*INTERNATIONAL BACCALAUREATE*

***Mathematics: applications and interpretation***

# *MAI*

***EXERCISES [MAI 2.13-2.14]***

***POLYNOMIAL MODELS***

## A. Paper 1 questions (SHORT)

**1.** [Maximum mark: 9]

Consider the arithmetic sequence 13, 18, 23, 28, 33, 38, ...

The terms of the sequence lie on the line L.

(a) Find

*(i)* the $n$-th term $u_n$ of the sequence in terms of $n$

(ii) the equation of the line L in the form $y = mx + c$. [3]

(b) Write down a comment about the gradient $m$ of the line L in the context of the arithmetic sequence. [1]

(c) Determine whether the points A(-1, 3), B(10, 58), C(0.2, 9) lie on the line. [3]

(d) Write down a comment about the $y$-coordinates of A, B, C in the context of the arithmetic sequence. [2]

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**2.** [Maximum mark: 8]

Consider the point series

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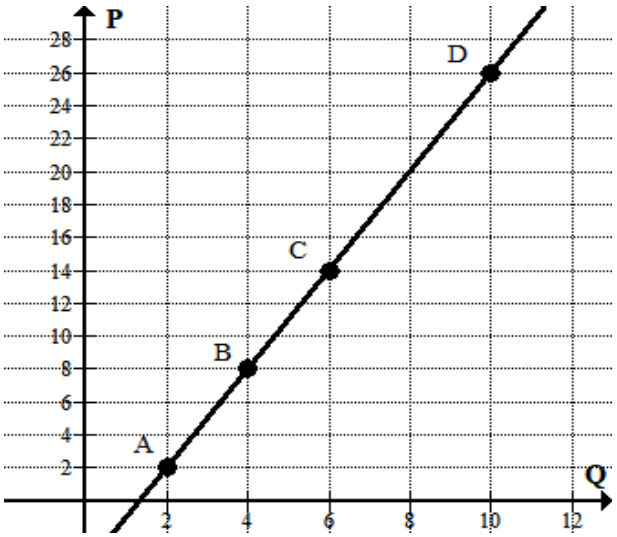
|---|---|---|---|---|
| $Q$ | 2 | 4 | 6 | 10 |
| $P$ | 2 | 8 | 14 | 26 |

The following diagram shows that the four points of the series line on a line L.




Select the points A(2,2) and D(10,26) to find the linear equation of $P$ in terms of $Q$, by using two different methods.

(a) Find the gradient of the line segment [AD} and hence find the equation of L. [3]

(b) Given that the points A and D lie on the line $P = aQ + b$, write down two linear equations in $a$ and $b$ and **hence** find the equation of L. [3]

(c) Verify the two remaining points lie on L. [2]

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**3.** [Maximum mark: 8]

Express each of the following quadratics in the form $y = ax^2 + bx + c$ based on the information given.

(a) $y = 2x^2 + 3x + c$ with $y$-intercept (0, 7). [1]

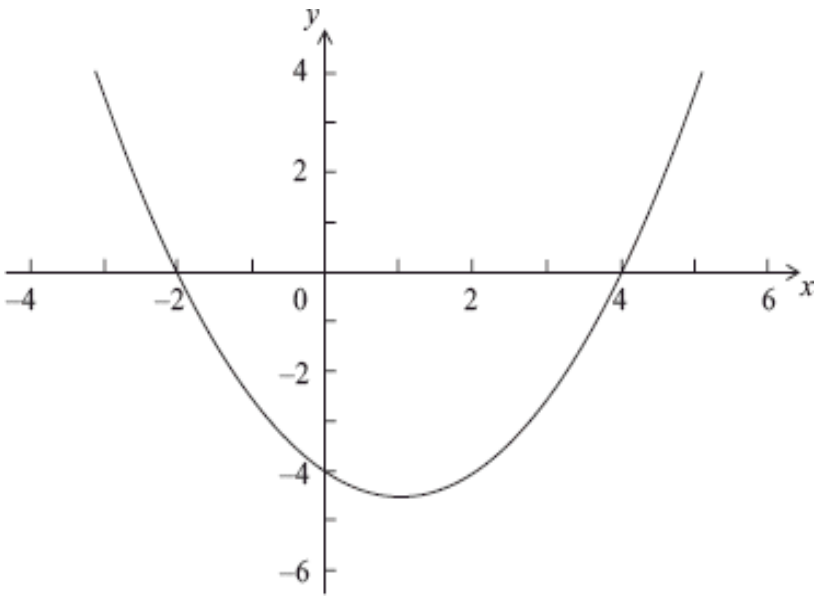
(b) $y = 2(x - p)(x - q)$ with $x$-intercepts (1, 0) and (5, 0). [2]

(c) $y = 2(x - h)^2 + k$ with vertex (3, 5). [2]

(d) $y = a(x - 3)^2 + 2$ with $y$-intercept (0, 20). [3]

**4.** [Maximum mark: 8]

Let $f(x) = p(x-q)(x-r)$. Part of the graph of $f$ is shown below.




The graph passes through the points (-2, 0), (0, -4) and (4, 0).

(a) Write down the value of $q$ and of $r$. [2]

(b) Write down the equation of the axis of symmetry. [1]

(c) Find the value of $p$. [3]

(d) Express the function in the form $f(x) = ax^2 + bx + c$. [2]

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**5.** [Maximum mark: 8]

Consider two different quadratic functions of the form $f(x) = 4x^2 - qx + 25$. The graph of each function has its vertex on the $x$-axis.

(a) Find both values of $q$. [3]

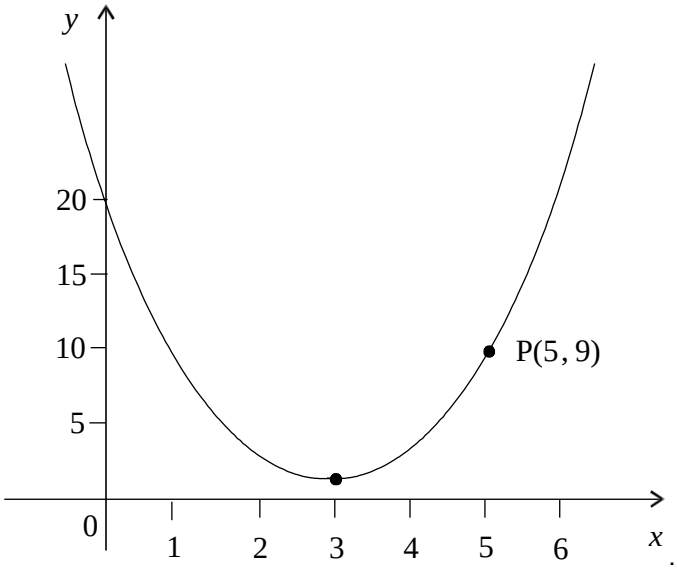
(b) For the greater value of $q$, solve $f(x) = 0$. [1]

(c) Find the coordinates of the point of intersection of the two graphs. [2]

(d) Sketch the two graphs on the same diagram. [2]

**6.** [Maximum mark: 7]

The diagram shows part of the graph of the curve $y = a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$.




(a) The vertex is at the point $(3, 1)$. Write down the value of $h$ and of $k$. [2]

(b) Write down the equation of the axis of symmetry. [1]

(b) The point $P(5, 9)$ is on the graph. Show that $a = 2$. [3]

(c) Hence show that the equation of the curve can be written as $y = 2x^2 - 12x + 19$. [1]

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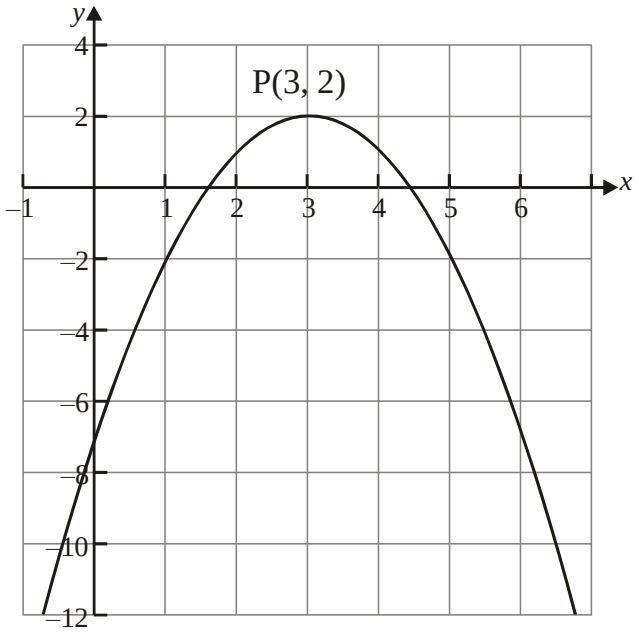
**7.** [Maximum mark: 6]

The function $f(x)$ is defined as $f(x) = -(x-h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is $\mathrm{P}(3, 2)$.




(a) Write down the value of (i) $h$ (ii) $k$. [2]

(b) Write down the possible values of $f(x)$. [2]

(c) Given that $f(2) = a$, write down the other value of $x$ such that $f(x) = a$. [1]

(d) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$. [1]

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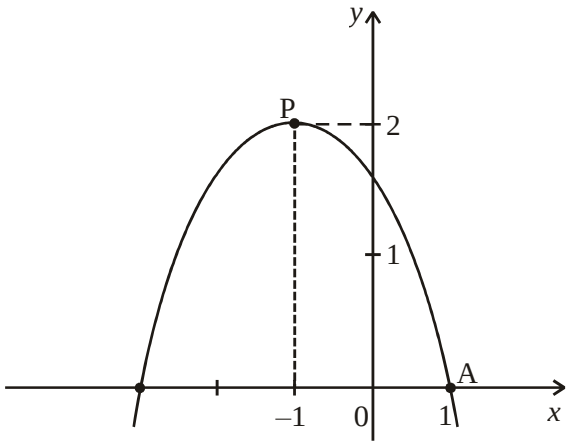
**8.** [Maximum mark: 8]

The diagram shows part of the graph of $y = a\,(x - h)^2 + k$. The graph has its vertex at $\mathrm{P}$, and passes through the point $\mathrm{A}$ with coordinates $(1, 0)$.




(a) Write down the value of (i) $h$ (ii) $k$ [2]

(b) Calculate the value of $a$. [3]

(c) Express the quadratic function in the forms

(i) $y = a\,(x - p)\,(x - q)$

(ii) $y = ax^2 + bx + c$. [3]

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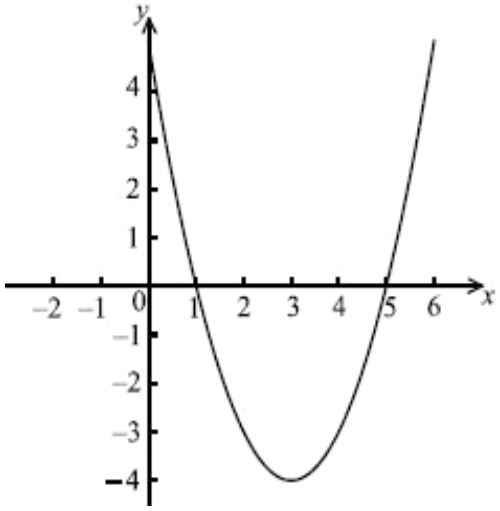
**9.** [Maximum mark: 8]

The following diagram shows part of the graph of a quadratic function, with equation in the form $f(x) = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.




(a) Write down

*(i)* the value of $p$ and of $q$

(ii) the equation of the axis of symmetry of the curve. [2]

(b) Find the equation of the function in the form $f(x) = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$. [2]

(c) Express the quadratic function in the form $f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}$. [2]

(d) Solve the inequality $f(x) < 0$. [2]

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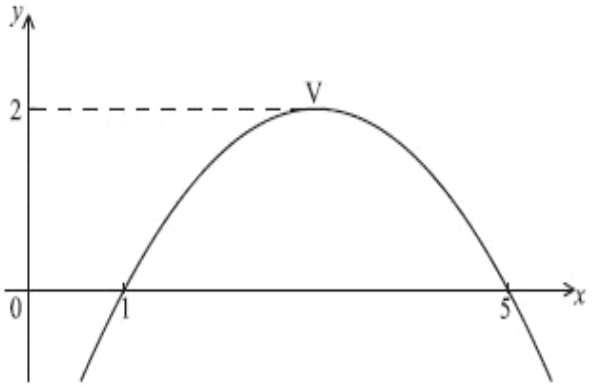
**10.** [Maximum mark: 7]

Part of the graph of the function $f(x) = d(x-m)^2 + p$ is given in the diagram below.

The $x$-intercepts are $(1, 0)$ and $(5, 0)$. The vertex is $V(m, 2)$.




(a) Write down the value of (i) $m$ (ii) $p$ [2]

(b) Find $d$. [2]

(c) Express the quadratic function in the form $f(x) = ax^2 + bx + c$. [2]

(d) Factorize $f(x)$. [1]

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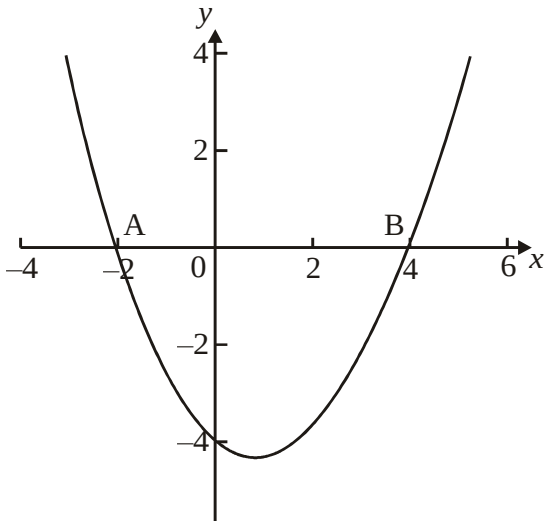
**11.** [Maximum mark: 8]

The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the $x$-axis at $A(-2, 0)$ and $B(4, 0)$. The curve of $y = f(x)$ is shown in the diagram below.




(a) Write down the value of $p$ and of $q$. [2]

(b) Given that the point $(6, 8)$ is on the curve, find the value of $a$. [2]

(c) Write the equation of the curve in the form $y = ax^2 + bx + c$. [1]

(d) (i) Solve the inequality $y > 0$.

(ii) Given that $x$ is a positive integer, write down the smallest value of $x$ for which $y > 0$. [3]

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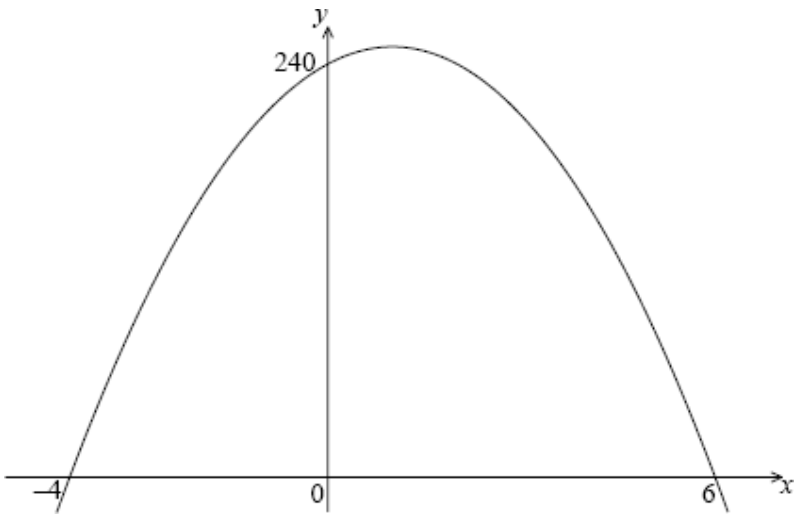
**12.** [Maximum mark: 8]

The following diagram shows part of the graph of a quadratic function $f$.




The $x$-intercepts are at $(-4, 0)$ and $(6, 0)$ and the $y$-intercept is at $(0, 240)$.

(a) Write down $f(x)$ in the form $f(x) = a(x - p)(x - q)$. [3]

(b) Find another expression for $f(x)$ in the form $f(x) = a(x - h)^2 + k$. [4]

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [1]

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**13.** [Maximum mark: 8]

Let $f(x) = ax^2 + bx + c$ where $a$, $b$ and $c$ are rational numbers.

(a) The point $P(-4, 3)$ lies on the curve of $f$. Show that $16a - 4b + c = 3$. [2]

(b) The points $Q(6, 3)$ and $R(-2, -1)$ also lie on the curve of $f$.

Write down two other linear equations in $a$, $b$ and $c$. [2]

(c) **Hence** find $f(x)$. [2]

(d) Write $f(x)$ in the form $f(x) = a(x - h)^2 + k$, where $a$, $h$ and $k$ are rational numbers. [2]

**14.** [Maximum mark: 5]

The following point series lies on a quadratic model $P = aQ^2 + bQ + c$

| $Q$ | **1** | **2** | **3** |
|---|---|---|---|
| $P$ | **4** | **7** | **14** |

(a) Write down three linear equations on $a, b, c$. [3]

(b) Hence find the quadratic model. [2]

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**15.** [Maximum mark: 7]

The following point series lies on a quadratic model $P = aQ^2 + bQ + c$

| $Q$ | **–2** | **1** | **2** |
|---|---|---|---|
| $P$ | **0** | **0** | **12** |

Find the quadratic model by using two different methods.

(a) By writing down three linear equations on $a, b, c$ and solving the system. [4]

(b) By using factorization and the fact that two of the points are $Q$-intercepts. [3]

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**16.** [Maximum mark: 10]

The following point series lies on a quadratic model $P = aQ^2 + bQ + c$

| $Q$ | **0** | **3** | **6** |
|---|---|---|---|
| $P$ | **1** | **10** | **1** |

(a) By sketching a graph determine

(i) the vertex of the quadratic. (ii) the $P$-intercept. [3]

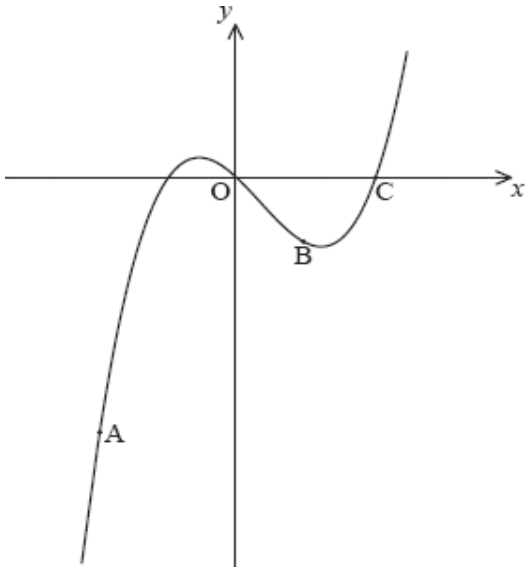
The quadratic model can be found by using two different methods.

(b) Write down three linear equations on $a, b, c$ and hence find the model. [4]

(c) Use your answer in part (a) to find the vertex form $P = a(Q - h)^2 + k$ and then expand to find the quadratic model. [3]

**17.** [Maximum mark: 9]

Consider the function $f(x) = px^3 + qx^2 + rx$. Part of the graph is shown below.




The graph passes through the origin O and the points A(–2, –8), B(1, –2) and C(2, 0).

(a) Find three linear equations in $p$, $q$ and $r$. [3]

(b) Hence find the value of $p$, of $q$ and of $r$. [2]

(c) Write down the three roots of the function $f$. [2]

(d) Solve the inequality $f(x) \geq 0$. [2]

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**18.** [Maximum mark: 8]

For the point series

| $x$ | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| $y$ | **3** | **5** | **9** | **13** |

two linear models are suggested.

$L_1$: $y = 3x$ [which passes through (1,3) and (3,9)]

$L_2$: $y = 3.4x - 1$

(a) Complete the following table of the Squared Residuals and $SS_{res}$ (sum of the squared residuals)

| $x$ | $y$ | **Model $L_1$** | | **Model $L_2$** | |
|---|---|---|---|---|---|
| | | $y_1 = 3x$ | $(y - y_1)^2$ | $y_2 = 3.4x - 1$ | $(y - y_2)^2$ |
| **1** | **3** | 3 | 0 | 2.4 | 0.36 |
| **2** | **5** | 6 | 1 | 5.8 | |
| **3** | **9** | | | | 0.04 |
| **4** | **13** | | | | |
| | | **SUM→** | | **SUM→** | |

[5]

(b) Write a comment about the appropriateness of the two models. [1]

(c) Find the best fit line by using linear regression on your GDC. [2]

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## B. Paper 2 questions (LONG)

**19.** [Maximum mark: 17]

Consider the points A(1,5), B(3, 11), C(6, 20)

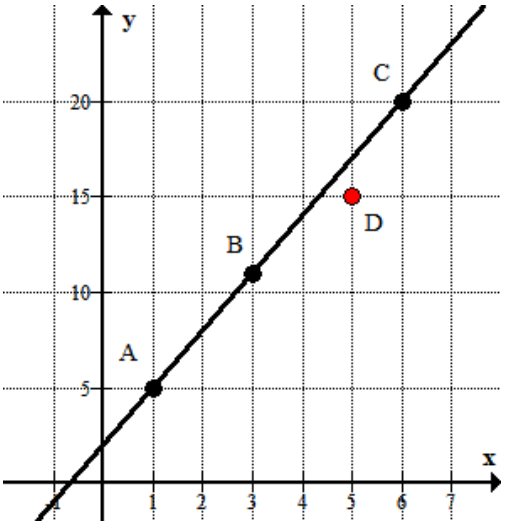
(a) Find

(i) the gradient of the line segment [AB].

(ii) the equation of the line (AB) in the form $y = mx + c$. [4]

(b) Show that the three points A, B and C are collinear. [2]

The following diagram shows the three points A, B, C and the line (AB). An additional point D(5,15) is added.




(c) Show that the point D does not lie on the line (AB). [2]

(d) Explain why the line (AB) is a satisfactory model for the point series A, B, C, D. [2]

(e) Use your GDC to find the best fit line L for the point series A, B, C, D. Give the coefficients in 3s.f. [2]

(f) Complete the following table. Give the squared residuals in 4dp.

| $x$ | $y$ | Estimations | | Squared Residuals | |
|---|---|---|---|---|---|
| | | $y_1$ by line AB) | $y_2$ by line L | $(y-y_1)^2$ | $(y-y_2)^2$ |
| **1** | **5** | 5 | | 0 | |
| **3** | **11** | 11 | | 0 | |
| **5** | **15** | | | | |
| **6** | **20** | | | | |
| **$SS_{res}$ = SUM OF THE SQUARED RESIDUALS →** | | | | | |

[5]

(g) Write down a comment about the two linear models found. [1]

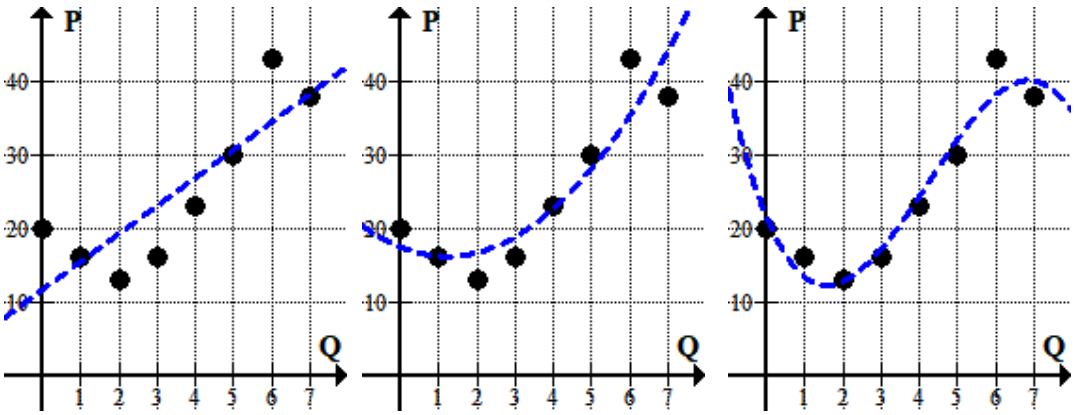
**20.** [Maximum mark: 10]

Consider the point series

| $Q$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P$ | 20 | 16 | 13 | 16 | 23 | 30 | 43 | 38 |

The following diagrams show three regression models for this point series.

(I) (II) (III)




(a) Using your GDC and regression find the three models:

*(i)* The linear model (I): $P = aQ + b$

(ii) The quadratic model (II): $P = aQ^2 + bQ + c$

(iii) The cubic model (III): $P = aQ^3 + bQ^2 + cQ + d$ .

Give all the coefficients in 3.s.f. [4]

(b) Find the percentage error of each model for the estimation of the $P$-intercept. [3]

(c) Write down a comment for the appropriateness of the three models. [1]

(d) Suggest a prediction for the value of $P$ when $Q = 8$. [2]

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