*INTERNATIONAL BACCALAUREATE*

***Mathematics: analysis and approaches***

# *MAA*

***EXERCISES [MAA 5.7]***

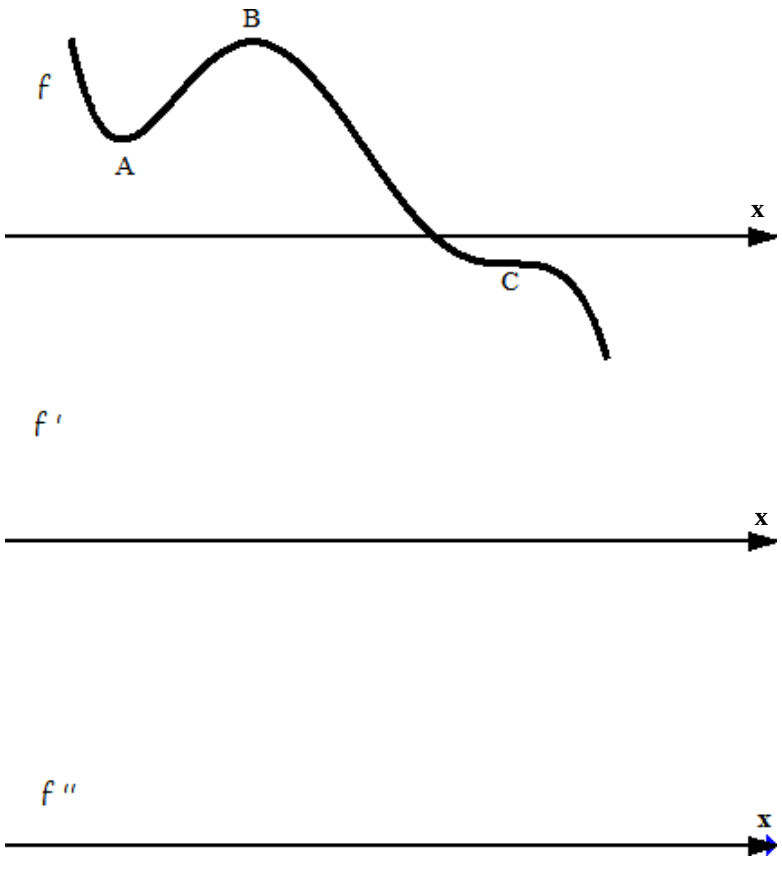
***THE GRAPH OF f ′***

## O. Practice questions

***1.*** [Maximum mark: 6] ***[without GDC]***

The diagram below shows the graph of $y = f(x)$. There is a local minimum at A, a local maximum at B, and a stationary point of inflexion at C.

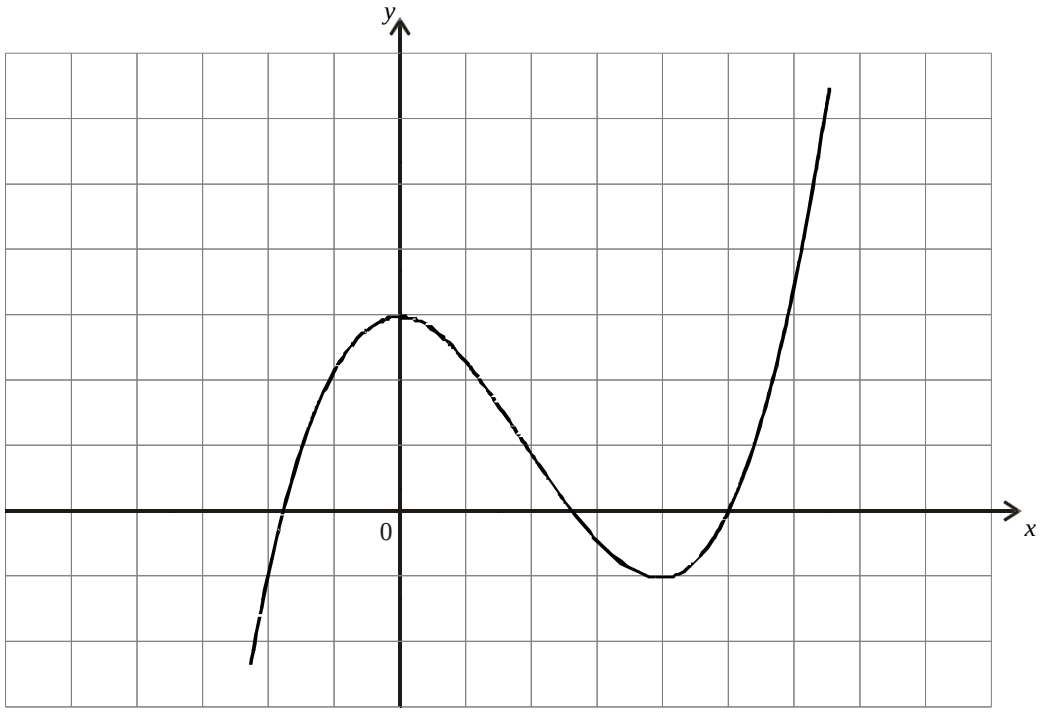
Sketch below the graph of the original function, the graphs of $y = f'(x)$ and $y = f''(x)$.



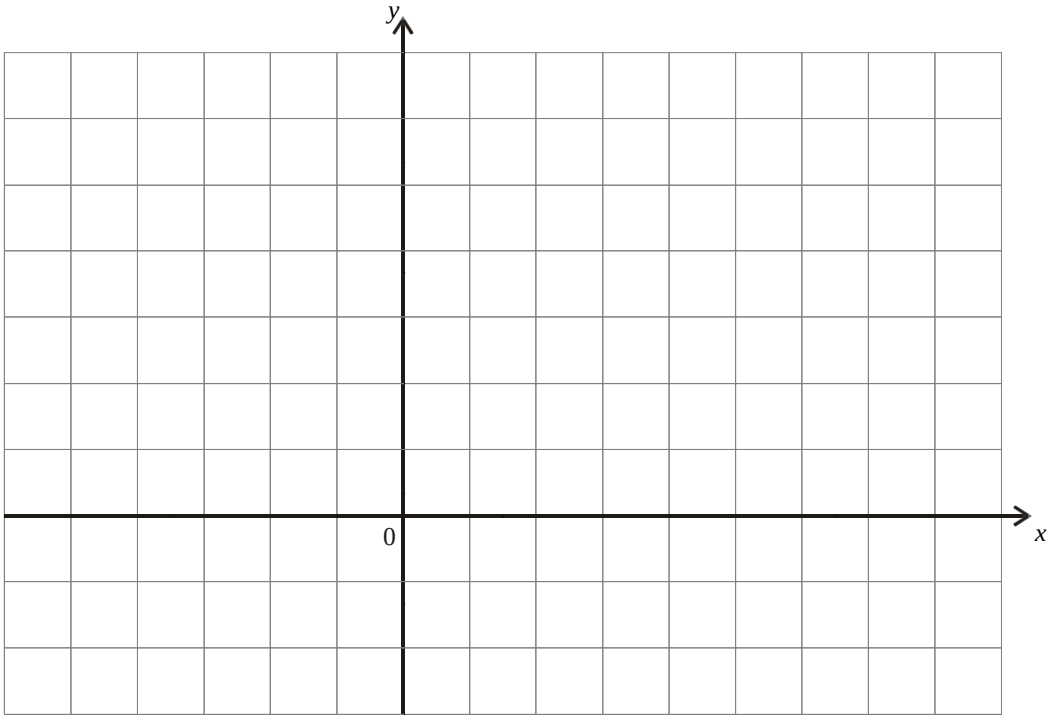
## A. Past paper questions (*SHORT*)

***2.*** [Maximum mark: 5] ***[without GDC]***

The diagram shows the graph of $y = f(x)$.




On the grid below sketch the graph of $y = f'(x)$.



***3.*** [Maximum mark: 6] ***[without GDC]***

**Figure 1** shows the graphs of the functions $f_1,\ f_2,\ f_3,\ f_4$.

**Figure 2 includes** the graphs of the derivatives of the functions shown in **Figure 1**, *eg* the derivative of $f_1$ is shown in diagram (d).

**Figure 1**

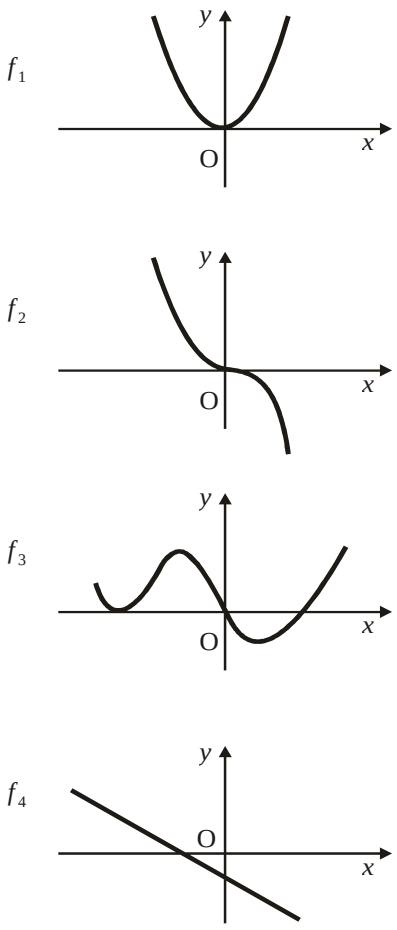
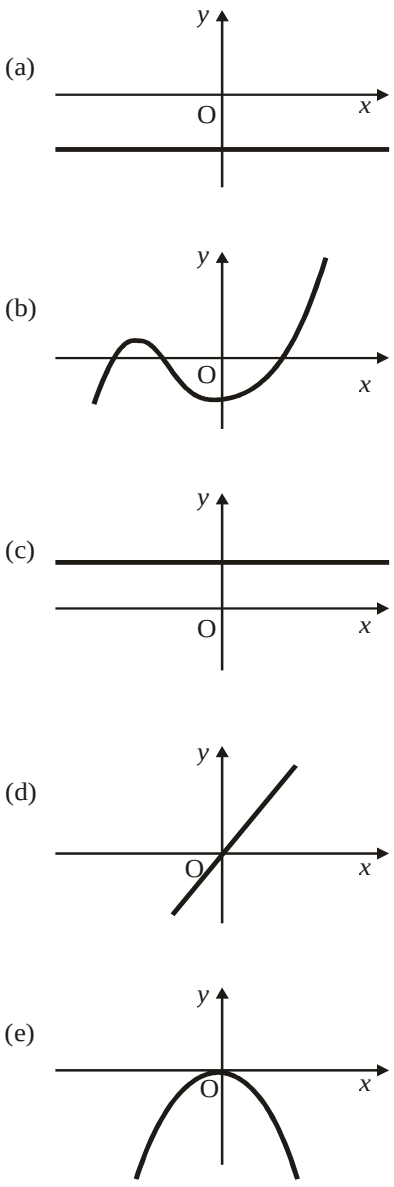



**Figure 2**

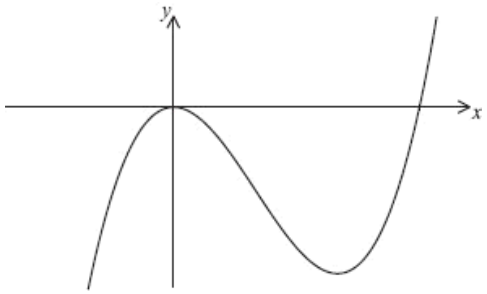



Complete the table below by matching each function with its derivative.

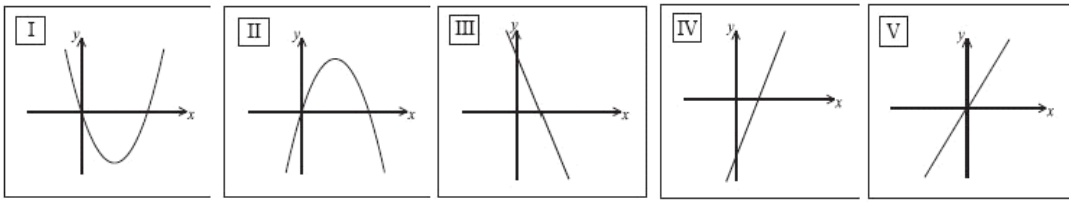
| Function | Derivative diagram |
|---|---|
| $f_1$ | (d) |
| $f_2$ | |
| $f_3$ | |
| $f_4$ | |

***4.*** [Maximum mark: 4] ***[without GDC]***

The following diagram shows the graph of a function $f$.




Consider the following diagrams.



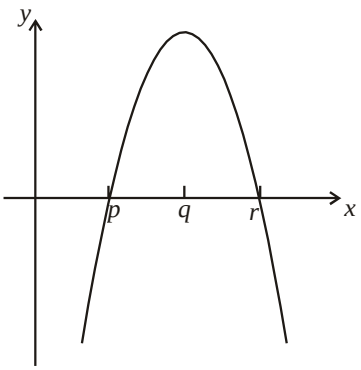

Complete the table below, noting the corresponding diagram for $f'(x)$ and $f''(x)$:

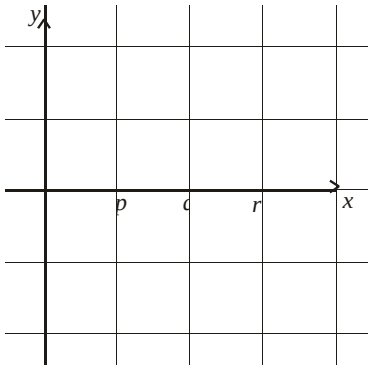
| Graph | $f'(x)$ | $f''(x)$ |
|---|---|---|
| Diagram | | |

***5.*** [Maximum mark: 6] ***[without GDC]***

The first diagram below shows part of the graph of the **gradient** function $y = f'(x)$.

(a) On the grid to the right, sketch a graph of $f''(x)$, clearly indicating the $x$-intercept.




[2]

(b) Complete the table, for the graph of $y = f(x)$.

| | Maximum point on *f* | Inflexion point on *f* |
|---|---|---|
| $x$-coordinate | | |

[2]

(c) Justify your answer to part (b) (ii). [2]

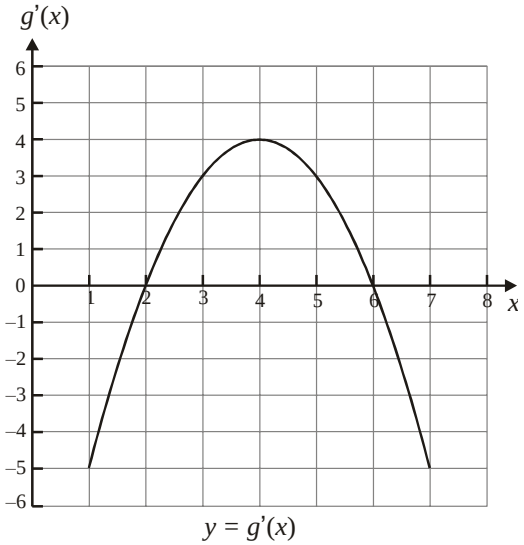
.....

.....

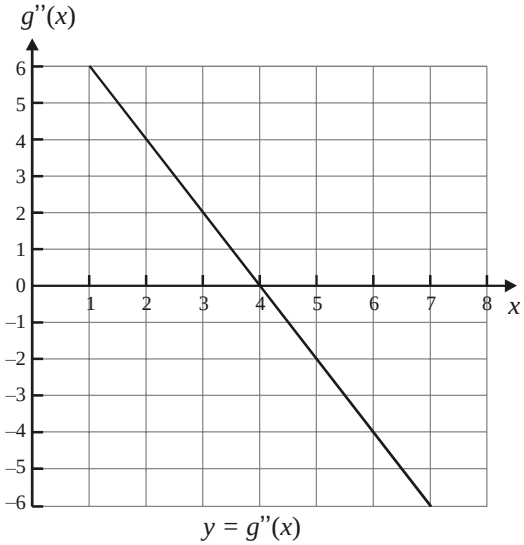
***6.*** [Maximum mark: 8] ***[without GDC]***

Let $y = g(x)$ be a function of $x$ for $1 \le x \le 7$. The graph of $g$ has an inflexion point at P, and a minimum point at M.

Partial sketches of the curves of $g'$ and $g''$ are shown below.




$y = g'(x)$




$y = g''(x)$

Use the above information to answer the following.

(a) Write down the $x$-coordinate of P, **and** justify your answer. [2]

(b) Write down the $x$-coordinate of M, **and** justify your answer. [2]

(c) Given that $g(4) = 0$, sketch the graph of $g$. Mark the points P and M. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

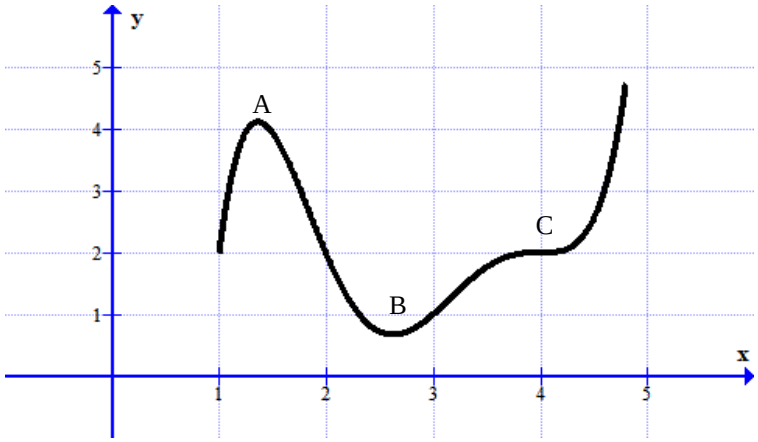
.....

.....

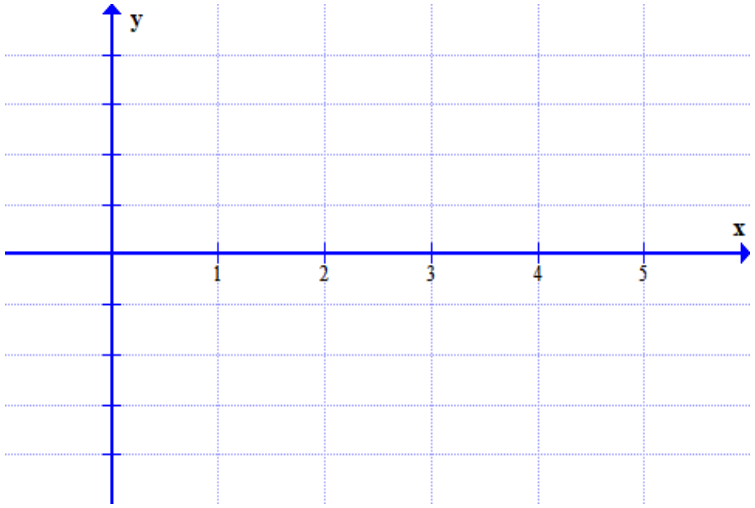
.....

***7.*** [Maximum mark: 6] ***[without GDC]***

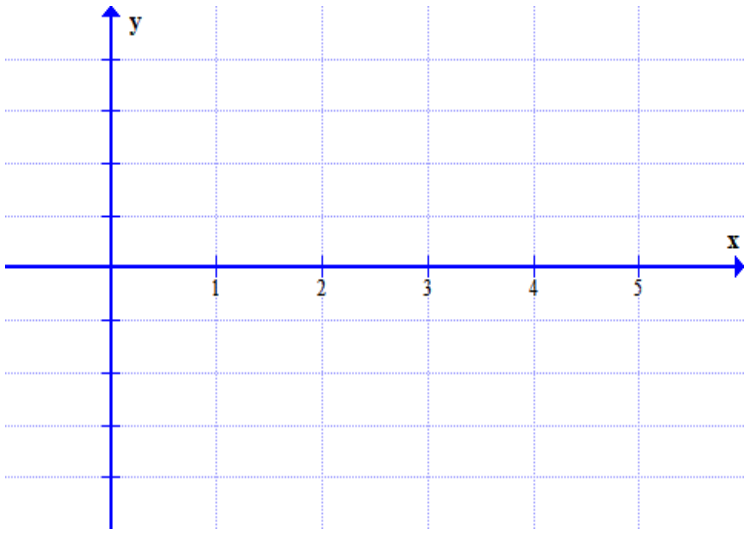
The diagram below shows the graph of $y = f(x)$. The tangent lines on this curve at points *A*, *B*, *C* are parallel to the $x$-axis.




(a) Sketch the graph of the function $y = f'(x)$, by indicating the $x$-intercepts. [3]



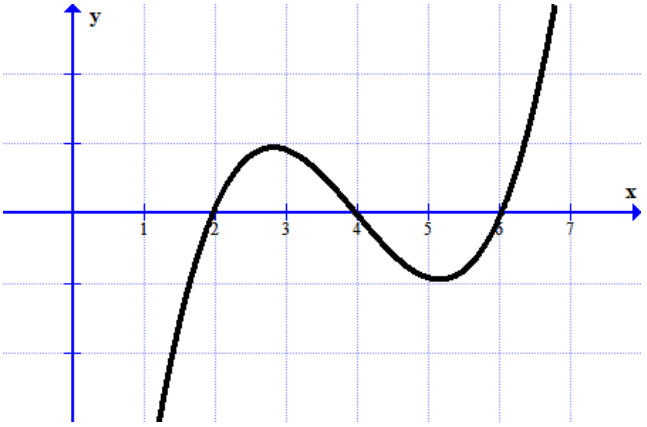

(b) Sketch the graph of the function $y = f'(x)$, by indicating the $x$-intercepts. [3]



***8.*** [Maximum mark: x] ***[without GDC]***

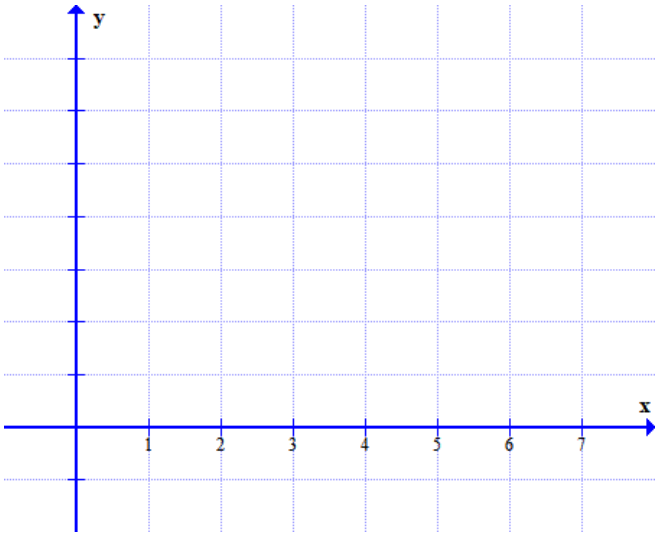
The diagram below shows the graph of $y = f'(x)$.

$y = f'(x)$



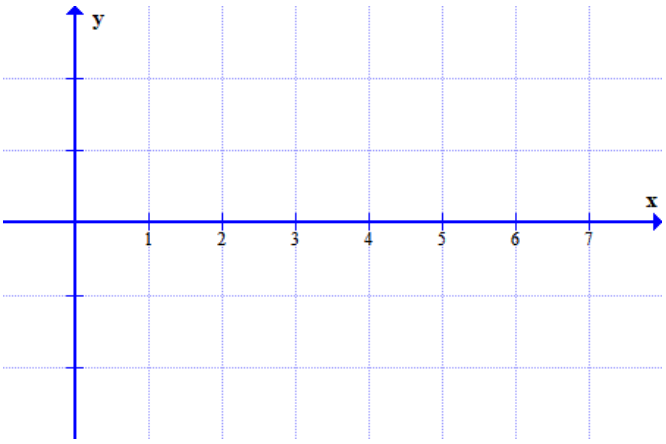

(a) Sketch the graph of the function $y = f(x)$ given that $f(x) > 0$ for any $x$.

$y = f(x)$



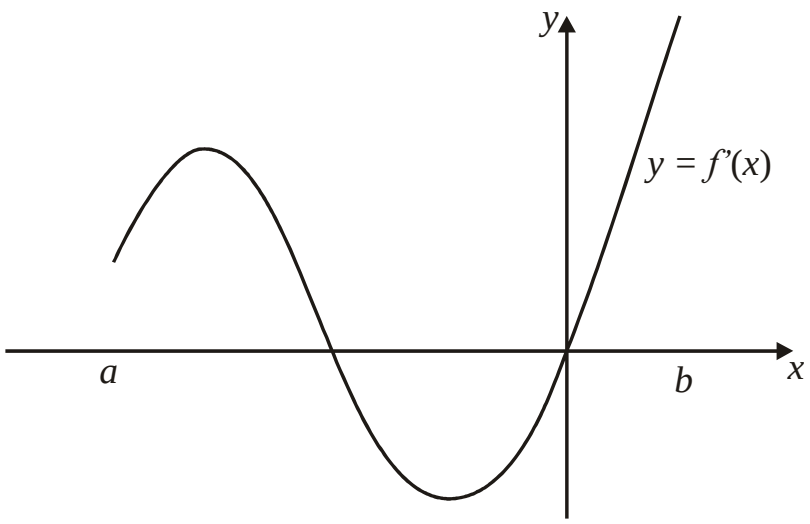

(b) Sketch the graph of the function $y = f''(x)$.

$y = f'(x)$

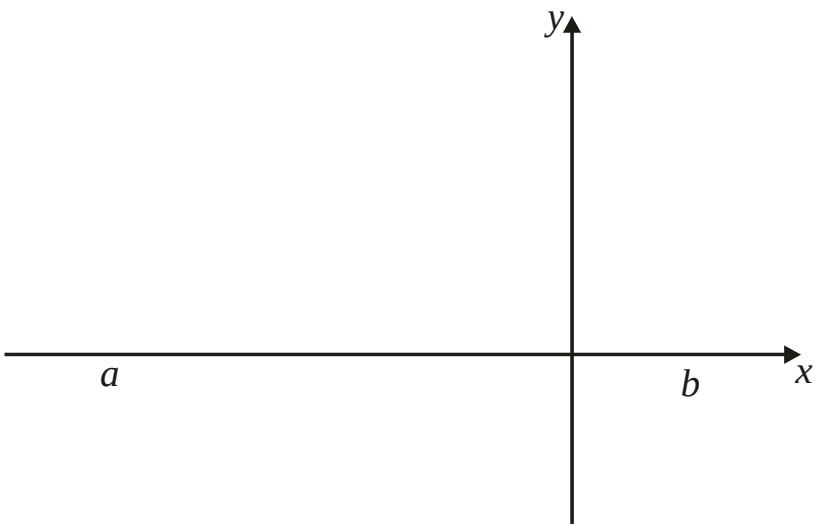


***9.*** [Maximum mark: 4] ***[without GDC]***

The diagram shows a sketch of the graph of $y = f'(x)$ for $a \le x \le b$.

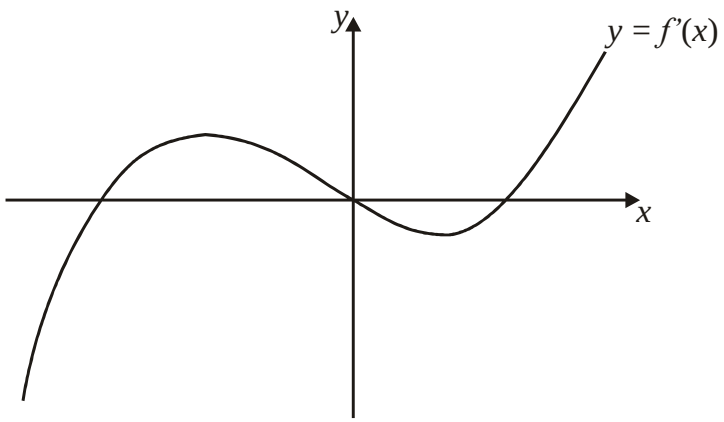



On the grid below, which has the same scale on the $x$-axis, draw a sketch of the graph of $y = f(x)$ for $a \le x \le b$, given that $f(0) = 0$ and $f(x) \ge 0$ for all $x$. On your graph you should clearly indicate any minimum or maximum points, or points of inflexion.



***10.*** [Maximum mark: 5] ***[without GDC]***

The diagram shows the graph of $y = f'(x)$.




Indicate, and label clearly, **on the graph** the points where $y = f(x)$ has

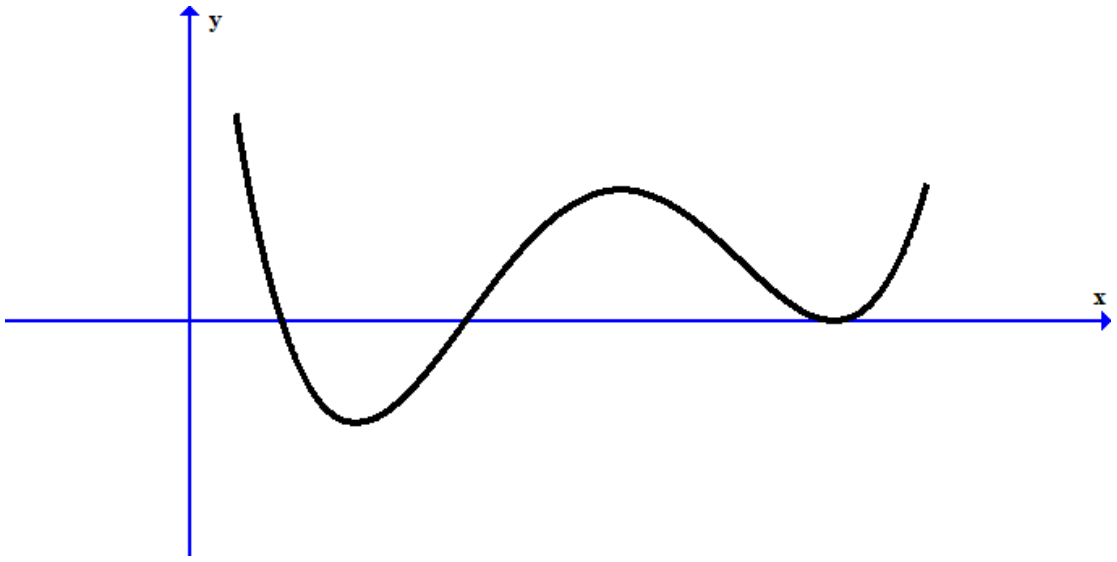
(i) minimum points;

(ii) maximum points;

(iii) points of inflexion.

***11.*** [Maximum mark: 6] ***[without GDC]***

The diagram shows the graph of $y = f'(x)$.

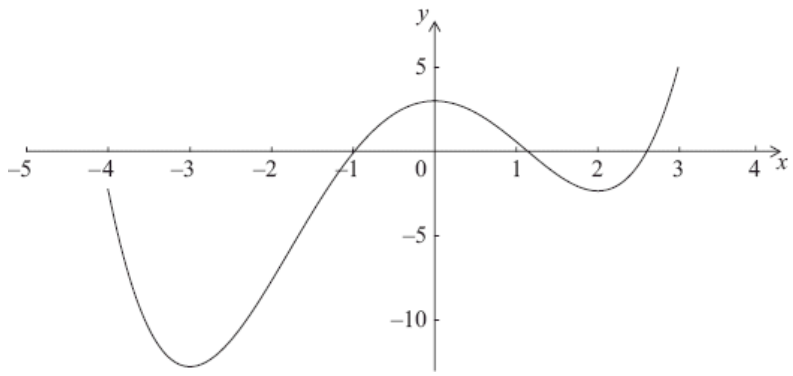



Indicate, and label clearly, **on the graph** the points where $y = f(x)$ has

- minimum points (**max)**
- maximum points (**min)**
- points of inflexion (**poi)**
- stationart point of inflexion (**s-poi)**

***12.*** [Maximum mark: 8] ***[without GDC]***

A function $f$ is defined for $-4 \le x \le 3$. The graph of $f$ is given below.




The graph has a local maximum when $x = 0$, and local minima when $x = -3$, $x = 2$.

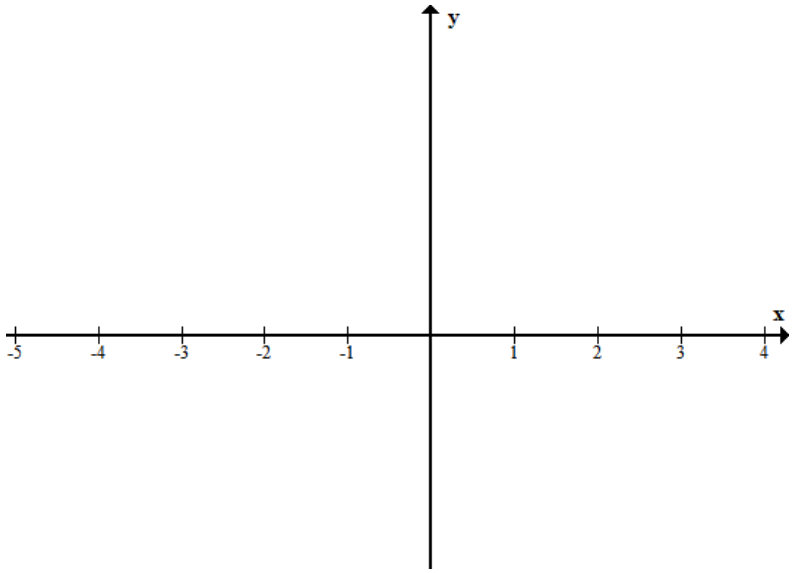
(a) Write down the $x$-intercepts of the graph of the **derivative** function, $f'$. [2]

(b) Write down all values of $x$ for which $f'(x)$ is positive. [2]

(c) At point D on the graph of $f$, the $x$-coordinate is $-0.5$.

Explain why $f''(x) < 0$ at D. [2]

(d) Sketch the graph of $f''$. [2]




.....

.....

.....

.....

.....

.....