

**EXERCISES [MAI 2.17]**  
**SINUSOIDAL MODEL**  
**SOLUTIONS**

**A. Paper 1 questions (SHORT)**

1.

| Function                                                               | Amplitude | Period | Range                |
|------------------------------------------------------------------------|-----------|--------|----------------------|
| $f(x) = \sin x^\circ$<br>$f(x) = \cos x^\circ$                         | 1         | 360    | $-1 \leq y \leq 1$   |
| $f(x) = \sin(10x)^\circ$<br>$f(x) = \cos(10x)^\circ$                   | 1         | 36     | $-1 \leq y \leq 1$   |
| $f(x) = \sin(36x)^\circ$<br>$f(x) = \cos(36x)^\circ$                   | 1         | 10     | $-1 \leq y \leq 1$   |
| $f(x) = \sin(180x)^\circ$<br>$f(x) = \cos(180x)^\circ$                 | 1         | 2      | $-1 \leq y \leq 1$   |
| $f(x) = 2 \sin(10x)^\circ$<br>$f(x) = 2 \cos(10x)^\circ$               | 2         | 36     | $-2 \leq y \leq 2$   |
| $f(x) = \sin(10x)^\circ + 50$<br>$f(x) = \cos(10x)^\circ + 50$         | 1         | 36     | $49 \leq y \leq 51$  |
| $f(x) = 2 \sin(10x)^\circ + 50$<br>$f(x) = 2 \cos(10x)^\circ + 50$     | 2         | 36     | $48 \leq y \leq 52$  |
| $f(x) = -2 \sin(10x)^\circ$<br>$f(x) = -2 \cos(10x)^\circ$             | 2         | 36     | $-2 \leq y \leq 2$   |
| $f(x) = -2 \sin(10x)^\circ + 50$<br>$f(x) = -2 \cos(10x)^\circ + 50$   | 2         | 36     | $48 \leq y \leq 52$  |
| $f(x) = 100 - 20 \sin(10x)^\circ$<br>$f(x) = 100 - 20 \cos(10x)^\circ$ | 20        | 36     | $80 \leq y \leq 120$ |

2. (a) central axis  $y = 100$ , amplitude = 80, period = 2

(b)  $B = \frac{360}{\text{Period}} = \frac{360}{2} = 180$ , It is of type +sin

$$f(x) = 60 \sin(180x)^\circ + 100,$$

(c) When  $x = 1.5$ ,  $y = 40$ . When  $x = 1.6$ ,  $y = 42.9$ .

(d) 0.232

3. (a) central axis  $y = 50$ , amplitude = 30, period = 180

(b)  $B = \frac{360}{\text{Period}} = \frac{360}{180} = 2$ , It is of type  $-\sin$

$$f(x) = -30 \sin(2x)^\circ + 50,$$

(c)  $x = 20.9$ ,  $x = 60.1$

(d)  $20.9 \leq x \leq 60.1$

4. (a) central axis  $y = 50$ , amplitude = 30, period = 10

(b)  $B = \frac{360}{\text{Period}} = \frac{360}{10} = 36$ , It is of type  $+\cos$

$$f(x) = 30 \cos(36x)^\circ + 50,$$

(c)  $x = 1.34$ ,  $x = 8.66$ ,  $x = 11.3$ ,  $x = 18.7$

(d)  $0 \leq x \leq 1.34$  or  $8.66 \leq x \leq 11.3$  or  $18.7 \leq x \leq 20$ .

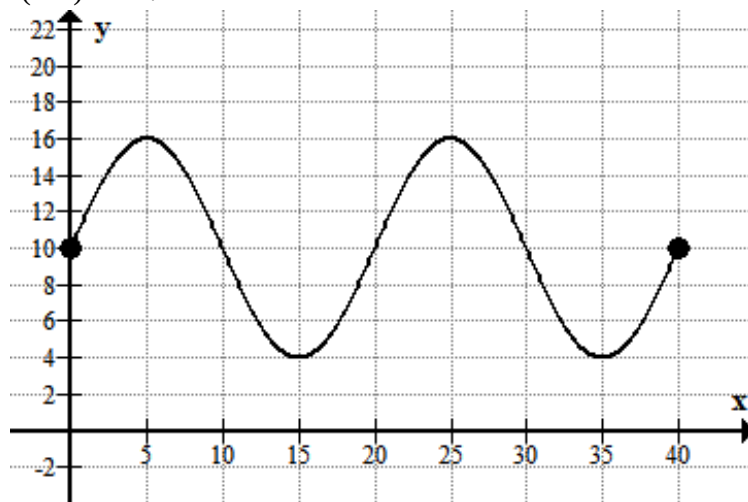
5. (a) central axis  $y = 13$ , amplitude = 5, period = 20

(b)  $B = \frac{360}{\text{Period}} = \frac{360}{20} = 18$ , It is of type  $-\cos$

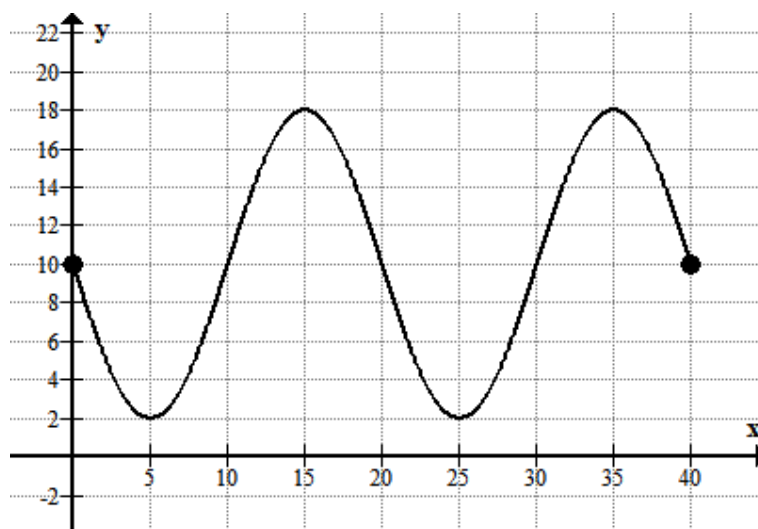
$$f(x) = -5 \cos(18x)^\circ + 13.$$

(c)  $x = 0$ ,  $x = 20$ ,  $x = 40$ .

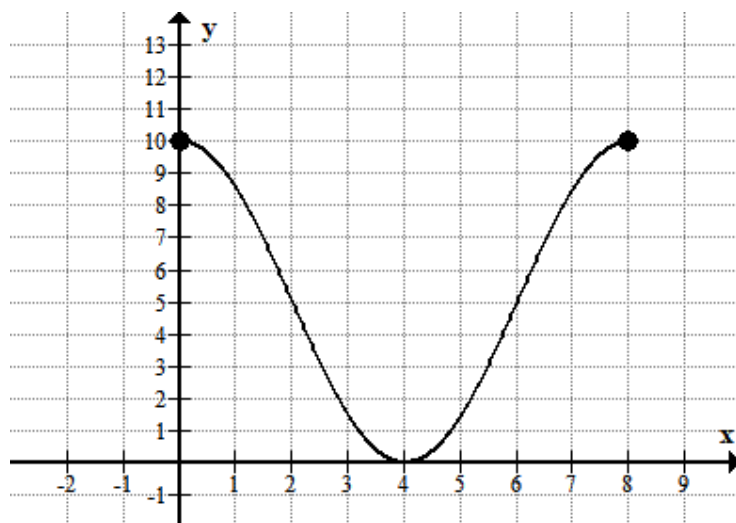
6.  $f(x) = 6 \sin(18x)^\circ + 10$ ,  $0 \leq x \leq 40$



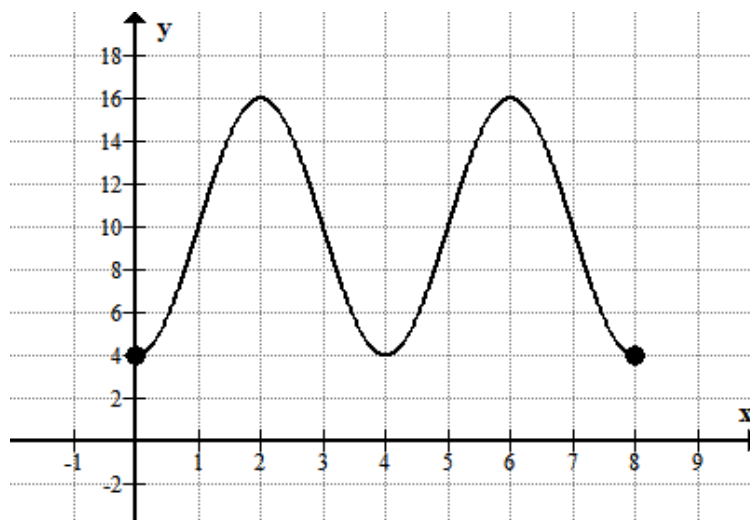
7.  $f(x) = -8 \sin(18x)^\circ + 10$ ,  $0 \leq x \leq 40$



8.  $f(x) = 5 \cos(45x)^\circ + 5, 0 \leq x \leq 8$



9.  $f(x) = -6 \cos(90x)^\circ + 10, 0 \leq x \leq 8$

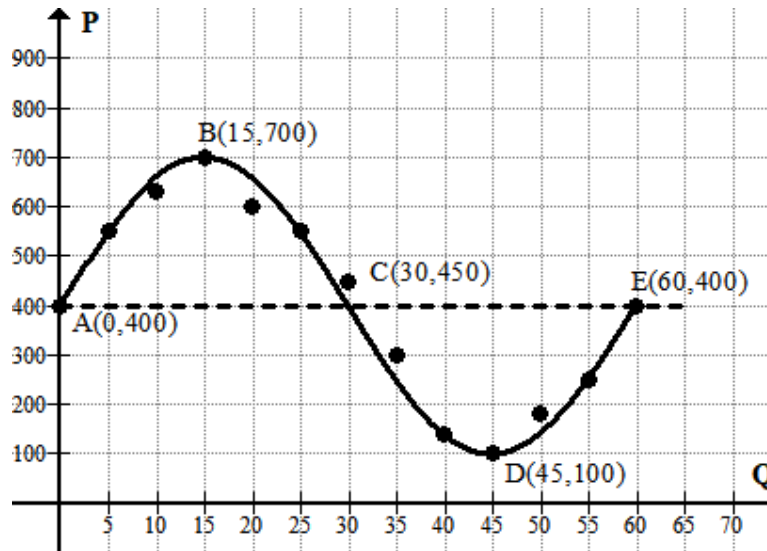


## B. Paper 2 questions (LONG)

10. (a) (i)  $P = 400$   
 (ii) amplitude = 300  
 (iii) Period =  $\frac{360}{6} = 60$   
 (b)  $B = \frac{360}{\text{Period}} = \frac{360}{60} = 6$ , It is of type  $+\sin$

$$P = 300 \cos(6Q)^\circ + 400.$$

(c)



- (d) When  $Q = 30$ ,  $P_A = 300 \sin(6 \times 30)^\circ + 400 = 400$ ,  $P_E = 450$

$$\text{Percentage error} = \left| \frac{P_A - P_E}{P_E} \right| \times 100\% = \left| \frac{400 - 450}{450} \right| \times 100\% = 11.1\%$$

- (e) (i) When  $Q = 90$ ,

(ii) Extrapolation: We are not certain if the data continue periodically.

11. (a) When  $t = 1$ ,  $l = 33 + 5 \cos 720 = 38$   
 (b) (i)  $l_{\min} = 33 - 5 = 28$  (ii)  $l_{\max} = 33 + 5 = 38$   
 (c)  $33 = 33 + 5 \cos 720t \Rightarrow t = 1/8$   
 (d) period =  $\frac{360}{720} \left( \frac{1}{2} \right) = \frac{1}{2}$   
 (e) (i) 10 times (ii) 20 times