EXERCISES [MAA 4.10]

BINOMIAL DISTRIBUTION

SOLUTIONS

O. Practice questions

1. (a)

exactly 4 heads	0.273
exactly 3 heads	0.219
3, 4 or 5 heads	0.711
no heads	0.00391
always heads	0.00391
at most 2 heads	0.145
at least 3heads	0.855

(b)

E(X)	4	Var(X)	2

2. $np = 10$ and $np(1-p) = 6$. Hence $10(1-p) = 6 \Leftrightarrow p = 0.4$ and $n = 25$

3. B(160 , 0.015)

(a) $\mathrm{E}(X) = 160 \times 0.015 = 2.4$, $\mathrm{Var}(X) = 160 \times 0.015 \times 0.985 = 2.364$

(b) $\mathrm{P}(X = 0) = 0.0891$

(c) $\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 0.911$ or directly $\mathrm{P}(X \geq 1) = 0.911$

(d) For the mode we compare the neighbouring values to the expected value

$\mathrm{P}(X = 2) = 0.263$ $\mathrm{P}(X = 3) = 0.211$ so the mode is 2

Hence the required probability is $\mathrm{P}(X = 2) = 0.263$

4. (a) $\mathrm{E}(X) - \mathrm{Var}(X) = 0.8 \Leftrightarrow 0.4n - 0.24n = 0.8 \Leftrightarrow 0.16n = 0.8 \Leftrightarrow n = 5$

(b) $n = 11$

(c) $n = 15$

(d) $\mathrm{E}(X^2) = \mathrm{Var}(X) + \mathrm{E}(X)^2 \Leftrightarrow n(0.4)(0.6) + (0.4n)^2 = 18.4$, hence n=10

A. Exam style questions (SHORT)

5. $B(n, p)$ with $n = 5$ and $p = \frac{1}{2}$

(a) $\mathrm{P}(X = 3) = 0.3125 \ldots = 0.313$

(b) $\mathrm{P}(X \geq 1) = 0.969$

6. $B(n, p)$ with $n = 7$ and $p = 0.18$

(a) $\mathrm{P}(X = 2) = 0.252$

(b) $\mathrm{P}(X \geq 2) = 0.368$

7. $B(n, p)$ with $n = 100$ and $p = 0.04$

(a) mean = $np = 100 \times 0.04 = 4$

(b) $\mathrm{P}(X = 6) = 0.105$

(c) $\mathrm{P}(X \geq 1) = 0.983$

8. $X \sim B(100,0.02)$

(a) $E(X) = 100 \times 0.02 = 2$

(b) (i) $P(X = 3) = 0.182$

(c) $P(X > 1) = 0.597$

9. $p(\text{Red}) = \frac{35}{40} = \frac{7}{8}$ $p(\text{Black}) = \frac{5}{40} = \frac{1}{8}$

(a) $B(n, p)$ with $n = 8$, $p = \frac{1}{8}$

(i) $p(\text{one black}) = P(X = 1) = 0.393$ to 3 s.f. (ii) $p(\text{at least one black}) = P(X \geq 1) = 0.656$

(b) 400 draws: expected number of blacks = $\frac{400}{8} = 50$

10. $X \sim B(n, p)$ with $n = 5$ and $p = \frac{1}{3}$

Therefore $P(X = 3) = 0.165$

11. (a) Probability = 0.138

(b) Probability = $(0.6)^2 \times 0.4 = 0.144$ $\left(\text{or } \frac{18}{125}\right)$

12. (a) X is $B(10, 0.25)$

$E(X) = 10 \times 0.25 = 2.5$

(b) $P(X \leq 2) = 0.526$

13. X is Binomial $n = 5$ $p = 0.4$

$P(X \leq 3) = 0.913$ to 3 s.f.

14. (a) $B(n, p)$ with $n = 3$, $p = \frac{1}{3}$

(i) $P(X = 3) = 0.0370$ or $P(3H) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

(ii) $P(X = 3) = 0.222$ or $P(2H, 1T) = 3\left(\frac{1}{3}\right)^2 \frac{2}{3} = \frac{2}{9}$

(b) (i) expected number of heads = $np = \left(\frac{1}{3} \times 12\right) = 4$

(ii) 4 heads, so 8 tails

E(winnings) = $4 \times 10 - 8 \times 6$ (= 40 – 48) = –\$ 8

15. $B(n, p)$ with $n = 20$, $p = \frac{1}{4}$

(a) $E(X) = 20 \times \frac{1}{4} = 5$

(b) $P(X = 5) = 0.202$ to 3 s.f.

(c) $P(X < 5) = 0.415$ to 3 s.f. [less than five means $P(X \leq 4)$]

16. $B(n, p)$ with $n = 7$, $p = \frac{1}{5}$

(a) $P(X = 4) \cong 0.0287$

(b) $P(X \geq 4) \cong 0.0333$

(c) $Y \sim B(n, p)$ with $n = 10$, $p = 0.0333...$

$P(Y = 0) \cong 0.712$ **(or (** $(1 - 0.0333...)^{10} \cong 0.712$

17. $X \sim B(n, p)$ with $n = 1800$, $p = \frac{2}{3}$

(a) $E(X) = np = 1200$

(b) $SD(X) = \sqrt{np(1-p)} = \sqrt{1200 \times \frac{1}{3}} = 20$

$Y \sim B(n, p)$ with $n = 18$, $p = \frac{2}{3}$

(c) $P(Y > 9) = P(Y \geq 10) \cong 0.892$

18. (a) P(all ten cells fail) = 0.107 (or 0.8^{10})

(b) (satellite is still operating at the end of one year if $X \geq 1$

$P(X \geq 1) = 0.893$ (or 1 – 0.107= 0.893)

19. (a) mean = $10 \times 0.4 = 4$

(b) check $P(X = 3) = 0.214$, $P(X = 4) = 0.251$, $P(X = 5) = 0.201$

so mode = 4

(c) variance = $10 \times 0.4 \times 0.6 = 2.4$

(d) st. dev = $\sqrt{2.4} = 1.55$

20. (a) mean = $10 \times \frac{1}{4} = 2.5$

(b) check $P(X = 2) = 0.281$, $P(X = 3) = 0.250$

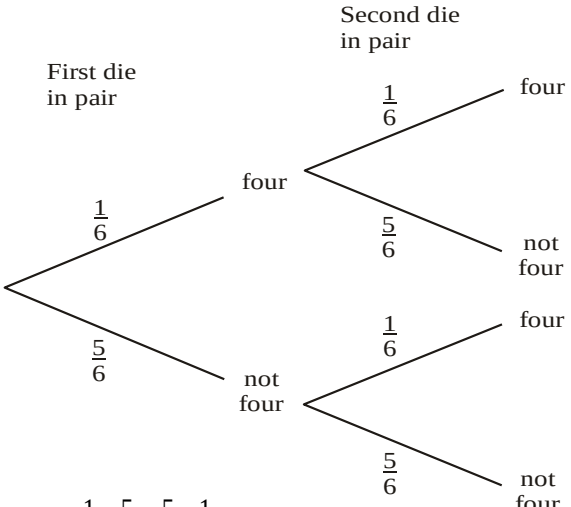
so mode = 2

(c) variance = $10 \times \frac{1}{4} \times \frac{3}{4} = \frac{15}{8} = 1.875$

(d) st. dev = $\sqrt{1.875} = 1.37$

B. Exam style questions (LONG)

21. (a)



(b) $P(E) = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \left(= \frac{5}{36} + \frac{5}{36} \right) = \frac{10}{36} \left(= \frac{5}{18} \text{ or } 0.278 \right)$

(c) $X \sim B\left(5, \frac{5}{18}\right)$

$P(X = 3) = 0.112$ [in fact $\binom{5}{3}\left(\frac{5}{18}\right)^3\left(\frac{13}{18}\right)^2 = 0.112$]

(d) $P(X \geq 3) = 0.135$

22. $B(n, p)$ with $n = 10$, $p = \frac{1}{4}$

(a) $E(X) = 10 \times \frac{1}{4} = 2.5$

(b) $P(X = 6) = 0.0162$

(c) $P(X \geq 2) = 0.756$

(d) Since $E(X) = 2.5$ the mode is 2 or 3

Using GDC

x	$P(X = x)$
1	0.188
2	0.282
3	0.250

From these values the most likely number of yellow ribbons is 2.

(e) The probability that a ribbon is yellow remains constant $\left(= \frac{1}{4}\right)$

23. $B(n, p)$ with $n = 20$, $p = 0.3$

(a) Mean = $20 \times 0.3 = 6$ Variance = $20 \times 0.3 \times 0.7 = 4.2$

(b) (i) $P(X = 5) = 0.179$ (ii) $P(4 \leq X \leq 8) = 0.780$

(c) 0.3

(d) $0.7 \times 0.7 \times 0.3 = 0.147$