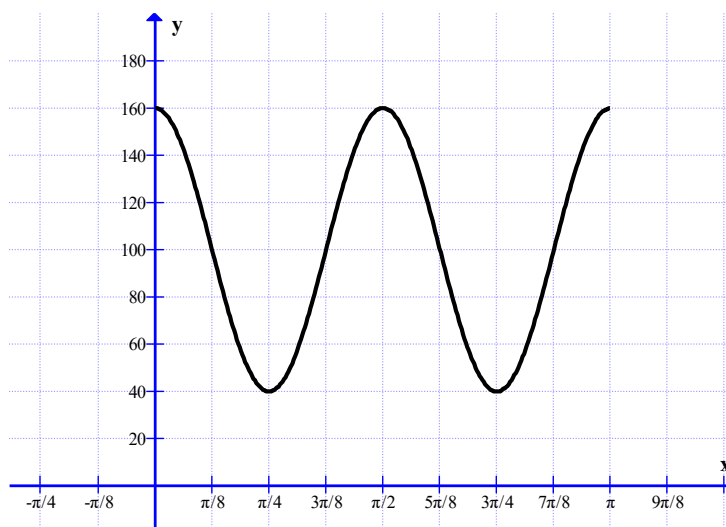
(ii) $f(x) = 80 \cos 4(x - \frac{\pi}{8}) + 100$, ($D = \frac{\pi}{8}$ is the position of the maximum)

(iii) $f(x) = -80 \cos 4(x - \frac{3\pi}{8}) + 100$ ($D = \frac{3\pi}{8}$ is the position of the minimum)

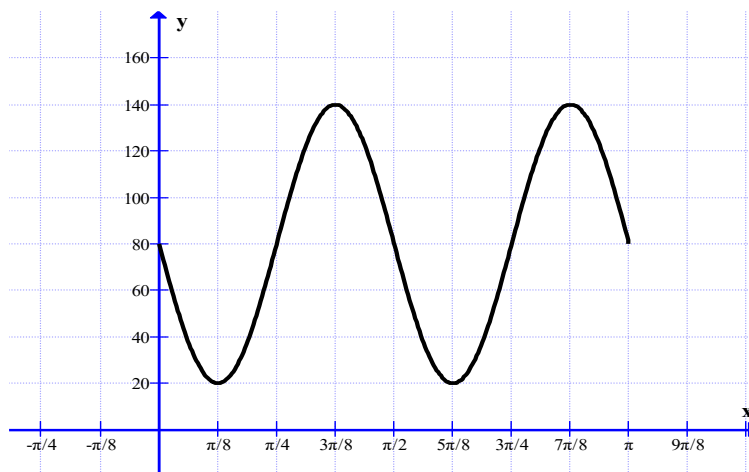
3. $f(x) = 60 \sin 4x + 100$, $0 \leq x \leq \pi$



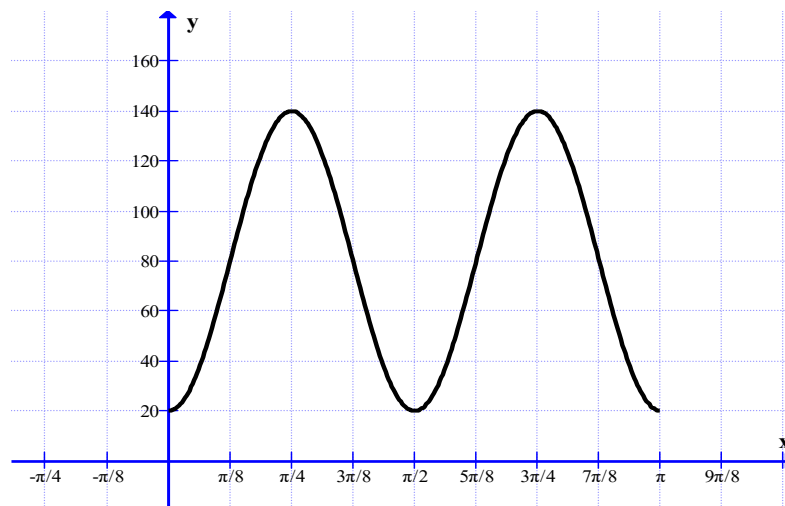
4. $f(x) = 60 \cos 4x + 100$, $0 \leq x \leq \pi$



5. $f(x) = -60 \sin 4x + 80$, $0 \leq x \leq \pi$



6. $f(x) = -60 \cos 4x + 80$, $0 \leq x \leq \pi$

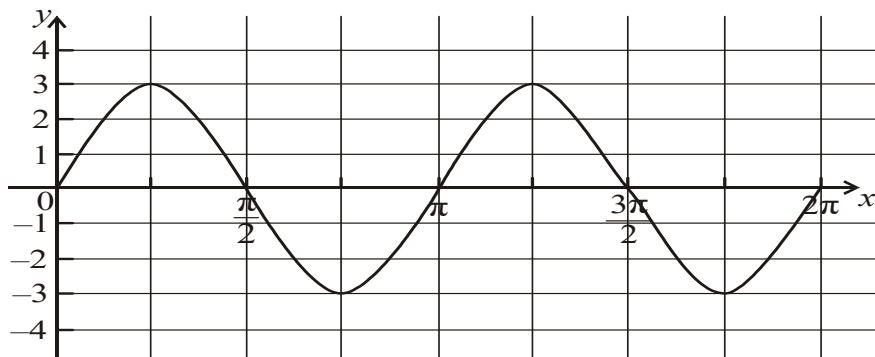


- (i) $k = 140$ (ii) $k = 20$ (iii) $20 < k < 140$ (iv) $k < 20$ or $k > 140$

7. From sketch of graph $y = 4\sin\left(3x + \frac{\pi}{2}\right)$
or by observing $|\sin \theta| \leq 1$. $k > 4, k < -4$

8. (a) (i) amplitude $= \frac{7+3}{2} = 5 \Rightarrow p = -5$
(ii) period $= 8 \Rightarrow q = 0.785 \left(= \frac{2\pi}{8} = \frac{\pi}{4} \right)$
(iii) $r = \frac{7-3}{2} \Rightarrow r = 2$
(b) $k = -3$ (accept $y = -3$)

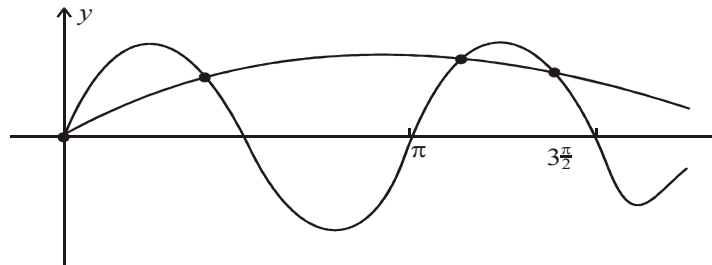
9. (a) period $= \pi$
(b)



- (c) 4 (solutions) (intersection points with line $y = 2$ on graph)

10. (a) $p = 30$
(b) Period $= \frac{2\pi}{q} = \frac{\pi}{2} \Rightarrow q = 4$

11. (a) (i) -1
(ii) 4π (accept 720°)
(b)



number of solutions: 4

12. $3 = p + q \cos 0 \Rightarrow 3 = p + q$
 $-1 = p + q \cos \pi \Rightarrow -1 = p - q$
(i) $p = 1$
(ii) $q = 2$

13. METHOD 1

The value of cosine varies between -1 and $+1$. Therefore:

$$t = 0 \Rightarrow a + b = 14.3$$

$$t = 6 \Rightarrow a - b = 10.3$$

$$\Rightarrow a = 12.3 \quad b = 2$$

$$\text{Period} = 12 \text{ hours} \Rightarrow \frac{2\pi(12)}{k} = 2\pi \Rightarrow k = 12$$

METHOD 2

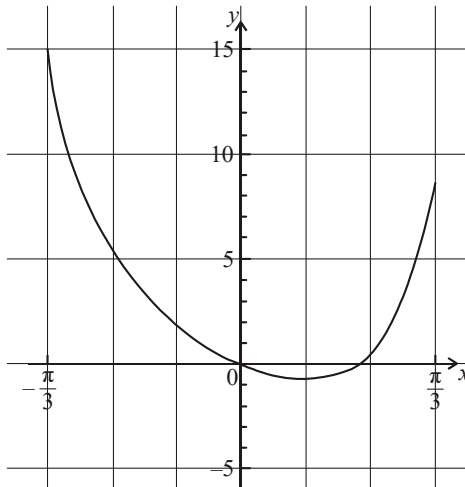
From graph: Midpoint = $a = 12.3$

Amplitude = $b = 2$

$$\text{Period} = \frac{2\pi}{\frac{2\pi}{k}} = 12 \Rightarrow k = 12$$

14. $a = 4, b = 2, c = \frac{\pi}{2} \left(\text{or } \frac{3\pi}{2} \text{ etc} \right)$

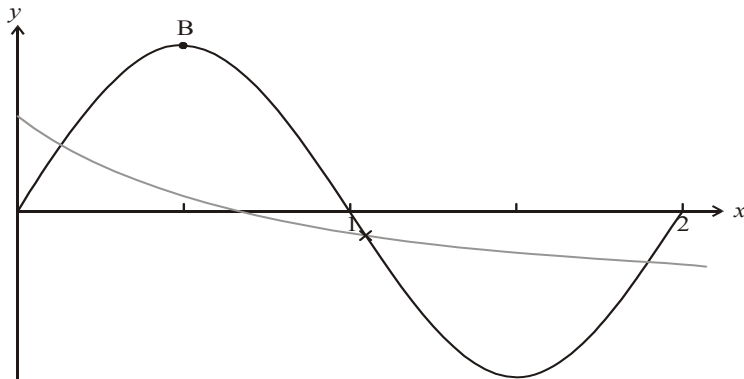
15. (a)



(b) $x = -0.207$ $x = 0.772$ *passing through (0, 0), range approximately -1 to 15.*

16. (a) $b = 6$

(b)

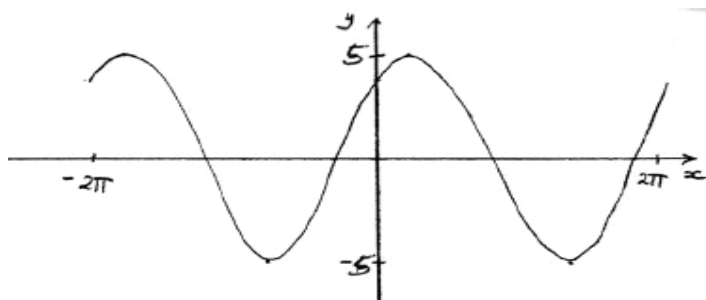


(c) $x = 1.05$ (accept (1.05, -0.896)) (no additional solutions)

B. Paper 2 questions (LONG)

17. (a) (i) $Q = \frac{1}{2}(14.6 - 8.2) = 3.2$
(ii) $P = \frac{1}{2}(14.6 + 8.2) = 11.4$
(b) $10 = 11.4 + 3.2\cos\left(\frac{\pi}{6}t\right)$
 $t = 3.8648$. $t = 3.86(3 \text{ s.f.})$
(c) (i) By symmetry, next time is $12 - 3.86... = 8.135... \Rightarrow t = 8.14 (3 \text{ s.f.})$
(ii) From above, first interval is $3.86 < t < 8.14$
This will happen again, 12 hours later, so $15.9 < t < 20.1$
18. (a) (i) 7
(ii) 1
(iii) 10
(b) (i) $A = \frac{18-2}{2} = 8$
(ii) $C = 10$
(iii) period = 12 $B = \frac{\pi}{6}$ (accept 0.524 or 30)
(c) $t = 3.52, t = 10.5$, between 03:31 and 10:29 (accept 10:30)
19. (a) $f(1) = 3$ $f(5) = 3$
(b) **EITHER** distance between successive maxima = period = $5 - 1 = 4$
OR period = $\frac{2\pi}{\frac{\pi}{2}} = 4$
(c) **EITHER** $A\sin\left(\frac{\pi}{2}\right) + B = 3$ and $A\sin\left(\frac{3\pi}{2}\right) + B = -1$
 $\Leftrightarrow A + B = 3, -A + B = -1$
 $\Leftrightarrow A = 2, B = 1$
OR Amplitude = $A = \frac{3 - (-1)}{2} = \frac{4}{2} = 2$
Midpoint value = $B = \frac{3 + (-1)}{2} = \frac{2}{2} = 1$
(d) $f(x) = 2 \Rightarrow 2\sin\left(\frac{\pi}{2}x\right) + 1 = 2 \Rightarrow x = \frac{1}{3} \text{ or } \frac{5}{3} \text{ or } \frac{13}{3}$
(e) (i) $k = -1$ (ii) $1 \leq k < 3$ (iii) $-1 < k < 1$ or $k = 3$ (iv) $k < -1$ or $k > 3$
20. (a) (i) $10 + 4\sin 1 = 13.4$
(ii) At 2100, $t = 21$
 $10 + 4\sin 10.5 = 6.48$
(b) (i) 14 metres
(ii) $14 = 10 + 4\sin\left(\frac{t}{2}\right) \Rightarrow t = \pi (=3.14)$
(c) (i) 4
(ii) $10 + 4\sin\left(\frac{t}{2}\right) = 7 \Rightarrow t = 7.98$
(iii) depth < 7 from $8 - 11 = 3$ hours, from 2030 – 2330 = 3 hours
therefore, total = 6 hours

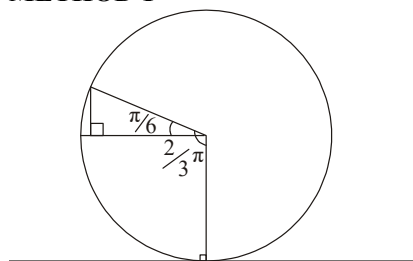
21. (a)



- (b) (i) 5 (ii) 2π (6.28) (iii) -0.927
 (c) $f(x) = 5 \sin(x + 0.927)$ (accept $p = 5, q = 1, r = 0.927$)
 (d) 3 s.f. values which round to $-5.6, 0.64$
 (e) $k = -5, k = 5$

22. (a) When $t = 1, l = 33 + 5 \cos 4\pi = 38$
 (b) (i) $l_{\min} = 33 - 5 = 28$ (ii) $l_{\max} = 33 + 5 = 38$
 (c) $33 = 33 + 5 \cos 4\pi t \Rightarrow t = 1/8$
 (d) period = $\frac{2\pi}{4\pi} = \frac{1}{2}$
 (e) (i) 10 times (ii) 20 times

23. (a) arc AB = $r\theta = 7.85$ (m)
 (b) Area of sector AOB $A = \frac{1}{2} r^2 \theta = 58.9$ (m²)
 (c) **METHOD 1**



$$\text{angle} = \frac{\pi}{6} (30^\circ)$$

$$\text{height} = 15 + 15 \sin \frac{\pi}{6} = 22.5 \text{ (m)}$$

- (d) (i) $h\left(\frac{\pi}{4}\right) = 15 - 15 \cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) = 25.6$ (m)
 (ii) $h(0) = 15 - 15 \cos\left(0 + \frac{\pi}{4}\right) = 4.39$ (m)

(iii) **METHOD 1**

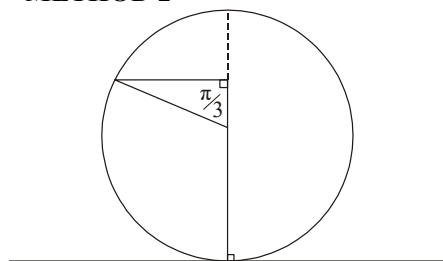
Highest point when $h = 30$

$$30 = 15 - 15 \cos\left(2t + \frac{\pi}{4}\right) \Leftrightarrow t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$$

METHOD 2

Using graph $t = 1.18$

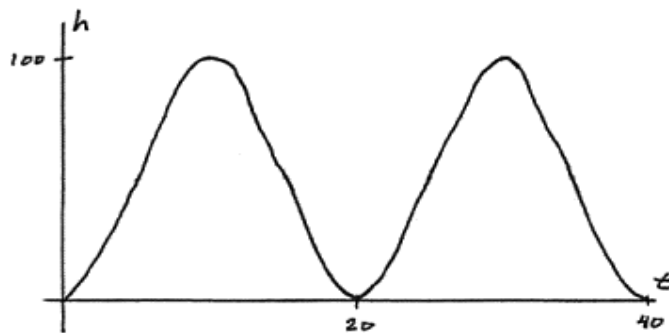
METHOD 2



$$\text{angle} = \frac{\pi}{3} (60^\circ)$$

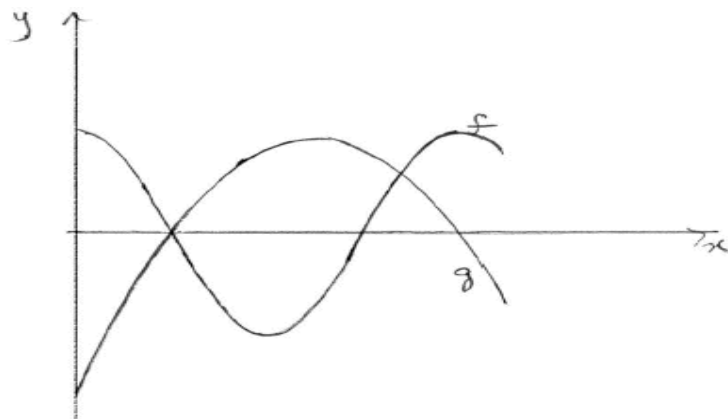
$$\text{height} = 15 + 15 \cos \frac{\pi}{3} = 22.5 \text{ (m)}$$

24. (a) (i) 100 (metres)
(ii) 50 (metres)
(b) (i) Symmetry with $h(2) = 9.5$
 $h(8) = 100 - 9.5 = 90.5$
(ii) $h(21) = h(1) = 2.4$
(c)



- (d) $b = \frac{2\pi}{20} \left(= \frac{\pi}{10} \right)$ (accept $b = 18$ if working in degrees)
 $a = -50, c = 50$

25. (a)



- (b) (i) $(2,0)$ (or $x = 2$)
(ii) period = 8
(iii) amplitude = 5
(c) (i) $(2, 0), (8, 0)$ (or $x = 2, x = 8$)
(ii) $x = 5$ (must be an equation)
(d) intersect when $x = 2$ and $x = 6.79$