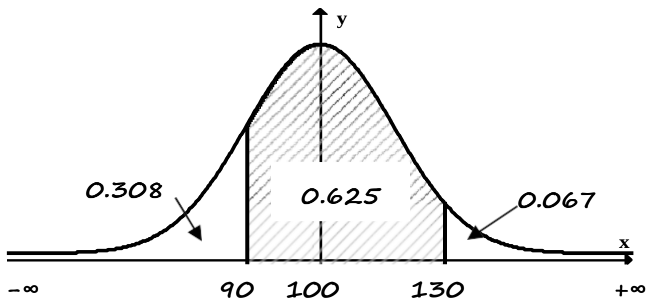
EXERCISES [MAA 4.11]
NORMAL DISTRIBUTION
SOLUTIONS

O. Practice questions

1. (a) $P(X < 90) = 0.308$ $P(90 < X < 130) = 0.625$ $P(X > 130) = 0.067$

(b)



2. (a) (use tail left, area 0.8) $a = 116.8$

(b) (use tail right, area 0.3) b=110.5

(c) (use tail central, area 0.4) c =89.5 d= 110.4

(d) (use tail central, area 0.5) Q_1 =86.5 and Q_3 = 113.5

3. (a) The standardised value of 100 is 0

(b) The standardised value of 90 is -0.5

The standardised value of 130 is 1.5

There are three ways to find them:

EITHER by observation

90 is 0.5 st. deviations below the mean, 130 is 1.5 st. deviations above the mean

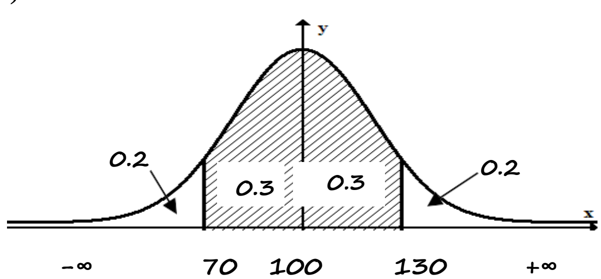
OR using the formula $Z = \dfrac{X-\mu}{\sigma} = \dfrac{X-100}{20}$

For 90, $Z = \dfrac{90-100}{20} = -0.5$, For 130, $Z = \dfrac{130-100}{20} = 1.5$

OR by GDC

Ncd (lower = 90, upper = 130), EXE gives the standardised values as well.

4. (a)



(b) (i) $P(X < 130) = 0.8$ (ii) $P(X < 70) = 0.2$

(iii) $P(100<X < 130) = 0.3$ (iv) $P(70<X < 130) = 0.6$

5. $\sigma = 10$, $1.12 \times 10 = 11.2$

(a) $100 + 11.2 = 111.2$

(b) $100 - 11.2 = 88.8$

6. $\dfrac{X-\mu}{\sigma} = z \Leftrightarrow \dfrac{130-\mu}{20} = 0.842$ (using standardised InvN with area 0.8 and tail left)

$\Leftrightarrow \mu = 113.2$

7. $\dfrac{X-\mu}{\sigma} = z \Leftrightarrow \dfrac{130-100}{\sigma} = 0.842$ (using standardised InvN with area 0.8 and tail left)

$\Leftrightarrow \sigma = 35.6$

8. We use $\dfrac{X-\mu}{\sigma} = z$ twice.

$\dfrac{60-\mu}{\sigma} = -0.84162 \Leftrightarrow 60-\mu = -0.84162\sigma \Leftrightarrow \mu - 0.84162\sigma = 60$

$\dfrac{130-\mu}{\sigma} = 1.28155 \Leftrightarrow 130-\mu = 1.28155\sigma \Leftrightarrow \mu + 1.28155\sigma = 130$

System gives $\mu = 87.7$ and $\sigma = 33.0$

9. (a) $\mathrm{P}(X = 102) = 0$

(b) $\mathrm{P}(101.5 \le X < 102.5) = 0.0736$

10. (a) $\mathrm{P}(X > 102) = 0.34457.. \cong 0.345$

(b) (i) $\mathrm{P}(X > 102 \mid X > 100) = \dfrac{\mathrm{P}(X > 102)}{\mathrm{P}(X > 100)} = \dfrac{0.34457}{0.5} = 0.172$

(ii) $\mathrm{P}(X < 102 \mid X > 100) = \dfrac{\mathrm{P}(100 < X < 102)}{\mathrm{P}(X > 100)} = \dfrac{0.15542}{0.5} = 0.0777$

11. (a) $P(X > 230) = 0.02275$

(b) (i) $(0.02275)^2 = 0.000518$ (ii) $(0.02275) \times (1 - 0.02275) \times 2 = 0.0445$

(c) Y follows $B(n, p)$ with $n = 7$ n = 7, $p = 0.02275$

$P(Y = 2) = 0.00969$

A. Exam style questions (SHORT)

12. P(180 < X < 280) = 0.645

13. P $(27 \le X \le 34) =$ 0.910 (3 s.f.) (accept 0.911)

14. Let X be the random variable "the diameter of the disc," then $X \sim \mathrm{N}(10, 0.1^2)$.

Using a graphic display calculator, $\mathrm{P}(X < 9.8) = 0.0228$

15. P(0.74 < X < 0.95) = 0.586

16. (a) $a = 1026$

(b) $b = 1008.8$

(c) $c = 958$ and $d = 1042$

(d) IQR=1033.7 - 966.3 = 67.4

17. (a) 0.773

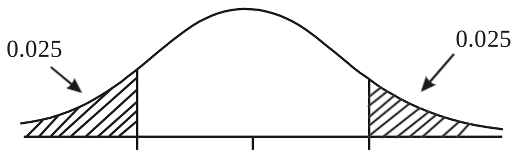
(b) $d = 161$

18. (a) 0.159

(b) $d = 136$

19. (a) $P(M \geq 350) = 0.0912$

(b)



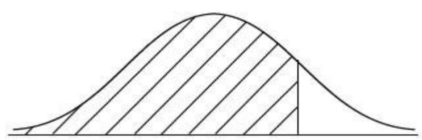
either tail left, tail right for each endpoint
or directly tail central with Area = 0.95

$251 < M < 369$

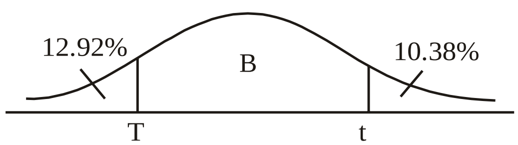
20. (a) 0.0668

(b) $k = 26.1$ kg

(c)



21. (a)



(b) $r = 6.56$ $t = 7.16$

22. (a) $P(90 < X < 125) = 0.701$

70.1 percent of the population (accept 70 percent).

(b) $P(X \geq 125) = 0.0478$

P(both persons having IQ ≥ 125) $= (0.0478)^2 = 0.00228$

23. (a) $M \sim N(750, 25^2)$

$P(M < 740 \text{ g}) = 0.345$

(b) $P(\text{both} < 740) = 0.345^2 = 0.119$

(c) $x = 737$ g

24. (a) 0.345 (34.5%)

(b) $P(2\,300 < X < 3\,300) = 0.645$

$P(\text{both}) = (0.645)^2 = 0.416$

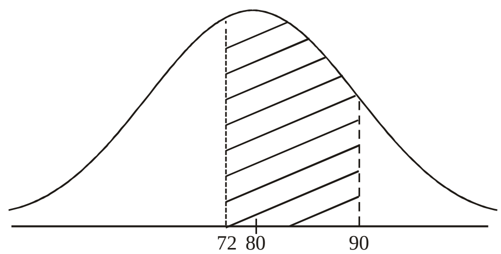
(c) $d = \$\,3\,325$ (= \$ 3 330 to 3 s.f.) (Accept \$3325.07)

25. $X \sim N(80, 8^2)$

(a) $P(X < 72) = 0.159$

(b) (i) $P(72 < X < 90) = 0.736$

(ii)



(c) $x = 66.0$ months

26. (a) $P(X > 45) = 0.8943... \cong 0.894$

(b) $P(X > 55 \mid X > 45) = \dfrac{P(X > 55)}{P(X > 45)} = \dfrac{0.1056..}{0.8943..} = 0.118$

27. $P(H > 197) = 0.159 = 15.9\%$

(b) 99% of heights under 209.6 = 210 cm (3 sf)

Height of standard doorway = 210 + 17 = 227 cm

28. (a) $P(X > 350) = 0.10564977... \cong 0.106$

$1000 \times 0.106 = 106$

(b) (i) $(0.1056)^2 = 0.0112$

(ii) $0.1056 \times (1 - 0.1056) \times 2 = 0.189$

(iii) $0.0112 + 0.189 = 0.2$

(c) Binomial with $n = 8$ and $p = 0.1056$

(i) $P(Y = 4) = 0.00557$ (ii) $P(Y \geq 1) = 0.591$

29. Due to symmetry, we are able to find the probabilities of this question (a diagram helps).

(a) $p = 0.94$

(b) $D = 10$

(c) $P(17 < H < 24) = 0.44$

E(trees) = $200 \times 0.44 = 88$

30. (a) P(weight >525 grams) = 0.0668

Expected number = $2000 \times 0.0668 = 134$

(b) Inverse Normal: $P(X < Q_3) = 0.75$ **OR** $P(X > Q_3) = 0.25$

$Q_3 = 484$

(c) A new variable Y is defined which follows $B(n,p)$ with $n = 40$ and $p = 0.0668$

(i) $P(Y = 0) = 0.0629$

(i) $P(Y \leq 3) = 0.723$

31. (a) $P(T < 40) = 0.762$ [use Ncd]

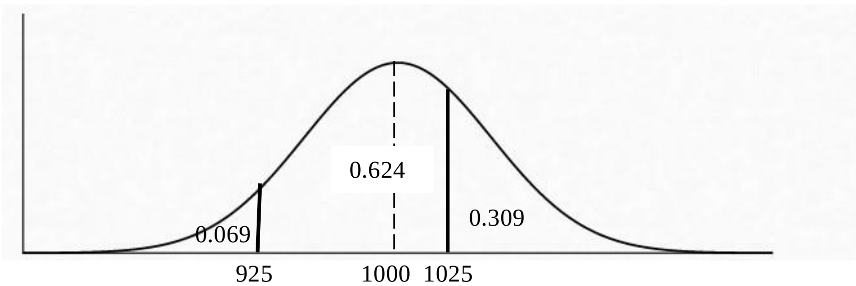
(b) $P(T < t) = 0.90 \Rightarrow t = 44.0$ (min) [use InvN]

(c) $X \sim B(10, 0.762\ldots)$

$P(X = 6) = 0.131$

32. (a) (i) $P(X < 925)=0.0668$ (ii) $P(925 < X < 1025)=0.624$ (iii) $P(X > 1025)=0.309$

(b)



(c) -1.5 and 0.5 respectively

(d) similar to the above, but the boundaries on *x*-axis are -1.5 and 0.5

33. (a) $P(H < 153) = 0.705$

Standardizing $\dfrac{153-\mu}{5} = 0.5388...$

$\mu = 150.30\ldots = 150$ (to 3sf)

(b) 0.128

34. $P(W < 220) = 0.88$

GDC – InvN gives $z = 1.175...$

$$z = \frac{x-\mu}{\sigma} \Leftrightarrow 1.175 = \frac{220-200}{\sigma} \Leftrightarrow \sigma = 17 \text{ (grams)}$$

35. (a) $0.84... = \dfrac{23.7-21}{\sigma} \Rightarrow \sigma = 3.21$

(b) (i) $P(X < 25.4) = 0.915$

(ii) By symmetry: $b = 21 - 4.4 \Rightarrow b = 16.6$

36. $P(Z<z) = 0.017$ $z = -2.12$

But $z = \dfrac{x-\mu}{\sigma} = \dfrac{1-1.02}{\sigma}$ where $x = 1$ kg

Therefore $\dfrac{1-1.02}{\sigma} = -2.12 \Leftrightarrow \sigma = 0.00943$ kg $= 9.4$ g

37. Let $E(X) = \mu$., $z_1 = 0.44$, $z_2 = 1.53$

$10 = \mu + 0.44\sigma$

$12 = \mu + 1.53\sigma$

$$\mu = \frac{1.53\times 10 - 0.44\times 12}{1.53 - 0.44} \Rightarrow E(X) = 9.19$$

38. $Z_1 = \dfrac{X-\mu}{\sigma} \Rightarrow \dfrac{100-\mu}{\sigma} = -0.9542 \Rightarrow \mu - 0.9542\sigma = 100$

$Z_2 = \dfrac{X-\mu}{\sigma} \Rightarrow \dfrac{200-\mu}{\sigma} = 0.1764 \Rightarrow \mu + 0.1764\sigma = 200$

$\mu = 184.4 \quad \sigma = 88.45$

39. $X \sim N(\mu, \sigma^2)$,

$\dfrac{3-\mu}{\sigma} = -0.8416, \quad \dfrac{8-\mu}{\sigma} = 1.282$

$3-\mu = -0.8416\sigma$

$8-\mu = 1.282\sigma$

$\sigma = 2.35, \qquad \mu = 4.99$

40. $P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$

$1.036 = \dfrac{90-\mu}{\sigma}, -1.175 = \dfrac{40-\mu}{\sigma}$

$\mu = 66.6, \; \sigma = 22.6$

41.

$P(X > 10.5) = 0.02$

$\Rightarrow \dfrac{10.5-\mu}{\sigma} = 2.05\ldots$

$P(X < 9.5) = 0.04$

$\Rightarrow \dfrac{9.5-\mu}{\sigma} = -1.75\ldots$

$10.5-\mu = 2.054\sigma$

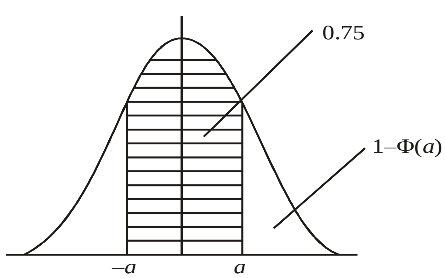
$9.5-\mu = -1.751\sigma$

$\Rightarrow 1 = 3.805\sigma$

$\Rightarrow \sigma = 0.263$

$\Rightarrow \mu = 9.96$

42.



From the diagram using inverse normal, tail central, area = 0.75

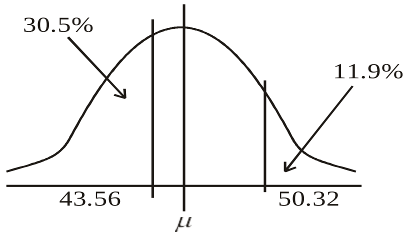
$a = 1.15$

43. $|X-25| < 3 \Leftrightarrow -3 < X-25 < 3 \Leftrightarrow 22 < X < 28$

St. deviation: $\sigma = 2$

$P(|X-25| < 3) = P(22 < X < 28) = 0.866$

44. (a)



$z_1 = 1.18$ $z_2 = -0.51$

$$\frac{50.32-\mu}{\sigma} = 1.18 \text{ and } \frac{43.56-\mu}{\sigma} = -0.51$$

Solving simultaneously

$\Rightarrow 50.32 = \mu + 1.18\sigma$ and $43.56 = \mu - 0.51\sigma$

$\Rightarrow 1.69\sigma = 6.76 \Rightarrow \sigma = 4 \Rightarrow \mu = 45.6$

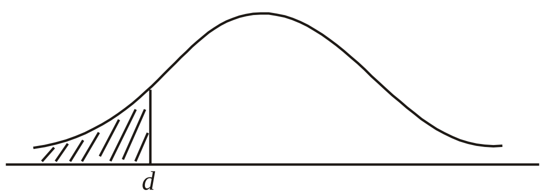
(b) $P(|X-\mu| < 5) = P(40.6 < x < 50.6) = 0.789$

B. Exam style questions (LONG)

45. $X \sim N(7, 0.5^2)$

(a) (i) $P(X < 8) = 0.977$ (ii) $P(6 < X < 8) = 0.95$

(b) (i)



(ii) $d = 6.18$

(c) $Y \sim N(\mu, 0.5^2)$, $P(Y < 5) = 0.2$

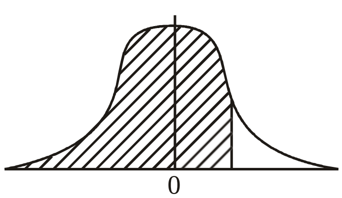
$$\frac{5-\mu}{0.5} = -0.8416 \qquad \mu = 5.42$$

46. (a) Let X be the lifespan in hours, $X \sim N(57, 4.4^2)$

(i) $a = -0.455$ (3 sf) $b = 0.682$ (3 sf)

(ii) (a) $P(X > 55) = 0.675$ (b) $P(55 \le X \le 60) = 0.428$ (3 sf)

(b) 90% have died $\Rightarrow$ shaded area = 0.9



$t = 62.6$ hours

47. (a) Area $A = 0.1$

(b) EITHER

Since $p\ (X \geq 12) = p\ (X \leq 8)$,

then 8 and 12 are symmetrical about the mean.

Thus mean $= \dfrac{8+12}{2} = 10$

OR

$\dfrac{12-\mu}{\sigma} = \dfrac{\mu-8}{\sigma} \Rightarrow 12 - \mu = \mu - 8 \Rightarrow \mu = 10$

(c) $\left(\dfrac{12-10}{\sigma}\right) = 1.282$

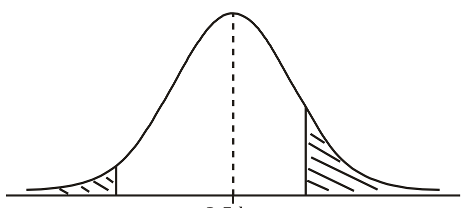
$\sigma = \dfrac{2}{1.282} = 1.56$ (3 sf)

(d) 0.739 (3 sf)

48. $W \sim \mathrm{N}(2.5, 0.3^2)$

(a) (i) 0.048 (ii) 0.159

(iii)



(iv) $P = 0.7936$

(b) (i) $X \sim \mathrm{B}(10, 0.7935...)$

$\mathrm{P}(X = 10) = (0.7935...)^{10}$ **OR** $\mathrm{P}(X = 10) = 0.0990$ (3 sf)

(ii) $X \sim \mathrm{B}(10, 0.7935...)$

$\mathrm{P}(X \geq 7) = 0.867$

49. (a) $\mathrm{P}(W < 25) = 0.0808$

(b) $\mu = 25.98 = 26.0$ (3 sf)

(c) Clearly, by symmetry $\mu = 25.5$

$\dfrac{25.0-25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma$

$\Rightarrow \sigma = 0.255$ kg

50. (a) $\mathrm{P}(X \leq 84) = 0.0524$

(b) $x = 81.4$ (accept 81)

(c) $\mathrm{P}(X \leq 84) = 0.12 \Rightarrow z = -1.1749...$

mean is 88.3 (accept 88)

51. Girls'height G ~ N(155,10^2), Boys'height B~ N(160,12^2)

(a) P(G > 170) = 0.0668

(b) x = 142

(c) r = 180 q = 140

(d) P(H > 170) = 0.60×0.0668 + 0.40×0.202 = 0.12088 = 0.121 (3 sf)

(e) $P(F \mid H > 170) = \dfrac{0.60 \times 0.0668}{0.121} = 0.332$

52. (a) $X \sim N(231, 1.5^2)$

$P(X < 228) = 0.0228$

(b) (i) $X \sim N(\mu, 1.5^2)$

$P(X < 228) = 0.002$ $\quad \dfrac{228-\mu}{1.5} = -2.878...$

$\mu = 232$ grams

(ii) $X \sim N(231, \sigma^2)$

$\dfrac{228-231}{\sigma} = -2.878...$

$\sigma = 1.04$ grams

(c) $X \sim B(100, 0.002)$

$P(X \geq 2) = 0.0174$

53. (a) (i) Let X be the random variable "the weight of a bag of salt".

Then $X \sim N(110, \sigma^2)$, where σ is the new standard deviation.

Given $P(X < 108) = 0.07$, let $Z = \dfrac{X-110}{\sigma}$

$\dfrac{-2}{\sigma} = -1.476 \Rightarrow \sigma = 1.355.$

(ii) Let the new mean be μ, then $X \sim N(\mu, 1.355^2)$.

$\dfrac{108-\mu}{1.355} = 1.751 \Rightarrow \mu = 110.37$

(b) If the mean is 110.37 g then

$X \sim N(110.37, 1.355)$, $P(A < X < B) = 0.8$

$A = 108.63$, $B = 112.11$

54. (a) B~N(1.63,0.16^2) , $P(B < 1.52) = 0.24588\ldots$ so 24.6%

(b) A~N(1.56,0.16^2) , $P(A < 1.52)$ 0.40129

P(d < 1.52) = 0.44 × 0.40129 + 0.56 × 0.24588 = 0.314 (3 s.f.)

(c) $P(\text{bolt produced by B} \mid d < 1.52) = \dfrac{0.56 \times 0.24588}{0.31426} = 0.438$

(d) $P(B > 1.83) = 0.10564$

$P(1.52 < B < 1.83) = 0.64846$

E(gain per bolt) = (0.24588)×(– 0.85) + (0. 64846)×(1.50) + (0.10564)×(0.50)= 0.81651

Expected gain = 8000 × 0.81651 = 6532 ($)

55. K = length of Karl's throw ~ N (59.50, 3.00^2)

I = length of Ian's throw. ~ N (60.33, 1.95^2)

(a) $P(K > 56) = 0.8783$ so 87.8%

(b) $P(I > x) = 0.80 \Rightarrow x = 58.69$ m

(c) (i) $Y \sim N(59.50, 3.00^2)$ $X \sim N(60.33, 1.95^2)$

$P(K \geq 65) = 0.0334$ $P(I \geq 65) = 0.00831$

Karl is more likely to qualify since $P(Y \geq 65) > P(X \geq 65)$

(ii) For both athletes Binomial with $n = 3$, $p_K = 0.0334$ $p_I = 0.00831$

P (Karl qualifies) = $P(X_K \geq 3) = 0.0969$

P(Ian qualifies) = $P(X_I \geq 3) = 0.0247$

P(both qualify) = (0.0969) (0.0247) = 0.00239