

EXERCISES [MAI 2.3-2.4]

FUNCTIONS

SOLUTIONS

A. Paper 1 questions (SHORT)

FUNCTIONS AND GRAPHS – USE OF GDC

1. (a) $f(0) = 7$ and $f(2) = 13$
 (b) $f(a) = 3a + 7$
 (c) $a = 1$
 (d) $b = 5$
 (e)

x	0	1	2	4	5
$f(x)$	7	10	13	19	22

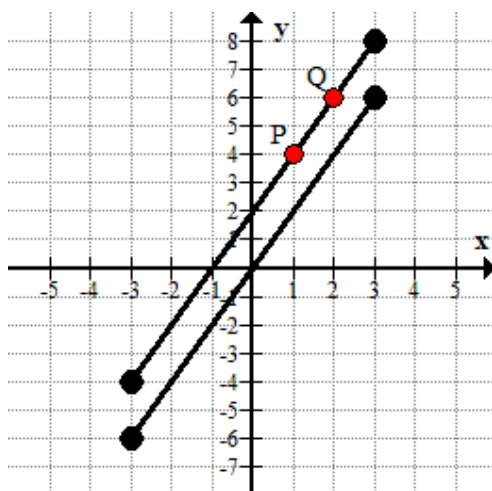
2. (a) $x \in [2, 6]$
 (b) $y \in [3, 5]$
 (c) (i) $f(2) = 3$ (ii) $f(4) = 4$
 (d) $a = 6$
 (e) minimum = 3, maximum = 5

3. (a) $x \in [2, 11]$
 (b) $y \in [2, 6]$
 (c) minimum = 2, maximum = 6
 (d)

x	2	4	6	9	11
$f(x)$	3	4	5	2	6

- (e) 4, 7, 10

4. (a)

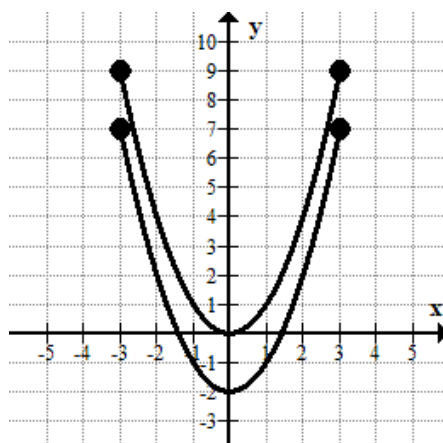


- (b) In the domain $-3 \leq x \leq 3$

Function	$f(x) = 2x$	$f(x) = 2x + 2$
Range	$-6 \leq y \leq 6$	$-4 \leq y \leq 8$

- (c) $a=1$, $b=6$

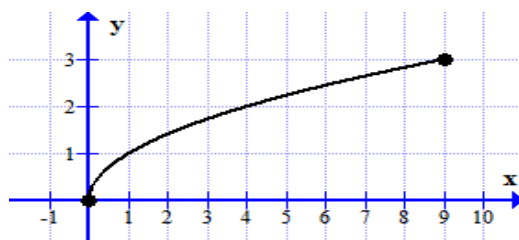
5. (a)



(b), (c), (d) In the domain $-3 \leq x \leq 3$ (for the intersection points you may use the GDC)

Function	Range	Points of intersection with $y = x$	Points of intersection with $y = 1$
$y = x^2$	$0 \leq y \leq 9$	$(0,0), (1,1)$	$(-1,1), (1,1)$
$y = x^2 - 2$	$-2 \leq y \leq 7$	$(-1, -1), (2,2)$	$(-\sqrt{3}, 1), (\sqrt{3}, 1)$ or $(-1.73, 1), (1.73, 1)$

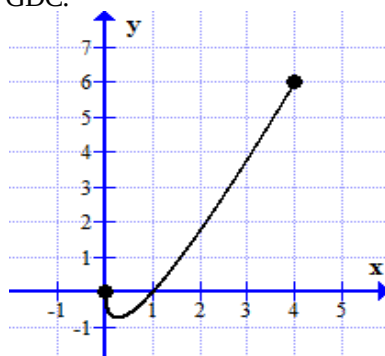
6. (a) Look at the graph in the GDC. Indicate the endpoints $(0,0)$ and $(9,3)$



(b) In the domain $0 \leq x \leq 9$ (for the intersection points you may use the GDC)

Range of f	$0 \leq y \leq 3$
Point of intersection with the line $y=1$	$(1,1)$
Points of intersection with the line $y=2$	$(4,2)$

7. (a) Look at the graph in the GDC.



Indicate the endpoints $(0,0)$ and $(4,6)$, the x -intercepts 0 and 1, and the minimum point.

(b)

Coordinates of end points	$(0,0)$ and $(4,6)$
Coordinates of minimum point	$(0.25, -0.75)$
Range	$[-0.75, 6]$

8. (b) For $f(x) = x^3$ $D_f: x \in \mathbb{R}$ $R_f: y \in \mathbb{R}$
 For $f(x) = x^4$ $D_f: x \in \mathbb{R}$ $R_f: y \in [0, +\infty)$

(c)

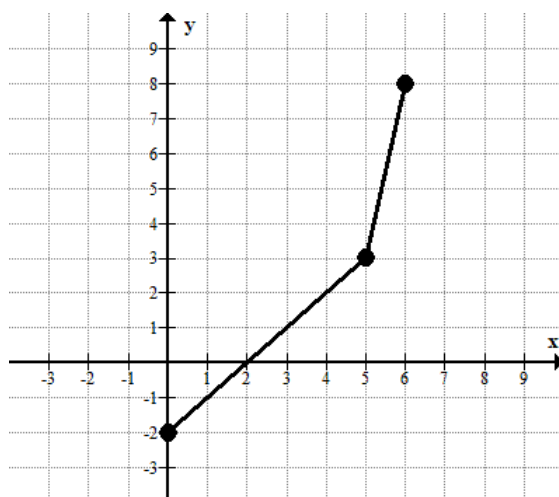
For $f(x) = x^3$	$D_f: 1 \leq x \leq 2$	$R_f: 1 \leq y \leq 8$
	$D_f: -1 \leq x \leq 2$	$R_f: -1 \leq y \leq 8$

For $f(x) = x^4$	$D_f: 1 \leq x \leq 2$	$R_f: 1 \leq y \leq 16$
	$D_f: -1 \leq x \leq 2$	$R_f: 0 \leq y \leq 16$

9. (a)

x	0	1	2	3	4	5	6
$f(x)$	-2	-1	0	1	2	3	8

(b)



(c) Domain: $x \in [0, 6]$, Range: $y \in [-2, 8]$

(d) y-intercept = -2, x-intercept = 2

10. $D_f: x \in \mathbb{R}$, $D_g: x \neq 1/2$, $D_h: x \leq 1/2$, $D_k: x < 1/2$,
 11. $D_f: x \in \mathbb{R}$, $D_g: x \neq \pm 2$, $D_h: x \leq -2$ or $x \geq 2$ $D_k: x < -2$ or $x > 2$
 12. $D_f: x \in [4, 5) \cup (5, 10)$ or $D_f: 4 \leq x < 5$ or $5 < x < 10$
 13. Domain: $x \geq -7$ Range: $y \geq 5$

14.

$f(x) = 2x^2 - 8x + 9, x \in \mathbb{R}$	$y \geq 1$
$f(x) = 2x^2 - 8x + 9, 0 \leq x \leq 2$	$1 \leq y \leq 9$
$f(x) = 2x^2 - 8x + 9, 0 \leq x \leq 5$	$1 \leq y \leq 19$
$f(x) = 2x^2 - 8x + 9, x \leq 0$	$y \geq 9$

15.

$f(x) = x^3 - 9x^2 + 15x + 29, x \in \mathbb{R}$	$y \in \mathbb{R}$
$f(x) = x^3 - 9x^2 + 15x + 29, x \geq 0$	$y \in [4, -\infty[$
$f(x) = x^3 - 9x^2 + 15x + 29, x \leq 0$	$y \in]-\infty, 29]$
$f(x) = x^3 - 9x^2 + 15x + 29, 0 \leq x \leq 6$	$y \in [4, 36]$
$f(x) = x^3 - 9x^2 + 15x + 29, 0 \leq x \leq 7$	$y \in [4, 36]$
$f(x) = x^3 - 9x^2 + 15x + 29, 0 \leq x \leq 8$	$y \in [4, 85]$

16. (a)

x	$f(x)$
0	5
5	10
10	15
12	17
20	25

x	$f^{-1}(x)$
5	0
10	5
15	10
17	12
25	20

(b) $f^{-1}(x) = x - 5$

(c) $Q = P - 5$

17. (a) $Q = \frac{P-5}{2}$

(b) $f^{-1}(x) = \frac{x-5}{2}$

18.

Original function	Inverse function
$f(x) = x + 5$	$f^{-1}(x) = x - 5$
$f(x) = x - 5$	$f^{-1}(x) = x + 5$
$f(x) = x + 100$	$f^{-1}(x) = x - 100$
$f(x) = 3x$	$f^{-1}(x) = x / 3$
$f(x) = x / 5$	$f^{-1}(x) = 5x$
$f(x) = x^3$	$f^{-1}(x) = \sqrt[3]{x}$
$f(x) = 3x + 100$	$f^{-1}(x) = \frac{x-100}{3}$

19. (a) (i) $f(1) = 3$ (ii) $f^{-1}(1) = 5$

(b) $x = 6$

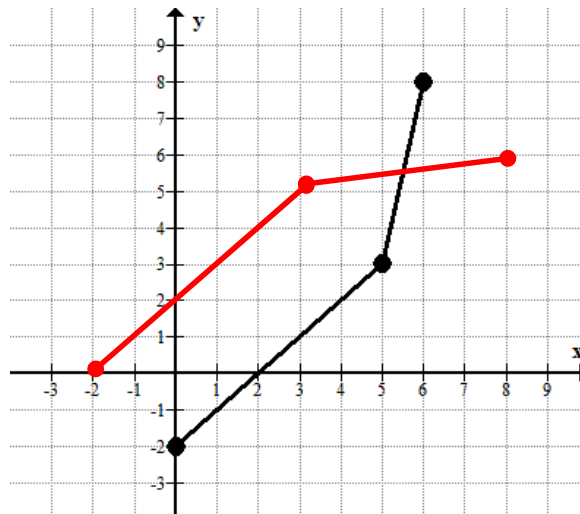
(c) $x = 4$

20. (a) (i) $f(0) = -2$ (ii) $f(2) = 0$ (iii) $f(4) = 2$

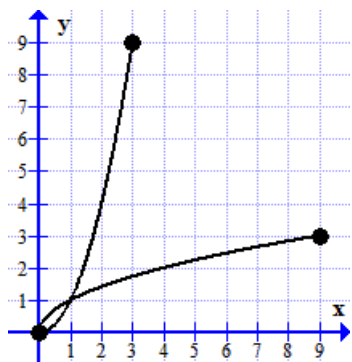
(b) (i) $f^{-1}(3) = 5$ (ii) $f^{-1}(8) = 6$ (iii) $f^{-1}(-1) = 1$ (iv) $f^{-1}(0) = 2$

(c) $x = 2$

(d) $x = 4$



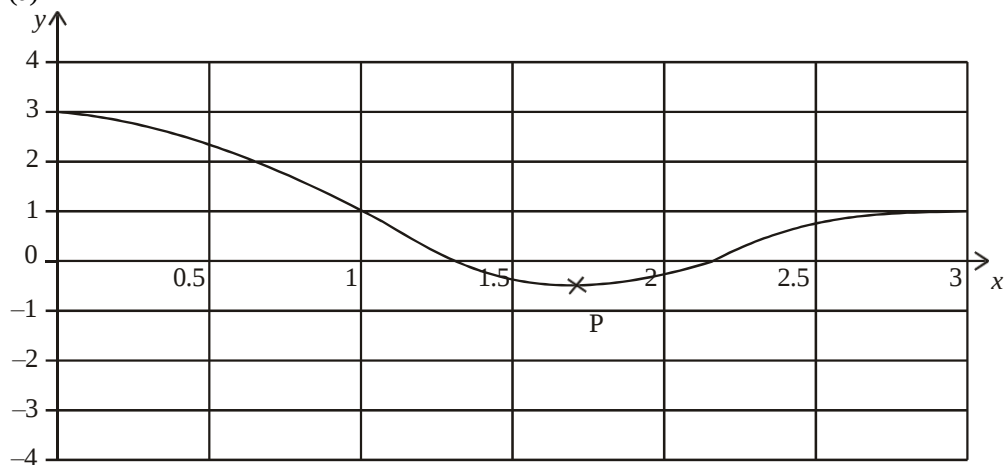
21. (a) parabola between $x=0$ and $x=3$
 (b) $0 \leq x \leq 3$ $0 \leq y \leq 9$
 (c) $y = \sqrt{x}$
 (d)



graph symmetric about the line $y = x$

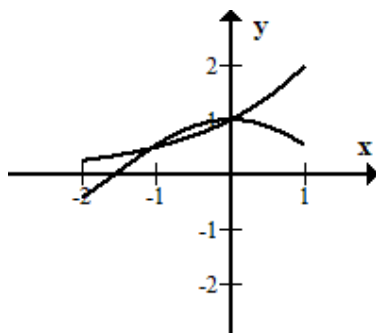
- (e) $0 \leq x \leq 9$ $0 \leq y \leq 3$

22. (a)

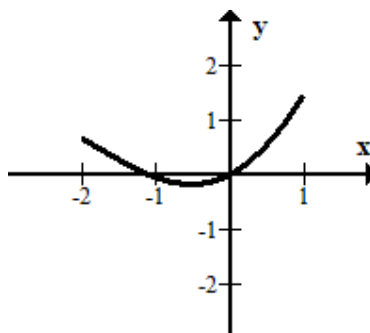


- (b) 1.26, 2.26
 (c) $-0.472 \leq y < 3$ (since the minimum point is (1.74, -0.472))

23. (a)

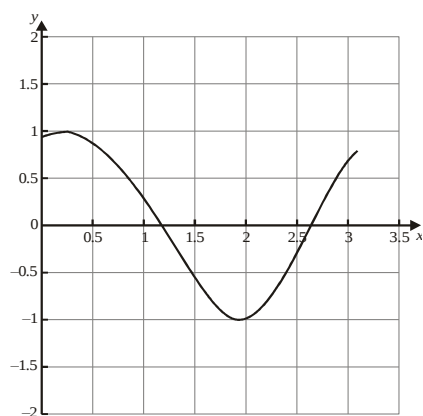


- (c)



- (b) $x = -1.29$
 (d) $x = -1.29$
 (e) $-1.29 < x < 0$

24. (a)

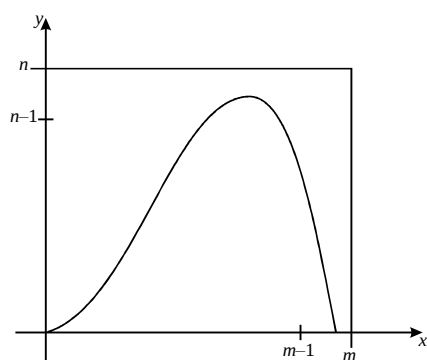


- (b) (Maximum) $x = 0.285\dots$ $x = 0.3$ (1 d.p.)
 (Minimum) $x = 1.856\dots$ $x = 1.9$ (1 d.p.)

25. (a) Maximum/minimum points at: 0.6075, 1.571, 2.534, 4.712

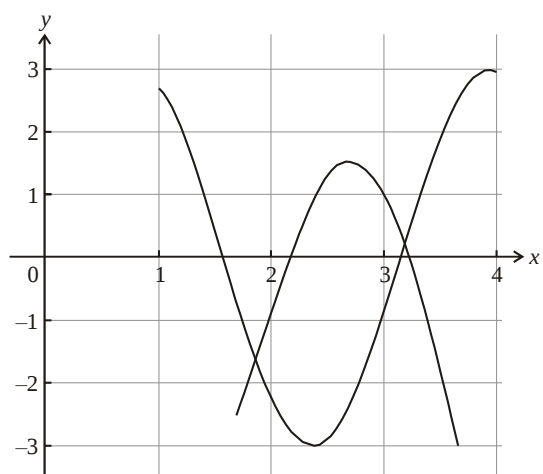
(c) $[0, 1]$

26.



- (a) From a graphic display calculator
 $y = 0 \Rightarrow x = 9.43$ (or x between 9 and 10) $\Rightarrow m = 10$
 (b) $y_{\max} = 5.46$ (or between 5 and 6)
 $\Rightarrow n = 6$

27. (a)



- (b) 3.19
 (c) $p = 1.89, q = 3.19$