[MAA(HL) 4.1-4.3] STATISTICS – BASIC CONCEPTS

SOLUTIONS

O. Practice questions

1. (a) $\bar{x} = \dfrac{1+2+3+4+5}{5} = \dfrac{15}{5} = 3$ (or directly 3 by symmetry)

(b) $\sigma^2 = \dfrac{(1-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(5-3)^2}{5} = \dfrac{4+1+0+1+4}{5} = 2$

OR

$\sigma^2 = \dfrac{1^2+2^2+3^2+4^2+5^2}{5} - 3^2 = \dfrac{1+4+9+16+25}{5} - 9 = 11-9 = 2$

(c) $\sigma = \sqrt{2}$

(d) $Q_1 = 1.5$, $Q_3 = 4.5$

IQR = 3

2. (a) $\bar{x} = \dfrac{2\times 10+5\times 20+3\times 30}{10} = \dfrac{210}{10} = 21$ (or directly 3 by symmetry)

(b) $\sigma^2 = \dfrac{2\times(10-21)^2+5\times(20-21)^2+3\times(30-21)^2}{10} = \dfrac{242+5+243}{10} = 49$

OR

$\sigma^2 = \dfrac{2\times 10^2+5\times 20^2+3\times 30^2}{10} - 21^2 = \dfrac{200+2000+2700}{10} - 441 = 490-441 = 49$

(c) $\sigma = 7$

(d) $Q_1 = 20$, $Q_3 = 30$

IQR = 10

3. (a) $\bar{x}_{new} = \dfrac{\sum_{i=1}^{n}(x_i+k)}{n} = \dfrac{\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}k}{n} = \dfrac{\sum_{i=1}^{n}x_i}{n} + \dfrac{\sum_{i=1}^{n}k}{n} = \bar{x} + \dfrac{kn}{n} = \bar{x} + k$

$$s_{new} = \sqrt{\dfrac{\sum_{i=1}^{n}((x_i+k)-(\bar{x}-k))^2}{n}} = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}} = s$$

(b) $\bar{x}_{new} = \dfrac{\sum_{i=1}^{n}(kx_i)}{n} = \dfrac{k\sum_{i=1}^{n}x_i}{n} = k\dfrac{\sum_{i=1}^{n}x_i}{n} = k\bar{x}$

$$s_{new} = \sqrt{\dfrac{\sum_{i=1}^{n}(kx_i-k\bar{x})^2}{n}} = \sqrt{\dfrac{k^2\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}} = k\sqrt{\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}} = ks$$

$s_{new} = ks \Rightarrow s^2{}_{new} = k^2 s^2$

so the original variance is multiplied by k^2.

A. Exam style questions (SHORT)

4. $\mu = \frac{1}{6}(1+1+2+3+4+5) = \frac{8}{3}$ (=2.67)

$\sigma = \frac{1}{6}(1+1+4+9+16+25) - \frac{64}{9} = \frac{20}{9}$ (=2.22)

5. $\bar{x} = \frac{2+3+6+9+a+b}{6} = \frac{20+a+b}{6} = 6 \Rightarrow a+b = 16$

$\sigma^2 = \frac{4+9+36+81+a^2+b^2}{6} - 6^2 = 10$

$\Rightarrow \frac{130+a^2+b^2}{6} = 46 \Rightarrow 130 + a^2 + b^2 = 276 \Rightarrow a^2 + b^2 = 146$

$\Rightarrow a^2 + (16-a)^2 = 146$

Hence, $a = 5$ or $a = 11$

And finally, $a = 5$ and $b = 11$

Note: You may use the alternative formula for the variance

variance = $\frac{\sum_{i=1}^{6}(x_i - 6)^2}{6} = \frac{4^2+3^2+0^2+3^2+(a-6)^2+(b-6)^2}{6} = 10$

$\Rightarrow (a-6)^2 + (b-6)^2 = 26$

$\Rightarrow (a-6)^2 + (10-a)^2 = 26$

Therefore $a = 5$, $b = 11$

6. (a) $\bar{x} = \frac{626}{20} = 31.3$

(b) $\frac{\sum x^2}{n} - \bar{x}^2 = \frac{19780.8}{20} - 31.3^2 = 9.35$

(c) $\frac{\sum_{x=1}^{20}(x_i - \mu)^2}{20} = 9.35 \Rightarrow \sum_{x=1}^{20}(x_i - \mu)^2 = 187$

7. $\sigma^2 = \frac{\sum_{x=1}^{10} x_i^2}{10} - \mu^2 \Rightarrow 2^2 = \frac{\sum_{x=1}^{10} x_i^2}{10} - 9^2 \Rightarrow \frac{\sum_{x=1}^{10} x_i^2}{10} = 85 \Rightarrow \sum_{x=1}^{10} x_i^2 = 850$

8. $6.9^2 = 47.61$

$47.61 = \frac{1341}{10} - \bar{x}^2$

$\bar{x}^2 = 86.49$

$\bar{x} = \pm 9.3$ (Do not penalise the absence of the minus)

9.

$$\frac{a+b+826}{22}=42 \quad \Rightarrow a+b=98$$

$$\frac{a^2+b^2+34132}{22}-42^2=32$$

$$\Rightarrow a^2+b^2=5380$$

Solving the system of equations

$a=32,\ b=66$ (or vice versa)

10. $$\frac{20+100+210+15a+25b}{100}=7 \Rightarrow 330+15a+25b=700$$

$$\Rightarrow 15a+25b=370$$

$$\frac{40+500+1470+15a^2+25b^2}{100}-49=5.7$$

$$\Rightarrow 2010+15a^2+25b^2=5470 \Rightarrow 15a^2+25b^2=3460$$

$$\Rightarrow 15a^2+25\left(\frac{370-15a}{25}\right)^2=460$$

Hence, $a=8$ and $b=10$

11. $$\frac{20+100+210+8a+10b}{60+a+b}=7 \Rightarrow 330+8a+10b=420+7a+7b$$

$$\Rightarrow a+3b=90 \qquad (1)$$

$$\frac{40+500+1470+64a+100b}{60+a+b}-49=5.7$$

$$\Rightarrow \frac{2010+64a+100b}{60+a+b}=54.7 \Rightarrow 2010+64a+100b=3282+54.7a+54.7b$$

$$\Rightarrow 9.3a+45.3b=1272 \qquad (2)$$

Hence, (1) and (2) give

$a=15$ and $b=25$