

[MAA 4.5-4.7] PROBABILITY I (VENN DIAGRAMS – TABLES)

SOLUTIONS

O. Practice questions

1. (a)

$n(A)$	15	$n(B)$	25	$n(A \cap B)$	10
$n(A')$	35	$n(B')$	25	$n(A \cup B)$	30
$n(A' \cap B)$	15	$n(A \cap B')$	5	$n(A' \cap B')$	20
$n(A' \cup B)$	45	$n(A \cup B')$	35	$n(A' \cup B')$	40

(b)

$P(A)$	15/50	$P(A')$	35/50	$P(A \cup B)$	30/50
$P(A' \cap B)$	15/50	$P(A' \cup B)$	45/50	$P(B' \cup A)$	35/50

(c)

$P(A B)$	10/25	$P(A' B)$	15/25	$P(B' A)$	5/15
$P(B A)$	10/15	$P(A B')$	5/25	$P(A' B')$	20/25

2. (a)

$P(A)$	0.5	$P(A')$	0.5	$P(A \cap B)$	0.2
$P(A \cup B)$	0.9	$P(A' \cap B)$	0.4	$P(A' \cup B)$	0.7

(b)

$P(A B)$	1/3	$P(A' B)$	2/3	$P(B' A)$	3/5
$P(B A)$	2/5	$P(A B')$	3/4	$P(A' B')$	1/4

3. (a)

$P(\text{Boy})$	30/80	$P(\text{Group C})$	15/80
$P(\text{Boy and Group C})$	10/80	$P(\text{Boy or Group C})$	35/80

(b)

$P(\text{Boy} \text{Group C})$	10/15	$P(\text{Group C} \text{Boy})$	10/30
$P(\text{Boy} \text{NOT Group C})$	20/65	$P(\text{NOT Group C} \text{Boy})$	20/30

4. (a)

$P(A)$	$a+c$	$P(A' \cap B)$	b
$P(A')$	$b+d$	$P(A' \cup B)$	$b+c+d$
$P(A \cap B)$	c	$P(A' \cap B')$	d
$P(A \cup B)$	$a+b+c$	$P(A' \cup B')$	$a+b+d$

(b)

$P(A B)$	$c / (b+c)$	$P(B' A)$	$a / (a+c)$
$P(B A)$	$c / (a+c)$	$P(B A')$	$b / (b+d)$
$P(A' B)$	$b / (b+c)$	$P(A' B')$	$d / (a+d)$
$P(A B')$	$a / (a+d)$	$P(B' A')$	$d / (b+d)$

(c) $P(A \cap B) = P(A)P(B) \Rightarrow c = (a+c)(b+c) \Rightarrow a = (2a)(2a)$
 $\Rightarrow a = 4a^2 \Rightarrow a = 0.25$, so $b = c = d = 0.25$

5. (a) $P(A) = 0.4$ $P(A \cup B) = 0.7$
 (b) $x = 0$.
 (c) $x = 0.2$
 (d) $x = 0.3$
 (e) $x = 0.1$

A. Exam style questions (SHORT)

6. (a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
 $65 = 30 + 50 - n(A \cap B) \Rightarrow n(A \cap B) = 15$ (may be on the diagram)
 $n(B \cap A') = 50 - 15 = 35$
 (b) $P(B \cap A') = \frac{n(B \cap A')}{n(U)} = \frac{35}{100} = 0.35$
7. (a) (i) $n(A \cap B) = 2$
 (ii) $P(A \cap B) = \frac{2}{36} \left(\text{or } \frac{1}{18} \right)$
 (b) $n(A \cap B) \neq 0$ (or equivalent)
8. (a) (i) $n = 0.1$
 (ii) $m = 0.2, p = 0.3, q = 0.4$
 (b) $P(B') = 0.6$
9. (a) (i) $p = 0.2$
 (ii) $q = 0.4$
 (iii) $r = 0.1$
 (b) $P(A | B') = \frac{2}{3}$
 (c) valid reason e.g. $\frac{2}{3} \neq 0.5, 0.35 \neq 0.3$ thus, A and B are not independent
10. (a) $p(A \cap B) = 0.6 + 0.8 - 1 = 0.4$
 (b) $p(A' \cup B') = p((A \cap B)') = 1 - 0.4 = 0.6$
11. (a) $\frac{19}{120}$ (≈ 0.158)
 (b) $35 - (8 + 5 + 7) (= 15)$
 Probability = $\frac{15}{120} \left(= \frac{3}{24} = \frac{1}{8} = 0.125 \right)$
 (c) Number studying = 76
 Number not studying = $120 - \text{number studying} = 44$
 Probability = $\frac{44}{120} \left(= \frac{11}{30} \approx 0.367 \right)$
12. (a) (i) $P(P | C) = \frac{20}{20+40} = \frac{1}{3}$
 (ii) $P(P | C') = \frac{30}{30+60} = \frac{1}{3}$
 (b) Investigating conditions, or some relevant calculations

P is independent of C , **with** valid reason

13. (a)

	Boy	Girl	Total
TV	13	25	38
Sport	33	29	62
Total	46	54	100

$$P(\text{TV}) = \frac{38}{100}$$

(b) $P(\text{TV} | \text{Boy}) = \frac{13}{46}$

14. (a)

	Males	Females	Totals
Unemployed	20	40	60
Employed	90	50	140
Totals	110	90	200

(b) (i) $P(\text{unemployed female}) = \frac{40}{200} = \frac{1}{5}$

(ii) $P(\text{male} | \text{employed person}) = \frac{90}{140} = \frac{9}{14}$

15. (a) 46/97

(b) 13/51

(c) 59/97

16. (a) (i) $P(\text{male or tennis}) = \frac{12}{20} \left(= \frac{3}{5} \right)$

(ii) $P(\text{not football} | \text{female}) = \frac{6}{11}$

(b) $P(\text{first not football}) = \frac{11}{20}$, $P(\text{second not football}) = \frac{10}{19}$

$$P(\text{neither football}) = \frac{11}{20} \times \frac{10}{19}$$

$$P(\text{neither football}) = \frac{110}{380} \left(= \frac{11}{38} \right)$$

17. (a) Independent (I)

(b) Mutually exclusive (M)

(c) Neither (N)

18. Using a table,

	ARGENTINE	NOT ARGENTINE	TOTAL
SPANISH	12	3	15
ENGLISH	3	3	6
TOTAL	15	6	21

$$p(S|A) = \frac{12}{15} = \frac{4}{5}$$

19. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

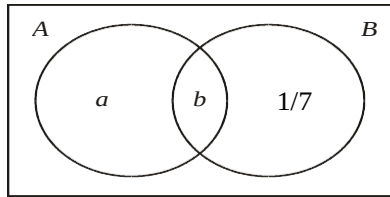
$$P(A \cap B) = \frac{1}{2} + \frac{3}{4} - \frac{7}{8} = \frac{3}{8}$$

(b) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{8}}{\frac{3}{4}} = \frac{1}{2}$

(c) Yes, the events are independent

EITHER $P(A|B) = P(A)$ **OR** $P(A \cap B) = P(A)P(B)$

20.



$$P(A'|B) = \frac{1}{3} \Rightarrow \frac{\frac{1}{7}}{b + \frac{1}{7}} = \frac{1}{3} \Rightarrow \frac{1}{7} = \frac{1}{3}b + \frac{1}{21} \Rightarrow \frac{1}{3}b = \frac{1}{21} - \frac{1}{7} \Rightarrow b = \frac{2}{7}$$

$$P(B) = \frac{2}{7} + \frac{1}{7} = \frac{3}{7}$$

21. (a) $P(A) = \frac{1}{11}$

(b) $P(B|A) = \frac{2}{10}$

(c) $P(A \cap B) = \frac{1}{11} \times \frac{2}{10} = \frac{2}{110}$

22. (a) Independent $\Rightarrow P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.8 = 0.24$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.8 - 0.24 = 0.86$

(c) No, **since** $P(A \cap B) \neq 0$ or $P(A \cup B) \neq P(A) + P(B)$

23. (a) $P(F) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

(b) $P(E \cup F) = P(E) + P(F) - (P(E \cap F)) = \frac{2}{3} + \frac{1}{2} - \frac{1}{3} = \frac{5}{6} (=0.833)$

24. (a) $\frac{3}{4}$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{2}{5} + \frac{3}{4} - \frac{7}{8} = \frac{11}{40} (0.275)$$

$$(c) \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{11}{40} \right)}{\left(\frac{3}{4} \right)} = \frac{11}{30} \quad (0.367)$$

$$25. (a) \quad P(A \cap B) = P(A) \times P(B) = 0.6x$$

$$(b) \quad (i) \quad P(A \cup B) = P(A) + P(B) - P(A)P(B) \\ \Leftrightarrow 0.80 = 0.6 + x - 0.6x \Leftrightarrow 0.2 = 0.4x \Leftrightarrow x = 0.5$$

$$(ii) \quad P(A \cap B) = 0.3$$

$$(c) \quad P(A \cap B) \neq 0$$

$$26. \quad \text{Let } P(A) = x \text{ then } P(B) = 3x \quad \text{and} \quad P(A \cap B) = P(A) \times 3P(A) = 3x^2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Leftrightarrow 0.68 = x + 3x - 3x^2$$

$$3x^2 - 4x + 0.68 = 0 \Leftrightarrow x = 0.2 \quad (x = 1.133, \text{ not possible})$$

$$P(B) = 3x = 0.6$$

$$27. (a) \quad P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{3}{11} + \frac{4}{11} - \frac{6}{11} = \frac{1}{11} \quad (0.0909)$$

$$(b) \quad \text{For independent events, } P(A \cap B) = P(A) \times P(B) = \frac{3}{11} \times \frac{4}{11} = \frac{12}{121} \quad (0.0992)$$

28.

$$(a) \quad (i) \quad \text{Use of } P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ 0.6 = 0.5 + 0.3 - P(A \cap B)$$

$$P(A \cap B) = 0.2$$

$$(ii) \quad P(A)P(B) = 0.15 \neq P(A \cap B)$$

Hence not independent

$$(b) \quad \text{Use of } P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{0.2}{0.5} \\ = 0.4$$

29. As $P(A|B) = P(A)$ then A and B are independent events

$$\text{Using } P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$$

$$\text{to obtain } 0.8 = 0.6 + P(B) - 0.6 \times P(B)$$

$$0.8 = 0.6 + 0.4P(B) \Rightarrow P(B) = 0.5$$

OR by using Venn diagram

30.

$$(a) \quad P(A \cap B) = P(A) \times P(B|A) \\ = \frac{1}{5} \times \frac{1}{4} \left(= \frac{1}{20} \right)$$

$$(b) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ \Rightarrow P(B) = \frac{7}{10} - \frac{1}{5} + \frac{1}{20} \\ = \frac{11}{20}$$

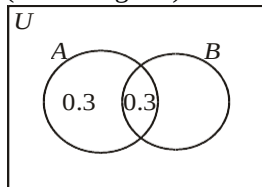
(c) **METHOD 1**

$$P(B) = \frac{11}{20} \text{ and } P(B|A) = \frac{1}{4}$$
$$P(B) \neq P(B|A)$$
$$\Rightarrow A \text{ and } B \text{ are not independent}$$

METHOD 2

$$P(A) \times P(B) = \frac{1}{5} \times \frac{11}{20} = \frac{11}{100} \text{ and } P(A \cap B) = \frac{1}{20}$$
$$P(A \cap B) \neq P(A) \times P(B)$$
$$\Rightarrow A \text{ and } B \text{ are not independent}$$

31. (Venn diagram)



$$P(A \cap B) = P(A)P(B) \quad 0.3 = 0.6 \times P(B)$$

$$P(B) = 0.5$$

$$\text{Therefore, } P(A \cup B) = 0.8$$

32. (a) $0.88 = 0.4 + P(B) - 0.4P(B)$

$$0.6P(B) = 0.48 \Rightarrow P(B) = 0.8$$

(b) $P(A \cup B) - P(A \cap B) = 0.88 - 0.32 = 0.56$

33. for independence $P(A \cap B) = P(A) \times P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$$

$$0.52 = P(A) + 2P(A) - 2P(A)P(A)$$

$$P(B) = 0.4$$

34. Total number of possible outcomes = 36

(i) $P(E) = P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + P(5, 5) + P(6, 6) = \frac{6}{36}$

(ii) $P(F) = P(6, 4) + P(5, 5) + P(4, 6) = \frac{3}{36}$

(iii) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$P(E \cap F) = \frac{1}{36}$$

$$P(E \cup F) = \frac{1}{36} + \frac{3}{36} - \frac{1}{36} \left(= \frac{8}{36} = \frac{2}{9}, 0.222 \right)$$

35. Sample space = {(1, 1), (1, 2) ... (6, 5), (6, 6)}

(This may be indicated in other ways, e.g. a grid or a tree diagram, partly or fully completed)

(a) $P(S < 8) = 21/36 = 7/12$

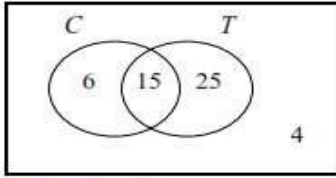
(b) $P(\text{at least one } 3) = \frac{11}{36}$

(c) $P(\text{at least one } 3 | S < 8) = 7/21 = \frac{1}{3}$

B. Exam style questions (LONG)

36.

(a) **METHOD 1**



$$P(T \cap C) = 0.3$$

METHOD 2

$$n(T \cup C) = n(T) + n(C) - n(T \cap C)$$

$$46 = 40 + 21 - n(T \cap C)$$

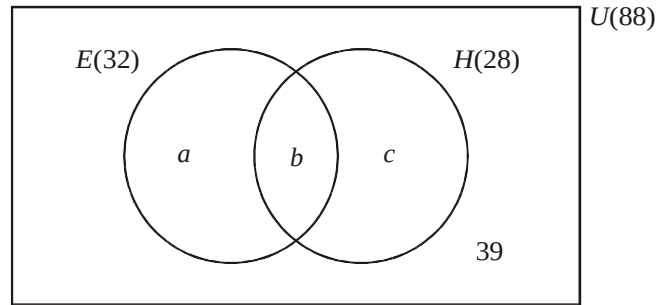
$$n(T \cap C) = 15$$

$$P(T \cap C) = 0.3$$

$$(b) \quad P(T|C) = \frac{P(T \cap C)}{P(C)}$$

$$= \frac{0.3}{0.42} \left(= \frac{5}{7}, 0.714 \text{ to 3 s.f.} \right)$$

37. (a)



$$n(E \cup H) = a + b + c = 88 - 39 = 49$$

$$n(E \cup H) = 32 + 28 - b = 49 \Leftrightarrow b = 11$$

$$a = 32 - 11 = 21$$

$$c = 28 - 11 = 17$$

$$(b) \quad (i) \quad P(E \cap H) = \frac{11}{88} = \frac{1}{8}$$

$$(ii) \quad P(H'|E) = \frac{P(H' \cap E)}{P(E)} = \frac{\frac{21}{88}}{\frac{32}{88}} = \frac{21}{32} \quad (= 0.656) \quad \text{Or directly} = \frac{21}{32}$$

$$(c) \quad (i) \quad P(\text{none in economics}) = \frac{56 \times 55 \times 54}{88 \times 87 \times 86} = 0.253$$

$$(ii) \quad P(\text{at least one}) = 1 - 0.253 = 0.747$$

OR

$$3 \left(\frac{32}{88} \times \frac{56}{87} \times \frac{55}{86} \right) + 3 \left(\frac{32}{88} \times \frac{31}{87} \times \frac{56}{86} \right) + \frac{32}{88} \times \frac{31}{87} \times \frac{30}{86} = 0.747$$

38. (a) $P(F \cup S) = 1 - 0.14 (= 0.86)$

$$P(F \cap S) = 0.93 - 0.86 = 0.07$$

Note: You can use *Venn Diagram*

(b) $P(F | S) = \frac{P(F \cap S)}{P(S)} = \frac{0.07}{0.62} = 0.113$

(c) F and S are **not** independent

EITHER

If independent $P(F | S) = P(F)$, $0.113 \neq 0.31$

OR

If independent $P(F \cap S) = P(F)P(S)$, $0.07 \neq 0.31 \times 0.62 (= 0.1922)$

(d) Let $P(F) = x$

$$P(S) = 2P(F) = 2x$$

$$P(F \cup S) = P(F)P(S) - P(F)P(S) \Leftrightarrow 0.86 = x + 2x - 2x^2 \Leftrightarrow 2x^2 - 3x + 0.86 = 0$$

$$x = 0.386, x = 1.11$$

$$P(F) = 0.386$$

39. (a) (i) $P(A) = \frac{80}{210} = \left(\frac{8}{21} = 0.381 \right)$

(ii) $P(\text{year 2 art}) = \frac{35}{210} = \left(\frac{1}{6} = 0.167 \right)$

(iii) No (the events are not independent)

EITHER $P(A \cap B) = P(A) \times P(B)$ (to be independent)

$$P(B) = \frac{100}{210} \left(= \frac{10}{21} = 0.476 \right) \text{ but } \frac{1}{6} \neq \frac{8}{21} \times \frac{10}{21}$$

OR $P(A) = P(A | B)$ (to be independent)

$$P(A | B) = \frac{35}{100} \text{ but } \frac{8}{21} \neq \frac{35}{100}$$

OR $P(B) = P(B | A)$ (to be independent)

$$P(B) = \frac{100}{210} \left(= \frac{10}{21} = 0.476 \right), P(B | A) = \frac{35}{80} \text{ but } \frac{35}{80} \neq \frac{100}{210}$$

(b) $n(\text{history}) = 85$

$$P(\text{year 1} | \text{history}) = \frac{50}{85} = \left(\frac{10}{17} = 0.588 \right)$$

(c) $\left(\frac{110}{210} \times \frac{100}{209} \right) + \left(\frac{100}{210} \times \frac{110}{209} \right) - \left(2 \times \frac{110}{210} \times \frac{100}{209} \right) = \frac{200}{399} (= 0.501)$

40. (a) (i) Venn diagram, 30

(ii) 45

(b) (i) $\frac{70}{100} \left(= \frac{7}{10} \right)$

(ii) $\frac{45}{70} \left(= \frac{9}{14} \right)$

(c) $P(A \cap B) = 0.3 \neq 0$

(d) $P(A \cap B) \neq P(A) \times P(B), \frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}, \frac{30}{55} \neq \frac{75}{100}$

OR $P(B|A) \neq P(B), \frac{30}{55} \neq \frac{75}{100}$

41. (a) (i) $s = 1$

(ii) $q = 5$

(iii) $p = 7, r = 3$

(b) (i) $P(\text{art}|\text{music}) = \frac{5}{8}$

(ii) **METHOD 1**

$P(\text{art}) = \frac{12}{16} \left(= \frac{3}{4} \right) \neq \frac{5}{8}$ the events are not independent

METHOD 2

$P(\text{art}) \times P(\text{music}) = \frac{96}{256} \left(= \frac{3}{8} \right) \neq \frac{12}{16} \times \frac{8}{16} = \frac{5}{16}$ the events are not independent

(c) $P(\text{first takes only music}) = \frac{3}{16}$

$P(\text{second takes only art}) = \frac{7}{15}$

$P(\text{music and art}) = \frac{3}{16} \times \frac{7}{15} = \frac{21}{240} \left(= \frac{7}{80} \right)$

42. (a) (i) 250 (ii) 166 (since $1000/6=166.67$) (iii) 83

(b) $250/1000 = 1/4$

(c) $166/1000$

(d) $83/1000$

(e) $250-83 = 167$ so $P=167/1000$.

(f) $250 + 166 - 83 = 333$ so $P=333/1000$

(g) $250 + 166 - 2 \times 83 = 250$ so $P=250/1000$.