[MAA 4.9] DISCRETE DISTRIBUTIONS IN GENERAL

SOLUTIONS

O. Practice questions

1. (a) 2.35 (b) mode =1 (c) median = 2 (d) $Q_1 = 1$ $Q_3 = 3.5$

2. $a = 0.2$ $b = 0.4$

3. $E(X) = 2.2$ E(Profit) = 0, so it is fair.

4. (a)

x	1	2	3
$P(X = x)$	1/6	2/6	3/6

(b) Find $E(X) = 7/3$

5. (a)

x	3	4	5
$P(X = x)$	4/10	3/10	3/10

(b) Find $E(X) = 3.9$

6.

$E(X)$	1.2
$E(X^2)$	2.2
$E(X^3)$	4.2
$Var(X)$	$2.2 - 1.2^2 = 0.76$
$Var(X)$	$1.2^2 \times 0.3 + 0.2^2 \times 0.2 + 0.8^2 \times 0.5 = 0.76$
MODE	2
MEDIAN	1.5
Q_1	0
Q_3	2
E(2X+1)	2E(X)+1=3.4
Var(2X+1)	4Var(X) = 2.96

The last two can also be found by constructing a new table for 2X+1

x	0	1	2
$2x+1$	1	3	5
Prob	0.3	0.2	0.5

7. (a) $n = 3$ (b) (ii) $E(X) = 18/7$ (ii) $E(X^2) = 7$ (iii) $Var(X) = 7 - \left(\frac{18}{7}\right)^2 = \frac{19}{49}$

8. We obtain three linear equations

(i) sum = 1 $\Rightarrow$ $a + b + c = 1$

(ii) $E(X) = 1$ $\Rightarrow b + 2c = 1$

(iii) From $Var(X) = 0.8$

Either $E(X^2) = Var(X) + \mu^2 = 1.8 \Rightarrow b + 4c = 1.8$

Or $E(X-\mu)^2 = 0.8 \Rightarrow a + c = 0.8$

In any case, the three equations give $a = 0.4$, $b = 0.2$, $c = 0.4$

A. Exam style questions (SHORT)

9. (a) $(0.4 + p + 0.2 + 0.07 + 0.02 = 1), \Rightarrow p = 0.31$

(b) $E(X) = 1(0.4) + 2(0.31) + 3(0.2) + 4(0.07) + 5(0.02) = 2$

10. (a) $0.1 + a + 0.3 + b = 1 \Rightarrow \quad a + b = 0.6$

(b) $0 \times 0.1 + 1 \times a + 2 \times 0.3 + 3 \times b$

$0 + a + 0.6 + 3b = 1.5$

$a + 3b = 0.9$

Solving simultaneously gives

$a = 0.45 \qquad b = 0.15$

11. (a) $0.2 + a + b + 0.25 = 1 (a + b = 0.55)$

$E(X) = a + 2b + 0.75 = 1.55$

$\Rightarrow a + 2b = 0.8$

$a = 0.3$ and $b = 0.25$

12. $\sum_{\text{all } x} P(X = x) = 1 \Rightarrow \frac{1}{5} + \frac{2}{5} + \frac{1}{10} + x = 1 \quad \Rightarrow x = \frac{3}{10}$

P(scoring six after two rolls) = $\left(\frac{1}{10} \times \frac{1}{10}\right) + 2 \times \left(\frac{2}{5} \times \frac{3}{10}\right) = \frac{1}{4}$

13. (a) $10k^2 + 3k + 0.6 = 1 \Rightarrow 10k^2 + 3k - 0.4 = 0 \Leftrightarrow \quad k = 0.1$ (by GDC)

(b) $E(X) = -1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3 = 1.5$

14. (a)

x	0	1	2	3	4
$P(X = x)$	k	2k	3k	4k	5k

(b) $k \times 1 + k \times 2 + k \times 3 + k \times 4 + k \times 5 = 15k = 1 \Leftrightarrow k = \frac{1}{15}$

(c) $E(X) = 0 \times \frac{1}{15} + 1 \times \frac{2}{15} + 2 \times \frac{3}{15} + 3 \times \frac{4}{15} + 4 \times \frac{5}{15} = \frac{40}{15} = \frac{8}{3}$

15. (a) $\sum P(X = x) = 1 \Rightarrow 4c + 6c + 6c + 4c = 1 \Rightarrow 20c = 1 \Rightarrow c = \frac{1}{20} \quad (= 0.05)$

(b) $E(X) = \sum x P(X = x) = (1 \times 0.2) + (2 \times 0.3) + (3 \times 0.3) + (4 \times 0.2) = 2.5$

16. $\sum_{\text{all } x} P(X = x) = 1 \Rightarrow k + \frac{2}{3}k + \left(\frac{2}{3}\right)^2 k + \left(\frac{2}{3}\right)^3 k + \ldots\ldots\ldots\ldots = 1$

$k\left(\frac{1}{1 - \frac{2}{3}}\right) = 1 \Rightarrow \quad k = \frac{1}{3}$

17. (a) Sum = 1.3 which is greater than 1

(b) $3k + 0.7 = 1 \Rightarrow k = 0.1$

(c) (i) $P(X=0) = \frac{0+1}{20} = \frac{1}{20}$

(ii) $P(X>0) = 1 - P(X=0) = \frac{19}{20}$ $\left(\text{or } \frac{4}{20} + \frac{5}{20} + \frac{10}{20}\right) = \frac{19}{20}$

Notice: in fact, the probability distribution is

x	0	3	4	9
$P(X = x)$	1/20	4/20	5/20	10/20

18. (a) (i) 1+2 or 2+1 Prob $= \frac{1}{2} \times \frac{1}{5} \times 2 = \frac{1}{5}$

(ii) 1+3 or 3+1 or 2+2 Prob $= \frac{1}{2} \times \frac{1}{5} \times 2 + \frac{1}{5} \times \frac{1}{5} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$

(b) Let X be the number of counters the player receives in return.

$E(X) = \sum p(x) \times x = 9$

$\Leftrightarrow \left(\frac{1}{2} \times 4\right) + \left(\frac{1}{5} \times 5\right) + \left(\frac{1}{5} \times 15\right) + \left(\frac{1}{10} \times n\right) = 9$

$\Leftrightarrow \frac{1}{10} n = 3 \Leftrightarrow n = 30$

B. Exam style questions (LONG)

19. (a) Probability of two 4s is $\frac{1}{16}$ (= 0.0625)

(b)

x	0	1	2
$P(X = x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

(c) $E(X) = \sum_{0}^{2} x P(X = x)$, $= 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16} \frac{8}{16} \left(= \frac{1}{2}\right)$

or $E(X) = np = 2 \times \frac{1}{4} = \frac{1}{2}$

(d)

x	0	1	2
amount	-2€	5€	20€
$P(X = x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Expected amount = $-2 \times \frac{9}{16} + 5 \times \frac{6}{16} + 20 \times \frac{1}{16} = 2$€

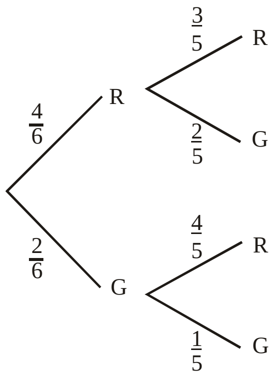
(e) $100 \times 2 = 200$ €

(f) 18 € implies 0 fours and 2 fours or vice versa

$\frac{9}{16} \times \frac{1}{16} \times 2 = \frac{9}{128}$

20. (a) $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10} = \frac{8}{10}$ (0.8)

(b) (i)



(ii) $P(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$

$P(Y = 1) = P(RG) + P(GR) \left(= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \right) = \frac{16}{30}$

$P(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$

Forming a distribution

y	0	1	2
$P(Y = y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

(c) $P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30} = \frac{27}{90} \left(\frac{3}{10}, 0.3 \right)$

(d) $P(1 \text{ or } 6 | RR) = P(A|RR) = \frac{P(A \cap RR)}{P(RR)} = \frac{1}{30} \div \frac{27}{90} = \frac{3}{27} \left(\frac{1}{9}, 0.111 \right)$

21. (a) (i) $P(B) = \frac{3}{4}$ (ii) $P(R) = \frac{1}{4}$

(b) $p = \frac{3}{4}$ $s = \frac{1}{4}$, $t = \frac{3}{4}$

(c) (i) $P(X = 3) = P$ (getting 1 and 2) $= \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

(ii) $P(X = 2) = \frac{1}{4} \times \frac{1}{4} + \frac{3}{4} \left(\text{or } 1 - \frac{3}{16} \right) = \frac{13}{16}$

(d) (i)

X	2	3
$P(X = x)$	$\frac{13}{16}$	$\frac{3}{16}$

(ii) $E(X) = 2\left(\frac{13}{16}\right) + 3\left(\frac{3}{16}\right) = \frac{35}{16}$

(e) win \$10 $\Rightarrow$ scores 3 one time, 2 other time

P(win \$10) = P(3) × P(2) + P(2) × P(3) = $2\left(\frac{13}{16} \times \frac{3}{16}\right), = \frac{78}{256} \left(= \frac{39}{128} \right)$

22. (a)

X_1	0	1	2
$P(X_1=x)$	$\frac{64}{100}$	$\frac{32}{100}$	$\frac{4}{100}$

$E(X_1) = \frac{40}{100} = 0.4$

(b)

X_2	0	1	2
$P(X_2=x)$	$\frac{56}{90}$	$\frac{32}{90}$	$\frac{2}{90}$

$E(X_2) = \frac{36}{90} = 0.4$

(c)

X_3	0	1	2
$P(X_3=x)$	$\frac{336}{720}$	$\frac{336}{720}$	$\frac{48}{720}$

$E(X_3) = \frac{432}{720} = 0.6$

(d)

X_4	1	2	3
$P(X_4=x)$	$\frac{4}{5}$	$\frac{8}{45}$	$\frac{1}{45}$

$E(X_4) = \frac{55}{45} = 1.22$

23. (a) 1, 2, 3, 4

(b) $P(Y = 1) = \frac{2}{5}$

$P(Y = 2) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

$P(Y = 3) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}$

$P(Y = 4) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{10}$

(c) $E(Y) = 1 \times \frac{2}{5} + 2 \times \frac{3}{10} + 3 \times \frac{1}{5} + 4 \times \frac{1}{10} = 2$

(d)

X	1	2	3
$P(X = x)$	3/5	3/10	1/10

24. (a) (i) P(Alan scores 9) = $\frac{1}{9}$ (= 0.111)

(ii) P(Alan scores 9 and Belle scores 9) = $\left(\frac{1}{9}\right)^2 = \left(\frac{1}{81}\right)$ = (= 0.0123)

(b) (i) P(Same score) = $\left(\frac{1}{36}\right)^2 + \left(\frac{2}{36}\right)^2 + \ldots + \left(\frac{6}{36}\right)^2 + \ldots + \left(\frac{2}{36}\right)^2 + \left(\frac{1}{36}\right)^2 = \frac{73}{648}$ (= 0.113)

(ii) P(A>B) = $\frac{1}{2}\left(1 - \frac{73}{648}\right) = \frac{575}{1296}$ (= 0.444)

(c) (i) P(One number $\le x$) = $\frac{x}{6}$ (with some explanation)

P($X \le x$) = P(All four numbers $\le x$) = $\left(\frac{x}{6}\right)^4$

(ii) P($X = x$) = P($X \le x$) – P($X \le x - 1$) = $\left(\frac{x}{6}\right)^4 - \left(\frac{x-1}{6}\right)^4$

x	1	2	3	4	5	6
P($X = x$)	$\frac{1}{1296}$	$\frac{15}{1296}$	$\mathbf{\frac{65}{1296}}$	$\mathbf{\frac{175}{1296}}$	$\mathbf{\frac{369}{1296}}$	$\frac{671}{1296}$

(iii) E(X) = $1 \times \frac{1}{1296} + 2 \times \frac{15}{1296} + \ldots + 6 \times \frac{671}{1296} = \frac{6797}{1296}$ (= 5.24)