INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.18]

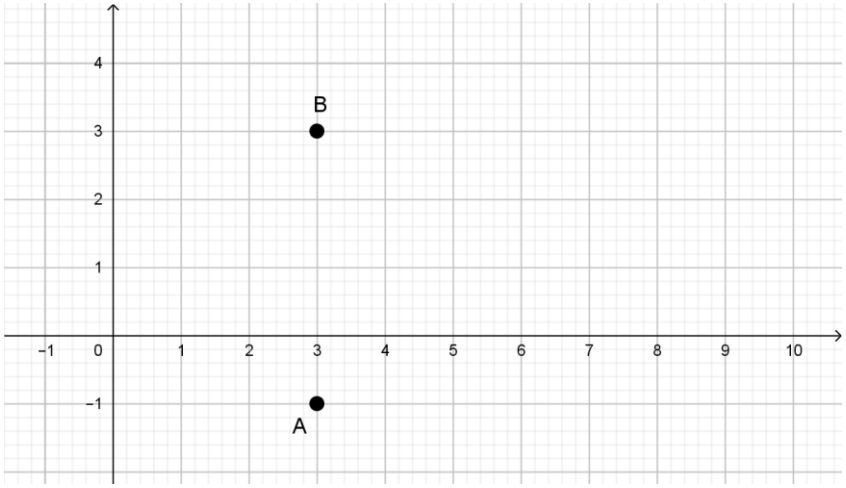
VORONOI DIAGRAM

A. Paper 1 questions (SHORT)

1. [Maximum mark: 5]

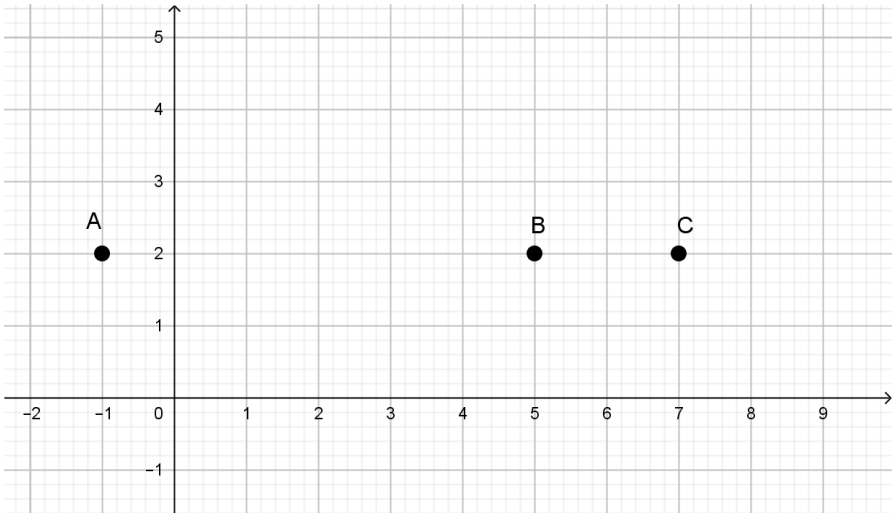
The following regions contain two and three sites respectively. Construct the Voronoi diagrams by writing down on each edge the equation of the corresponding line.

(a)



[2]

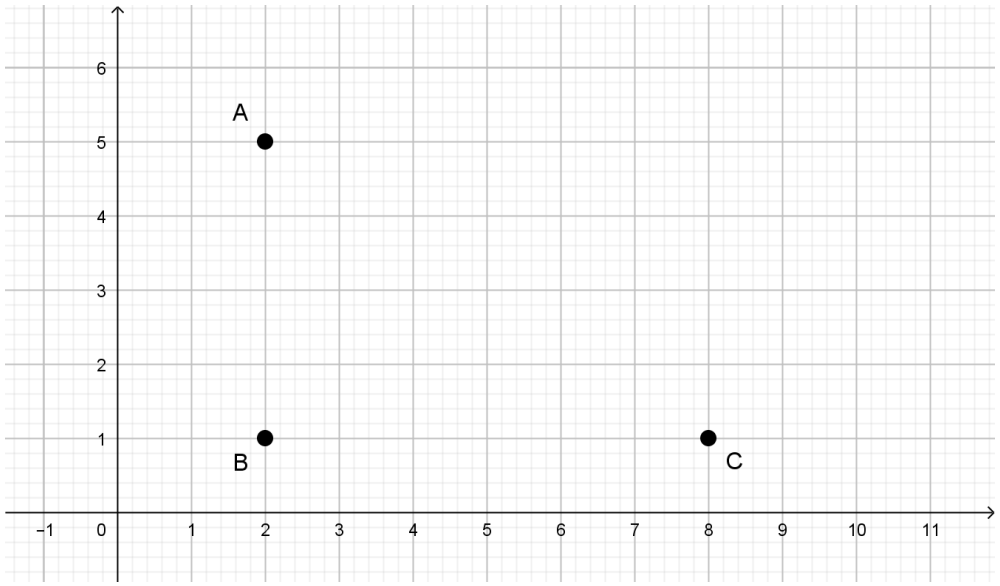
(b)



[3]

2. [Maximum mark: 9]

The following region contains three sites, A(2, 5), B(2, 1) and C(8,1).



(a) Write down the equation of the perpendicular bisector

(i) of the line segment [AB], (ii) of the line segment [BC]. [2]

(b) Find the equation of the perpendicular bisector of the line segment [AC]. [4]

(c) Hence construct the Voronoi diagram for the region above; indicate the coordinates of the vertex. [3]

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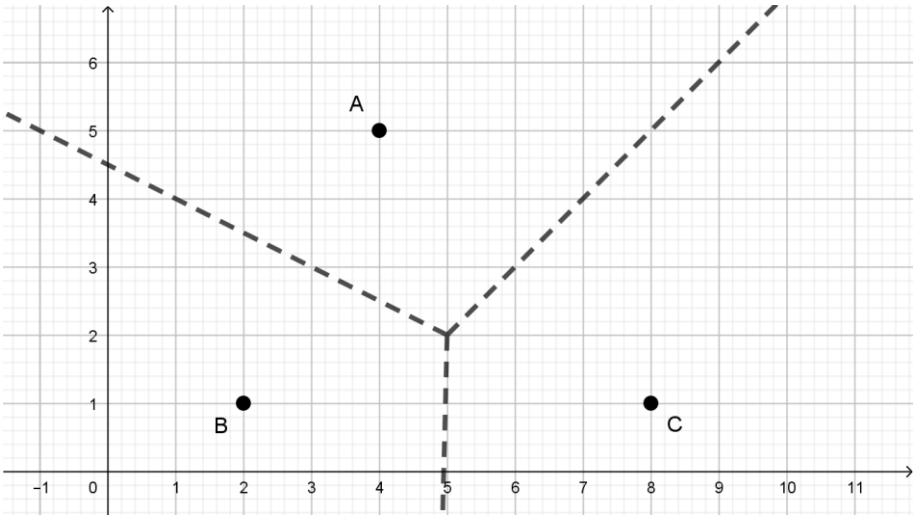
3. [Maximum mark: 8]

The Voronoi diagram for three cells, A(4, 5), B(2, 1) and C(8, 1) is shown below.



(a) Write down the equation of the edge between B and C. [1]

(b) Find the equation of the edge between A and B in the form $ax + by + d = 0$, where $a, b, d \in Z$. [4]

(c) Write down the coordinates of the only one vertex. [1]

(d) Given that the edge between A and C has equation $y = x + c$, find the value of c. [2]

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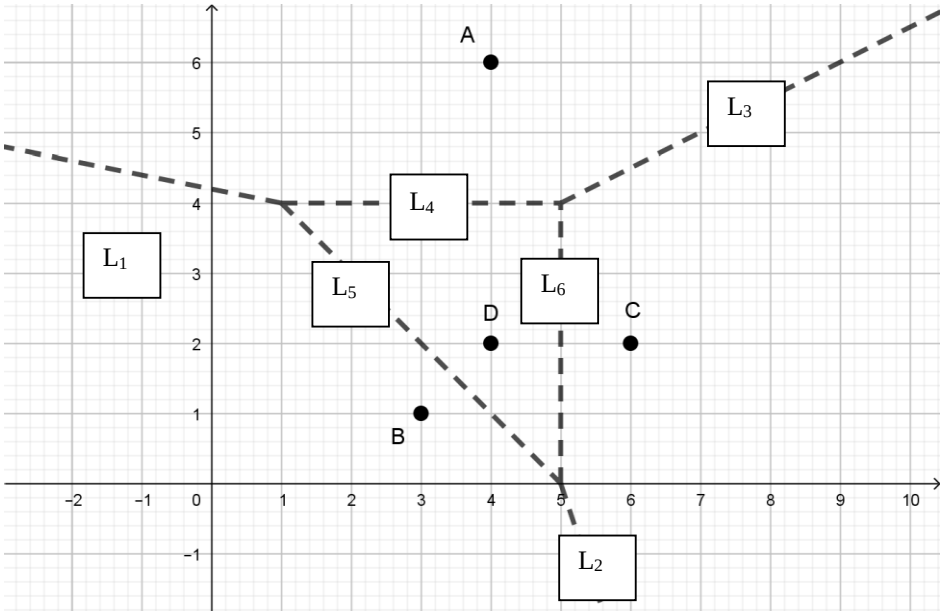
4. [Maximum mark: 8]

The Voronoi diagram for four sites, A(4, 6), B(3, 1), C(6, 2) and D(4, 2), is shown below.



(a) Find the equation of the edge L_3 between cells A and C in the form $y = mx + c$ [4]

(b) Complete the table of the edges below

Line	Equation
L_3	
L_1	$y = -0.2x + 4.2$
	$y = -x + 5$
	$y = -3x + 15$
	` $y = 4$
L_6	

[4]

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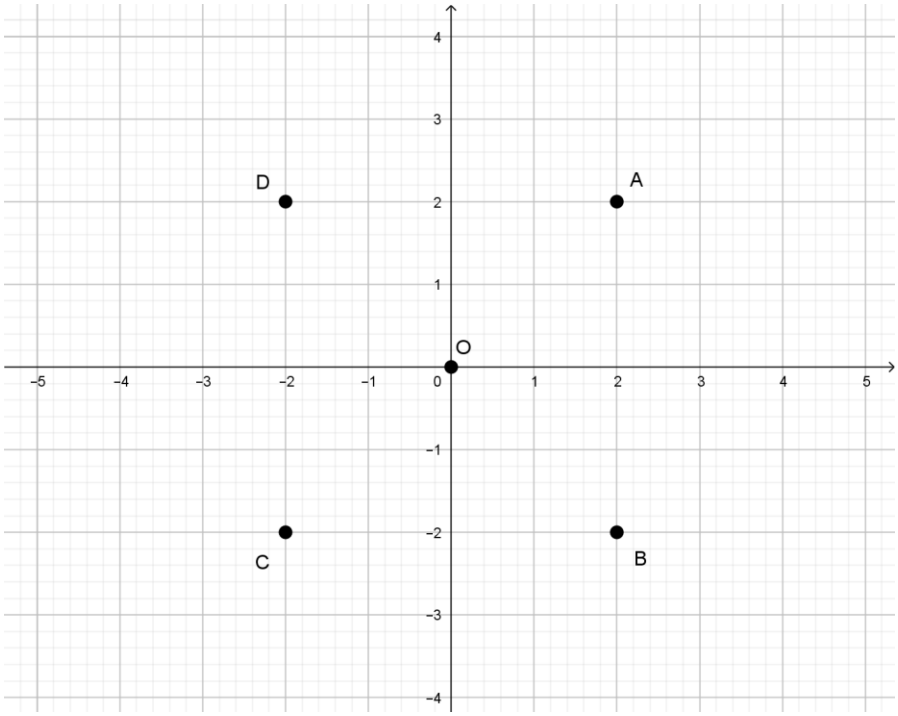
5. [Maximum mark: 8]

The diagram shows five sites are represented by the points O(0,0), A(2, 2), B(2, –2),C(–2, –2) and D(–2, 2).



(a) On the same diagram, complete the Voronoi diagram for the five sites. [3]

The border of cell O contains four edges.

(b) Write down the equations of the four edges. [3]

(c) Find the area of the cell O. [2]

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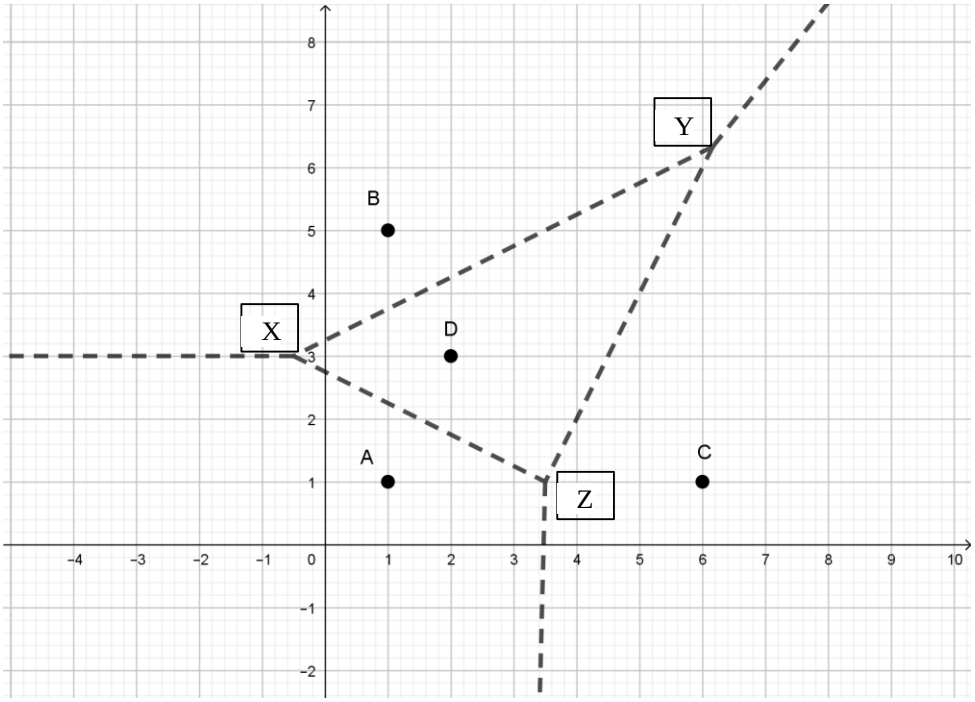
B. Paper 2 questions (LONG)

6. [Maximum mark: 16]

The Voronoi diagram for four sites, A(1, 1), B(1, 5), C(6, 1) and D(2, 3), is shown below.



The vertex Z is the midpoint of the line segment [AC].

(a) Write down the coordinates of the vertex Z. [1]

The equation of the edge [XY] is $-2x + 4y = 13$.

(b) Find the equation of the edge [ZY] in the form $ax + by = d$, where $a, b, d \in Z$. [4]

(c) **Hence**, find the coordinates of the vertex Y. [2]

(d) Find the **exact** value of the length ZY. [2]

Consider now the cell XYZ. The length XY is $\frac{10\sqrt{5}}{3}$.

(e) Show that the edges [XZ] and [ZY] are perpendicular. [3]

(f) Let $\theta = X\hat{Y}Z$.

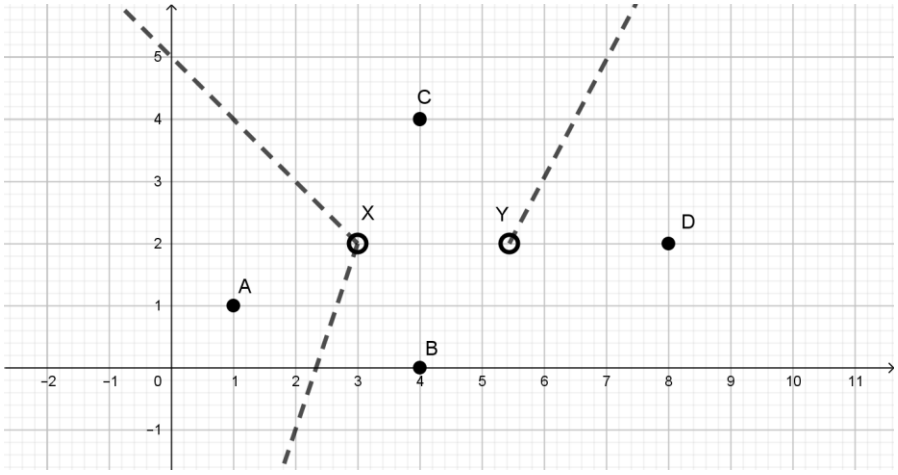
(i) Show that $\cos\theta = \frac{4}{5}$

(ii) Explain why $\sin\theta = \frac{3}{5}$ [4]

(g) Find the exact value of the area of the triangle XYZ. [2]

7. [Maximum mark: 14]

Part of the Voronoi diagram for four sites, A(1, 1), B(4, 0), C(4, 4) and D(8, 2) is shown below. The two vertices of the Voronoi diagram are shown by the points X and Y.



(a) Two edges are missing.

(i) Draw the two edges. (ii) Find their equations. [6]

(b) Write down the coordinates of the two vertices X and Y. [2]

(c) Find the exact value of the distance between the sites A and D. [2]

(d) Find the area of the quadrilateral ABCD. [4]

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