

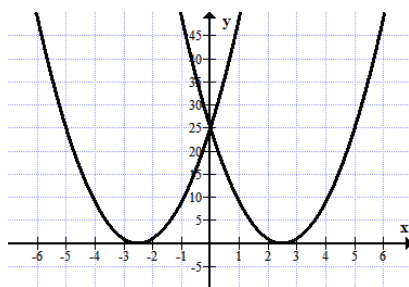
EXERCISES [MAI 2.13-2.14]

POLYNOMIAL MODELS

SOLUTIONS

A. Paper 1 questions (SHORT)

1. (a) (i) $u_n = 13 + (n-1) \times 5 = 5n + 8$ (ii) $y = 5x + 8$
 (b) the gradient of the line is the common difference of the sequence.
 (c) $5 \times (-1) + 8 = 3$, $5 \times 10 + 8 = 58$, $5 \times (0.2) + 8 = 9$: All three points lie on the line
 (d) Although all the points lie on the line, only 58 is a term of the sequence (it is the only one for which x is a positive integer).
2. (a) $m_{AD} = \frac{24}{8} = 3$
 $P - 2 = 3(Q - 2) \Rightarrow P = 3Q - 4$
 (b) $P = aQ + b$. For A(2,2): $2a + b = 2$
 For D(10,26): $10a + b = 26$
 Hence: $a = 3$, $b = -4$, So $P = 3Q - 4$
 (c) For B(4,8): $3 \times 4 - 4 = 8$, so it lies on the line.
 For C(6,14): $3 \times 6 - 4 = 14$, so it lies on the line.
3. (a) $y = 2x^2 + 3x + 7$
 (b) $y = 2(x-1)(x-5) \Rightarrow y = 2(x^2 - 6x + 5) \Rightarrow y = 2x^2 - 12x + 10$
 (c) $y = 2(x-3)^2 + 5 \Rightarrow y = 2(x^2 - 6x + 9) + 5 \Rightarrow y = 2x^2 - 12x + 23$
 (d) $a(0-3)^2 + 2 = 20 \Rightarrow 9a + 2 = 20 \Rightarrow 9a = 18 \Rightarrow a = 2$
 Hence, $y = 2(x-3)^2 + 2 \Rightarrow y = 2x^2 - 12x + 20$
4. (a) $q = -2$, $r = 4$ or $q = 4$, $r = -2$
 (b) $x = 1$
 (c) substituting $(0, -4)$ into the equation: $-4 = -8p \Rightarrow p = \frac{4}{8} = \frac{1}{2}$
 (d) $f(x) = \frac{1}{2}(x+2)(x-4) = \frac{1}{2}(x^2 - 2x - 8) = \frac{1}{2}x^2 - x - 4$
5. (a) **METHOD 1:** Discriminant $= 0 \Rightarrow q^2 - 4(4)(25) = 0 \Rightarrow q^2 = 400 \Rightarrow q = 20$, $q = -20$
METHOD 2: Using factorizing: $(2x-5)^2$ or $(2x+5)^2 \Rightarrow q = 20$, $q = -20$
 (b) $x = 2.5$
 (c) $(0, 25)$
 (d)



6. (a) Since the vertex is at (3, 1)
 $h = 3, k = 1$
 (b) $x = 3$
 (c) (5, 9) is on the graph $\Rightarrow 9 = a(5 - 3)^2 + 1 \Rightarrow 9 = 4a + 1 \Rightarrow a = 2$
 (d) $y = 2(x - 3)^2 + 1 = 2(x^2 - 6x + 9) + 1 = 2x^2 - 12x + 19$
7. (a) $h = 3, k = 2$
 (b) $y \leq 2$
 (c) due to symmetry: $f(4) = a$, hence $x = 4$.
 (d) $f(x) = -(x - 3)^2 + 2 = -x^2 + 6x - 9 + 2 = -x^2 + 6x - 7$
8. (a) (i) $h = -1$ (ii) $k = 2$
 (b) $a(1 + 1)^2 + 2 = 0 \Rightarrow a = -0.5$
 (c) $y = -0.5(x + 3)(x - 1)$
 (d) $y = -0.5(x + 3)(x - 1) = -0.5(x^2 + 2x - 3) = -0.5x^2 - x + 1.5$
 OR $y = -0.5(x + 1)^2 + 2 = -0.5(x^2 + 2x + 1) + 2 = -0.5x^2 - x + 1.5$
9. (a) (i) $p = 1, q = 5$ (or $p = 5, q = 1$)
 (ii) $x = 3$
 (b) For $x = 3, y = -4$, hence $f(x) = (x - 3)^2 - 4$ ($h = 3, k = -4$)
 (c) $f(x) = (x - 1)(x - 5) = x^2 - 6x + 5$
 (d) $f(x) < 0 \Rightarrow 1 < x < 5$
10. (a) (i) $m = 3$ (ii) $p = 2$
 (b) $0 = d(1 - 3)^2 + 2 \Rightarrow d = -\frac{1}{2}$
 (c) $f(x) = -\frac{1}{2}(x - 3)^2 + 2 = -\frac{1}{2}(x^2 - 6x + 9) + 2 = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$
 (d) $f(x) = -\frac{1}{2}(x - 1)(x - 5)$
11. (a) $p = -2, q = 4$ (or $p = 4, q = -2$)
 (b) $y = a(x + 2)(x - 4)$
 $8 = a(6 + 2)(6 - 4)$
 $8 = 16a$
 $a = \frac{1}{2}$
 (c) $y = \frac{1}{2}(x + 2)(x - 4) = \frac{1}{2}(x^2 - 2x - 8) = \frac{1}{2}x^2 - x - 4$
 (d) (i) $x < -2$ or $x > 4$. (ii) $x = 5$
12. (a) $f(x) = a(x + 4)(x - 6)$
 For $x = 0, y = 240: -24a = 240 \Rightarrow a = -10$
 $f(x) = -10(x + 4)(x - 6)$
 (b) Vertex at $x = 1, y = -10(1 + 4)(1 - 6) = 250$
 $f(x) = -10(x - 1)^2 + 250$
 (c) $y = -10(x - 1)(x - 1) + 250 = 240 + 20x - 10x^2$
 OR $y = -10(x^2 - 2x + 1) + 250 = 240 + 20x - 10x^2$

13. (a) substituting $(-4, 3)$: $3 = a(-4)^2 + b(-4) + c \Rightarrow 16a - 4b + c = 3$
 (b) $36a + 6b + c = 3$, $4a - 2b + c = -1$
 (c) $a = 0.25$, $b = -0.5$, $c = -3$ (accept fractions)
 $f(x) = 0.25x^2 - 0.5x - 3$
 (d) Vertex (min with GDC) at $(1, -3.25)$
 $f(x) = 0.25(x - 1)^2 - 3.25$
14. (a) $a + b + c = 4$
 $4a + 2b + c = 7$
 $9a + 3b + c = 14$
 (b) $P = 2Q^2 - 3Q + 5$
15. (a) $4a - 2b + c = 0$
 $a + b + c = 0$
 $4a + 2b + c = 12$
 Hence, $P = 3Q^2 + 3Q - 6$
 (b) $P = a(Q + 2)(Q - 1)$
 Since $(2, 12)$ lies on the line: $a(2 + 2)(2 - 1) = 12 \Rightarrow 4a = 12 \Rightarrow a = 3$
 Hence, $P = 3(Q + 2)(Q - 1) = 3(Q^2 + Q - 2) = 3Q^2 + 3Q - 6$
16. (a) (i) Due to symmetry the vertex is $V(3, 10)$, (ii) P -intercept $(0, 1)$
 (a) $0a + 0b + c = 1$
 $9a + 3b + c = 10$
 $36a + 6b + c = 1$
 Hence, $P = -Q^2 + 6Q + 1$
 (b) $P = a(Q - 3)^2 + 10$
 Since $(0, 1)$ lies on the line: $9a + 10 = 1 \Rightarrow a = -1$
 Hence, $P = -(Q - 3)^2 + 10 = -(Q^2 - 6Q + 9) + 10 = -Q^2 + 6Q + 1$
17. (a) $-8 = -8p + 4q - 2r$,
 $-2 = p + q + r$,
 $0 = 8p + 4q + 2r$
 (b) $p = 1$, $q = -1$, $r = -2$
 (c) $f(x) = x^3 - x^2 - 2x$,
 Roots: $x = -1$, $x = 0$, $x = 2$
 (d) $-1 \leq x \leq 0$ or $x \geq 2$.

18. (a)

x	y	Model L_1		Model L_2	
		$y_1 = 3x$	$(y - y_1)^2$	$y_2 = 3.4x - 1$	$(y - y_2)^2$
1	3	3	0	2.4	0.36
2	5	6	1	5.8	0.64
3	9	9	0	9.2	0.04
4	13	12	1	12.6	0.16
		SUM→	2	SUM→	1.2

- (b) Although L_1 passes through two of the points, the model L_2 is closer to the points,
 (c) The regression line is $y = 3.4x - 1$, that is the line L_2 .

B. Paper 2 questions (LONG)

19. (a) (i) $m_{AB} = \frac{6}{2} = 3$ (ii) $y - 5 = 3(x - 1) \Rightarrow y = 3x + 2$

(b) The point C(6,20) also lies on the line since

$$3 \times 6 + 2 = 20$$

Hence the three points lie on the same line.

(c) The point D(5,15) does not lie on the line since

$$3 \times 5 + 2 = 17 \neq 15$$

(d) It is a satisfactory model since it passes through 3 out of the four points, and it is close enough to the fourth point.

(e) $y = 2.83x + 2.14$

(f)

x	y	Estimations		Squared Residuals	
		y_1 by line AB)	y_2 by line L	$(y - y_1)^2$	$(y - y_2)^2$
1	5	5	4.97	0	0.0009
3	11	11	10.63	0	0.1369
5	15	17	16.29	4	0.5041
6	20	20	19.12	0	0.7744
SS _{res} = SUM OF THE SQUARED RESIDUALS →				4	1.4163

(g) Although the first model passes through 3 out of the four points, the second model is much better. It is totally closer to the points since, since SS_{res} is smaller.

20. (a) (i) $P = 3.80Q + 11.6$

(ii) $P = 0.839Q^2 - 2.08Q + 17.5$

(iii) $P = -0.376Q^3 + 4.79Q^2 - 12.4Q + 21.4$

(b) exact value = 20,

Linear model value = 11.6, Percentage error = $\left| \frac{11.6 - 20}{20} \right| \times 100\% = 42\%$

Quadratic model value = 17.5, Percentage error = $\left| \frac{17.5 - 20}{20} \right| \times 100\% = 12.5\%$

Cubic model value = 21.4, Percentage error = $\left| \frac{21.4 - 20}{20} \right| \times 100\% = 7\%$

(c) The cubic model seems to be much closer to the given points.

(d) For $Q = 8$, the cubic model gives $P = 36.2$. This seems to be the most reliable prediction. (The first two models give 42 and 54.6 respectively)