

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches
MAA

EXERCISES [MAA 5.18-5.19]
MORE INTEGRALS – FURTHER SUBSTITUTION

MORE INTEGRALS

O. Practice questions

1. [Maximum mark: 10] **[without GDC]**

Use the differentiation and integration formulas [from the formula booklet!] to find

Integral	Result
$\int 5^x dx$	
$\int (x^3 + 3^x) dx$	
$\int \sec^2 x dx$	
$\int (\tan^2 x + 1) dx$	
$\int \frac{5}{\cos^2 x} dx$	
$\int \operatorname{cosec}^2 x dx$	
$\int (\cot^2 x + 1) dx$	
$\int \frac{5}{\sin^2 x} dx$	
$\int \sec x \tan x dx$	
$\int \operatorname{cosec} x \cot x dx$	

2. [Maximum mark: 10] **[without GDC]**

Use the formula $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C$ to find the integrals

$\int \frac{7}{16 + x^2} dx$
$\int \frac{7}{1 + 16x^2} dx$
$\int \frac{7}{25 + 16x^2} dx$
$\int \frac{7}{2 + x^2} dx$
$\int \frac{7}{2 + 3x^2} dx$

3. [Maximum mark: 10] **[without GDC]**

Use the formula $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C$ to find the integrals

$\int \frac{7}{\sqrt{16 - x^2}} dx$
$\int \frac{7}{\sqrt{1 - 16x^2}} dx$
$\int \frac{7}{\sqrt{25 - 16x^2}} dx$
$\int \frac{7}{\sqrt{2 - x^2}} dx$
$\int \frac{7}{\sqrt{2 - 3x^2}} dx$

- (a) Factorise $x^2 - 10x + 24$ and thus express $\frac{2}{x^2 - 10x + 24}$ in partial fractions. [5]
- (b) Express $x^2 - 10x + 25$ as a perfect square $(x - h)^2$. [1]
- (c) Express $x^2 - 10x + 26$ in the vertex form $(x - h)^2 + k$. [2]
- (d) **Hence** find the integrals
- (i) $\int \frac{2}{x^2 - 10x + 24} dx$ (ii) $\int \frac{2}{x^2 - 10x + 25} dx$ (iii) $\int \frac{2}{x^2 - 10x + 26} dx$ [6]

[illegible]

A. Exam style questions (SHORT)

5. [Maximum mark: 6] **[without GDC]**

Find (i) $\int \frac{e^{3x} + 2e^x + 4}{3e^{2x}} dx$ (ii) $\int \frac{4^x + 2^x + 1}{2^x} dx$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6. [Maximum mark: 4] **[without GDC]**

Use appropriate trigonometric identities to modify and hence calculate the integrals

(i) $\int \tan^2 x dx$ (ii) $\int \cot^2 x dx$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7. [Maximum mark: 5] **[without GDC]**

Find the integrals (i) $\int \frac{3}{x^2+9} dx$ (ii) $\int \frac{3}{9x^2+1} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8. [Maximum mark: 5] **[without GDC]**

Find the integrals (i) $\int \frac{3}{\sqrt{9-x^2}} dx$ (ii) $\int \frac{3}{\sqrt{1-9x^2}} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

9. [Maximum mark: 5] **[without GDC]**

Find (i) $\int \frac{7}{(x-1)^2+4} dx$ (ii) $\int \frac{7}{4(x-1)^2+1} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

10. [Maximum mark: 5] **[without GDC]**

Find (i) $\int \frac{7}{\sqrt{4-(x-1)^2}} dx$ (ii) $\int \frac{7}{\sqrt{1-4(x-1)^2}} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11. [Maximum mark: 6] **[without GDC]**

Given that $b, c \in \mathbb{R}^+$, find the integrals (i) $\int \frac{a}{bx^2 + c} dx$ (ii) $\int \frac{a}{\sqrt{c - bx^2}} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

12. [Maximum mark: 7] **[without GDC]**

(a) Show that $\frac{x^2 + 3x + 12}{x(x + 2)^2} = \frac{3}{x} - \frac{2}{(x + 2)} - \frac{5}{(x + 2)^2}$ [3]

(b) Hence find $\int \frac{x^2 + 3x + 12}{x(x + 2)^2} dx$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 13.** [Maximum mark: 7] **[without GDC]**

Calculate the integral $\int \frac{6}{x^2 + 6x + 8} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 14.** [Maximum mark: 5] **[without GDC]**

Calculate the integral $\int \frac{6}{x^2 + 6x + 13} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

15. [Maximum mark: 7] **[without GDC]**

Calculate the integral $\int \frac{x+6}{x^2-6x+8} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

16. [Maximum mark: 5] **[without GDC]**

Calculate the integral $\int \frac{7}{\sqrt{-x^2-6x+7}} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

O. Practice questions

O. Practice questions

18. [Maximum mark: 12] *[without GDC]*

Use the substitution $u = \tan x$, to find the indefinite integrals

(i) $\int \tan^5 x \sec^2 x dx$ (ii) $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$ (iii) $\int \frac{\sec^2 x}{e^{\tan x}} dx$ (iv) $\int \frac{\sin^5 x}{\cos^7 x} dx$

[illegible]

19. [Maximum mark: 16] **[without GDC]**

Using the fact $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$, calculate the integrals

$\int \tan x dx$
$\int \cot x dx$
$\int \frac{2x}{x^2 + 7} dx$
$\int \frac{x}{3x^2 + 7} dx$
$\int \frac{x^2}{3x^3 + 7} dx$
$\int \frac{6x + 5}{3x^2 + 5x + 1} dx$
$\int \frac{e^x}{e^x + 5} dx$
$\int \frac{e^{2x}}{e^{2x} + 5} dx$

20. [Maximum mark: 5] **[without GDC]**

Find the indefinite integral $\int \frac{e^x}{e^{2x} + 1} dx$ by using the substitution $u = e^x$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

24. [Maximum mark: 6] **[without GDC]**

Calculate $\int \frac{1}{9+x^2} dx$, by using the substitution $x = 3 \tan \theta$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

25. [Maximum mark: 6] **[without GDC]**

Calculate $\int \frac{1}{\sqrt{9-x^2}} dx$, by using the substitution $x = 3 \sin \theta$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A. Exam style questions (SHORT)

27. [Maximum mark: 6] **[without GDC]**

By using an appropriate substitution find $\int \frac{\tan(\ln y)}{y} dy$, $y > 0$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

28. [Maximum mark: 6] **[without GDC]**

By using an appropriate substitution find $\int \frac{\cot(\ln y)}{y} dy$, $y > 0$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

29. [Maximum mark: 6] **[without GDC]**

Using the substitution $y = 2 - x$, or otherwise, find $\int \left| \frac{x}{2-x} \right|^2 dx$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

30. [Maximum mark: 5] **[without GDC]**

Using the substitution $u = \frac{1}{2}x + 1$, or otherwise, find the integral $\int x \sqrt{\frac{1}{2}x + 1} dx$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

34. [Maximum mark: 6] **[without GDC]**

(a) Show that $\frac{2x+4}{(x^2+4)(x-2)} = \frac{1}{x-2} - \frac{x}{x^2+4}$ [2]

(b) Hence find $\int \frac{2x+4}{(x^2+4)(x-2)} dx$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

35. [Maximum mark: 5] **[without GDC]**

Find (i) $\int \frac{e^x+1}{e^x} dx$ (ii) $\int \frac{e^x}{e^x+1} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

36. [Maximum mark: 6] **[without GDC]**

Find $\int \frac{e^x}{e^{2x} + 4} dx$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

37. [Maximum mark: 6] **[without GDC]**

Find $\int \frac{3^x}{9^x + 9} dx$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

39. [Maximum mark: 6] **[without GDC]**

By using an appropriate substitution find the integral $\int \sin^3 x dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

40. [Maximum mark: 5] **[without GDC]**

By using the substitution $u = \sin x$, find the integral $\int \sin^3 x \cos^3 x dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DEFINITE INTEGRALS

A. Exam style questions (SHORT)

41. [Maximum mark: 4] **[without GDC]**

Find $\int_0^2 \frac{1}{4+x^2} dx$

.....

.....

.....

.....

.....

.....

.....

.....

.....

42. [Maximum mark: 6] **[without GDC]**

Find k given that $\int_0^k \frac{1}{4+x^2} dx = \frac{\pi}{6}$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

44. [Maximum mark: 6] *[without GDC]*

[illegible]

