

INTERNATIONAL BACCALAUREATE
Mathematics: applications and interpretation
MAI

EXERCISES [MAI 2.7-2.8]
COMPOSITION – INVERSE FUNCTIONS

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A. Paper 1 questions (SHORT)

1. [Maximum mark: 10]

Let $f(x) = 10 - 2x$ and $g(x) = 5x$. Calculate

- (a) $(f \circ g)(x)$ and $(g \circ f)(x)$ [2]
- (b) $(f \circ f)(x)$ and $(g \circ g)(x)$ [2]
- (c) $f^{-1}(x)$ [2]
- (d) $g^{-1}(10)$ [2]
- (e) $(f^{-1} \circ g)(x)$ and $(g \circ f)^{-1}(x)$ [2]

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3. [Maximum mark: 8]

The tables below show some values of two functions f and g

x	1	2	3	4
$f(x)$	2	-3	-1	3

x	1	2	3	4
$g(x)$	5	3	1	-3

- (a) Write down the values of $g(3)$, $f^{-1}(3)$ [2]
 (b) Calculate $(f \circ g)(2)$ [2]
 (c) Calculate $(g \circ g)(3)$ [2]
 (d) Find a solution of the equation $(g \circ f)(x) = 3$ [2]

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4. [Maximum mark: 6]

The functions f and g are both defined in the interval $[-4,4]$ and g is invertible.

Some values of the functions are given below.

x	1	3	4
$f(x)$	3	2	1

x	1	3	4
$g(x)$	4	1	-3

- (a) Calculate $(g^{-1} \circ f)(4)$ [2]
 (b) Find a solution of the equation $(f \circ g)(x) = 1$ [3]
 (c) Write down the value of $(g^{-1} \circ g)(2)$ [1]

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5. [Maximum mark: 6]

Let $f(x) = 2x + 1$ and $g(x) = 3x^2 - 4$. Find

- (a) $f^{-1}(x)$; [2]
- (b) $(g \circ f)(-2)$; [2]
- (c) $(f \circ g)(x)$. [2]

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6. [Maximum mark: 4]

The function f is defined by $f: x \mapsto \sqrt{3-2x}$, $x \leq \frac{3}{2}$

Evaluate $f^{-1}(5)$.

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7. [Maximum mark: 6]

Let $g(x) = 3x - 2$, $h(x) = \frac{5x}{x-4}$, $x \neq 4$.

- (a) Find an expression for $(h \circ g)(x)$. Simplify your answer. [3]
 (b) Solve the equation $(h \circ g)(x) = 0$. [3]

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8. [Maximum mark: 6]

The functions f and g are defined by $f: x \mapsto 3x$, $g: x \mapsto x + 2$.

- (a) Find an expression for $(f \circ g)(x)$. [2]
 (b) Show that $f^{-1}(18) + g^{-1}(18) = 22$. [4]

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9. [Maximum mark: 5]

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$. [2]

(b) Find $(f \circ g)(4)$. [3]

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10. [Maximum mark: 5]

Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$. [2]

(b) Write down $g^{-1}(x)$. [1]

(c) Find $(f \circ g^{-1})(5)$. [2]

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11. [Maximum mark: 6]

Let $f(x) = \sqrt{x+4}$ $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.

(a) Find $(g \circ f)(3)$ [2]

(b) Find $f^{-1}(x)$ [3]

(c) Write down the domain of f^{-1} . [1]

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12. [Maximum mark: 5]

Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

(a) Find $(g \circ f)(-2)$. [2]

(b) Find $f^{-1}(x)$. [3]

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13. [Maximum mark: 4]

Two functions f, g are defined as follows:

$$f: x \mapsto 3x + 5$$

$$g: x \mapsto 2(1 - x)$$

Find

(a) $f^{-1}(2)$; [2]

(b) $(g \circ f)(-4)$. [2]

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14. [Maximum mark: 6]

Let $f(x) = 2^x$, and $g(x) = \frac{x}{x-2}$, ($x \neq 2$). Find

(a) $(g \circ f)(3)$; [3]

(b) $g^{-1}(5)$. [3]

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15. [Maximum mark: 7]

(a) Find the inverse of the function $f(x) = \frac{x}{x+5}$. [5]

(b) Given that $S = \frac{R}{R+5}$, express R in terms of S . [2]

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16. [Maximum mark: 6]

Consider the functions $f(x) = 2x$ and $g(x) = \frac{1}{x-3}$, $x \neq 3$.

(a) Calculate $(f \circ g)(x)$. [2]

(b) Calculate $(f \circ g)(4)$. [2]

Let $P = 2R$, and $R = \frac{1}{S-3}$.

(c) Express P in terms of S . [1]

(d) Given that $S = 4$, find the value of P . [1]

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18. [Maximum mark: 6]

Consider the functions $f: x \mapsto 4(x-1)$ and $g: x \mapsto \frac{6-x}{2}$.

(a) Find g^{-1} . [2]

(b) Solve the equation $(f \circ g^{-1})(x) = 4$. [4]

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19. [Maximum mark: 4]

(a) Given functions $f: x \mapsto x+1$ and $g: x \mapsto x^3$, find the function $(f \circ g)^{-1}$.

(b) Given $A = B+1$ and $B = C^3$, describe what $(f \circ g)^{-1}$ represents.

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24. [Maximum mark: 8]

The function f is defined as $f(x) = \frac{3x-4}{x+2}$, $x \neq -2$

- (a) Find an expression for $f^{-1}(x)$. [5]

- (b) Write down the domain of f^{-1} . [1]

- (c) **Hence**, given that $F = \frac{2D+4}{3-D}$, express D in terms of F . [2]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings on the paper.

25. [Maximum mark: 10]

- $$g = f^{-1} \circ (f \circ g)$$

(c) Let $(f \circ g)(x) = 10x + 11$ and $g(x) = 5x + 3$. By using the fact

$$f = (f \circ g) \circ g^{-1}$$

[illegible]

26. [Maximum mark: 6]

- (a) Given that $f \circ g = h$, express f in terms of g and h . [2]
- (b) Given that $f \circ g = h$, express g in terms of f and h . [2]
- (c) Given that $f \circ g \circ h = k$, express g in terms of f , h and k . [2]

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27. [Maximum mark: 8]

The function f is defined by $f: x \mapsto x^3$.

- (a) Find $f^{-1}(x)$. [2]
- (b) Given that $(f \circ g)(x) = x + 1$, find $g(x)$. [3]
- (c) Given that $(g \circ f)(x) = x + 1$, find $g(x)$. [3]

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29. [Maximum mark: 6]

Let f and g be the functions. Given that $(f \circ g)(x) = \frac{x+1}{2}$ and $g(x) = 2x-1$,

find $f(x-3)$.

[illegible]

32. [Maximum mark: 16]

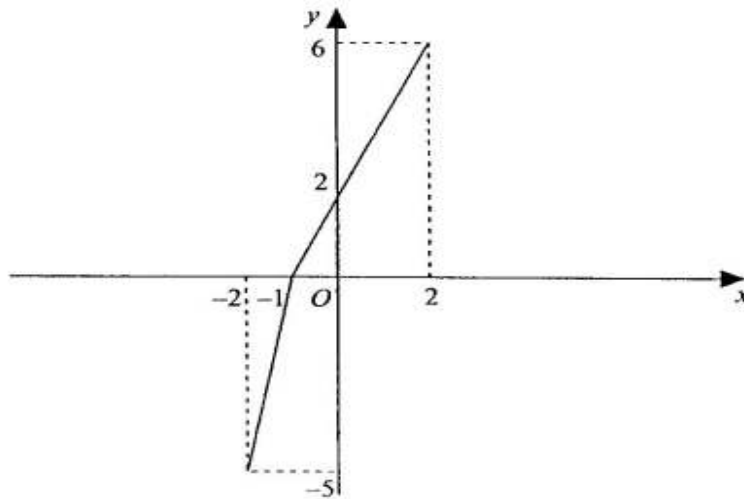
The function $f(x)$ is defined by $f(x) = \frac{3x-1}{x-3}$, $x \neq 3$.

(a) Show that f is a self-inverse function. [4]

(b) **Hence** find, in terms of k , the result of $(f \circ f)(k)$, where $k \neq 3$ [1]

(c) Given that $P = \frac{3Q-1}{Q-3}$, express Q in terms of P . [2]

The figure below shows a sketch of a one-to-one function $g(x)$ defined over the domain $-2 \leq x \leq 2$. The graph of $y = g(x)$ consists of two straight line segments and the range of $g(x)$ is $-5 \leq g(x) \leq 6$.



(d) Find the values of
 (i) $(f \circ g)(-2)$ (ii) $(g \circ f)(-5)$ (iii) $(g \circ g)(0)$. [6]

(e) On the same diagram above, sketch the graph of the inverse function $y = g^{-1}(x)$ and state its domain. [3]

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[illegible]

