

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches
MAA

EXERCISES [MAA 2.7]
ASYMPTOTES

O. Practice questions

1. [Maximum mark: 18] **[without GDC]**

Complete the following table by writing down the corresponding asymptotes

Function	Vertical asymptote	Horizontal asymptote
$f(x) = \frac{2x+7}{x-5}$	$x = 5$	$y = 2$
$f(x) = \frac{x+7}{2x-3}$		
$f(x) = \frac{-4x+1}{2x-6}$		
$f(x) = \frac{2}{x-5}$		
$f(x) = \frac{2}{x-5} + 1$		
$f(x) = \frac{2}{x-5} - 1$		
$f(x) = \frac{2x+7}{x-5} + 1$		
$f(x) = \frac{-4x+1}{2x-6} + 3$		
$f(x) = \frac{5-x}{5+x}$		
$f(x) = \frac{3x+134}{2x+5}$		

2. [Maximum mark: 12] *[with / without GDC]*

Let $f(x) = \frac{3x - 6}{x + 2}$

- x Write down the equations of the asymptotes. [2]
- xi Find the x - and y -intercepts. [2]
- xii Sketch the graph of the function. Indicate the information found in (a) and (b). [4]
- xiii Write down the domain and the range of the function. [2]
- xiv Find the point of intersection between the graph of f and the line $y = 1$. [2]

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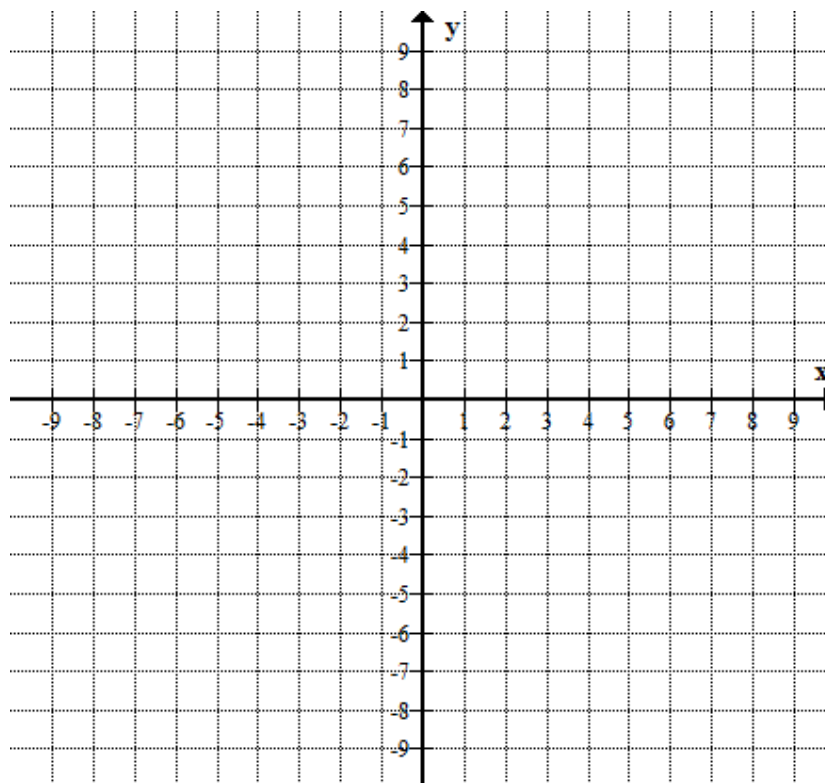
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3. [Maximum mark: 10] *[with / without GDC]*

Let $f(x) = \frac{3}{x-2}$

- (a) Write down the equations of the asymptotes. [2]
- (b) Find any x - or y -intercepts. [2]
- (c) Sketch the graph of the function. Indicate the information found in (a) and (b). [4]
- (d) Write down the domain and the range of the function. [2]

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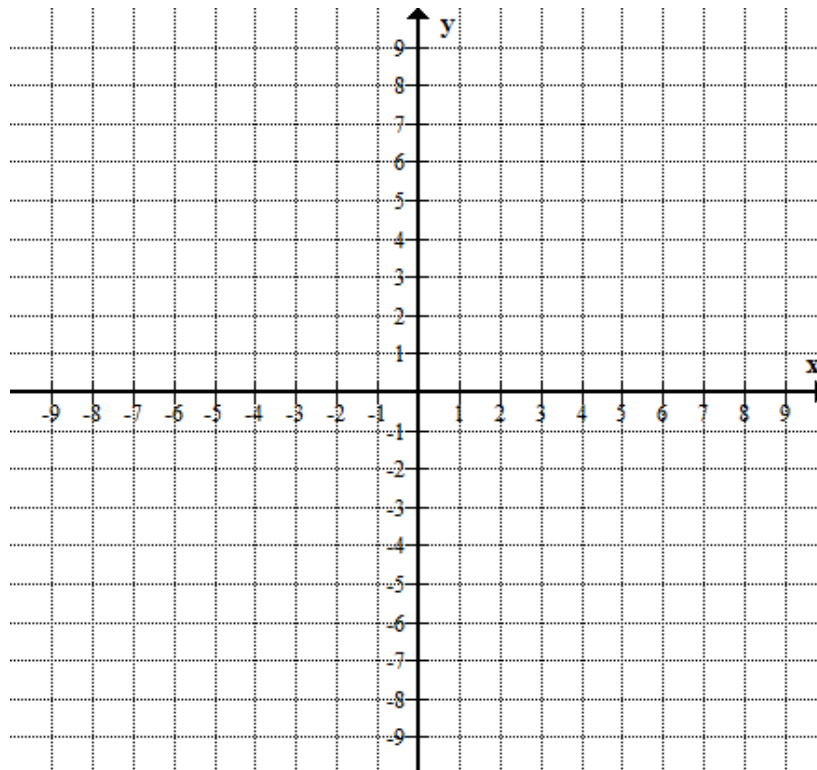
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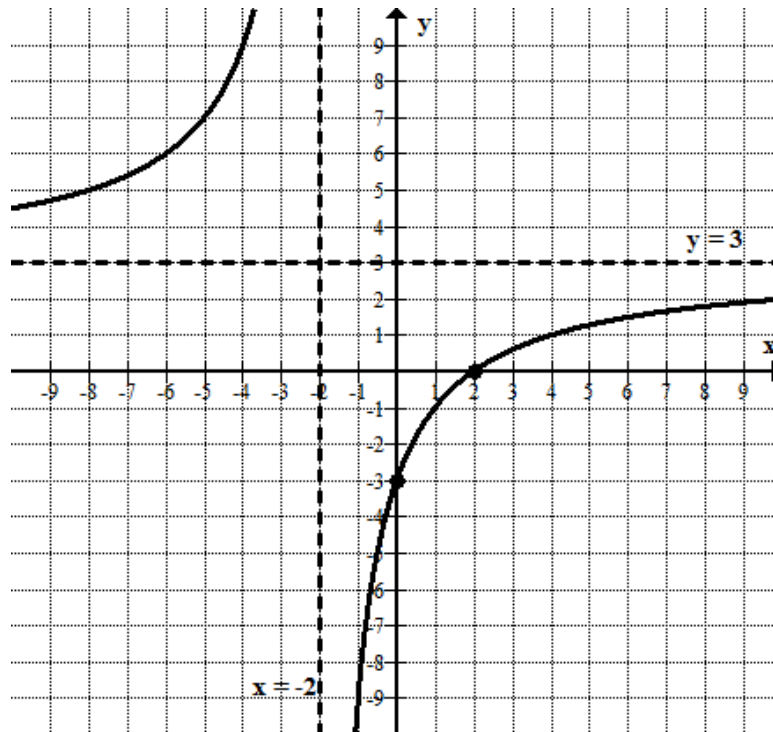
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4. [Maximum mark: 8] [without GDC]

The graph of the function $f(x) = \frac{ax+b}{x+c}$ is shown below.



- (a) Find the values of a, b, c . [4]
 (b) Find $f(4)$ and $f^{-1}(1)$. [2]
 (c) By observing the graph, solve the inequality $f(x) \leq 1$. [2]

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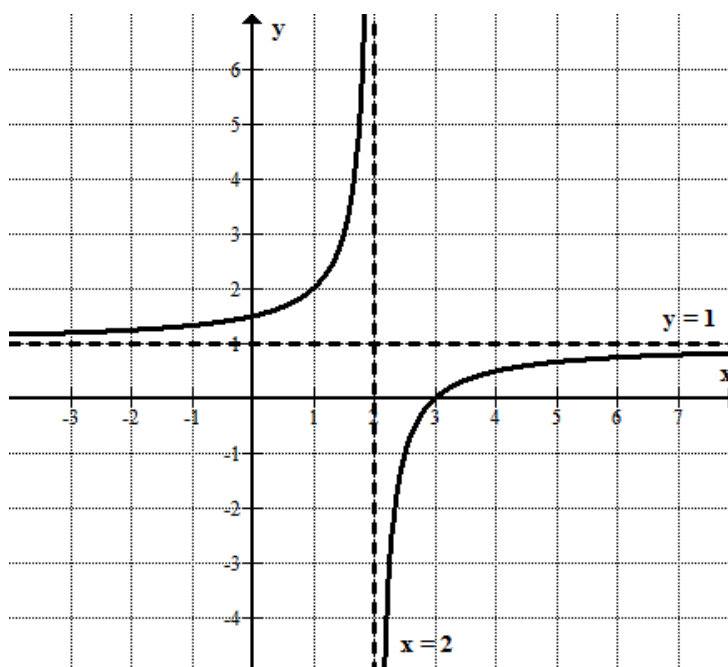
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5. [Maximum mark: 19] *[without GDC]*

The graph of the function $f(x) = \frac{x-6}{x-2}$ is shown below.



- (a) Write down the images of the horizontal asymptote $y = 1$ under the following transformations

[9]

$y = f(x) + 2$	$y = 3$	$y = f(x + 2)$	
$y = f(x) - 2$		$y = f(x - 2)$	
$y = 2f(x)$		$y = f(2x)$	
$y = f(x)/2$		$y = f(x/2)$	
$y = -f(x)$		$y = f(-x)$	

- (b) Write down the images of the vertical asymptote $x = 2$ under the following transformations

[10]

$y = f(x) + 2$		$y = f(x + 2)$	
$y = f(x) - 2$		$y = f(x - 2)$	
$y = 2f(x)$		$y = f(2x)$	
$y = f(x)/2$		$y = f(x/2)$	
$y = -f(x)$		$y = f(-x)$	

(b) Express the function $g(x) = \frac{x+10}{x+3}$ in the form $g(x) = A + \frac{B}{x+3}$. [2]

(c) Express the function $h(x) = \frac{2x+10}{x+3}$ in the form $h(x) = C + \frac{D}{x+3}$. [2]

(d) Complete the following table

	f	g	h
Vertical asymptote			
Horizontal asymptote			

[4]

[illegible]

7*. [Maximum mark: 6] *[without GDC]*

Let $g(x) = 3 + \frac{4}{x-2}$

(a) Write down the equations of the asymptotes of the graph of g . [2]

(b) Describe the sequence of transformations that map the graph of $f(x) = \frac{1}{x}$ into the graph of g . [4]

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8**. [Maximum mark: 7] *[without GDC]*

Let $h(x) = \frac{3x+1}{x-2}$

(a) Write down the equations of the asymptotes of the graph of h . [2]

(b) Describe the sequence of transformations that map the graph of $f(x) = \frac{1}{x}$ into the graph of h . [5]

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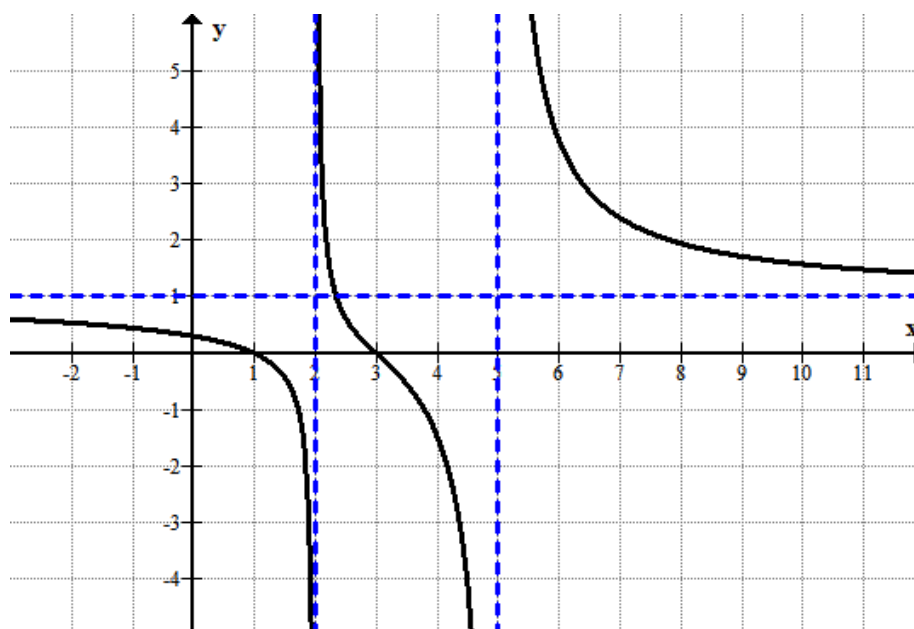
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9*. [Maximum mark: 7] [without GDC]

The graph of a function f is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

(b) Write down the equations of all vertical asymptotes. [1]

(c) Complete the following table

(1) $\lim_{x \rightarrow +\infty} f(x) =$	(4) $\lim_{x \rightarrow -\infty} f(x) =$
(2) $\lim_{x \rightarrow 2^-} f(x) =$	(5) $\lim_{x \rightarrow 2^+} f(x) =$
(3) $\lim_{x \rightarrow 5^-} f(x) =$	(6) $\lim_{x \rightarrow 5^+} f(x) =$

[3]

(d) State the results from question (c) which justify

(i) the horizontal asymptote

(ii) the vertical asymptotes.

[2]

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A. Exam style questions (SHORT)

10. [Maximum mark: 6] *[without GDC]*

Consider the graph of $f(x) = \frac{6x+1}{2x-4}$.

- (a) Write down the domain and the range of f . [2]
- (b) Write down the equation of
 - (i) the horizontal asymptote
 - (ii) the vertical asymptote. [2]
- (c) Find the x -intercept and the y -intercept. [2]

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11. [Maximum mark: 6] *[without GDC]*

Consider the graph of $f(x) = \frac{6}{2x-4}$.

- (a) Write down the domain and the range of f . [2]
- (b) Write down the equation of
 - (i) the horizontal asymptote
 - (ii) the vertical asymptote. [2]
- (c) Find the x -intercept and the y -intercept (if they exist). [2]

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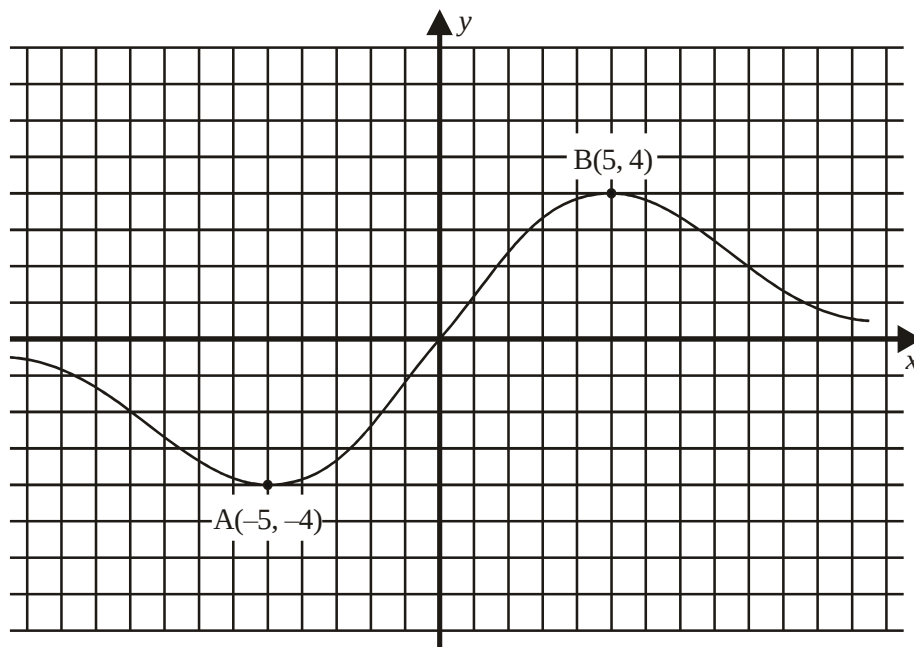
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12. [Maximum mark: 6] [without GDC]

The diagram shows the graph of $y = f(x)$, with the x -axis as an asymptote.



- (a) On the same axes, draw the graph of $y = f(x + 2) - 3$, indicating the coordinates of the images of the points A and B. [3]

- (b) Write down the equation of the asymptote

(i) for the graph of $y = f(x + 2)$.

(ii) for the graph of $y = f(x) - 3$.

(iii) for the graph of $y = f(x + 2) - 3$. [3]

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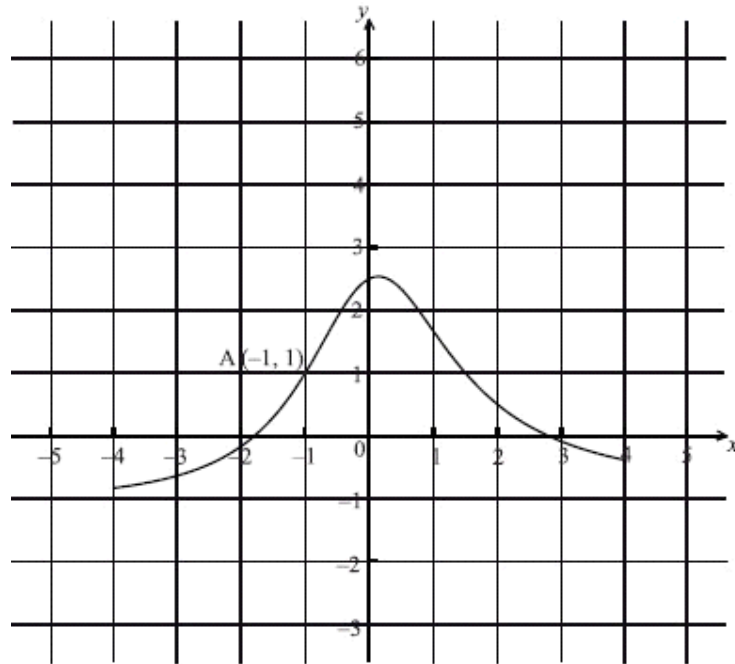
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13. [Maximum mark: 8] [without GDC]

The graph of a function f is shown in the diagram below. The point A $(-1, 1)$ is on the graph, and $y = -1$ is a horizontal asymptote.



- (a) Let $g(x) = f(x - 1) + 2$. On the diagram, sketch the graph of g . [3]
- (b) Write down the equation of the horizontal asymptote of g . [1]
- (c) Let A' be the point on the graph of g corresponding to point A. Write down the coordinates of A' . [2]
- (d) Write down the equation of the horizontal asymptote of
- (i) $y = f(2x)$. (ii) $y = 2f(x)$ [2]

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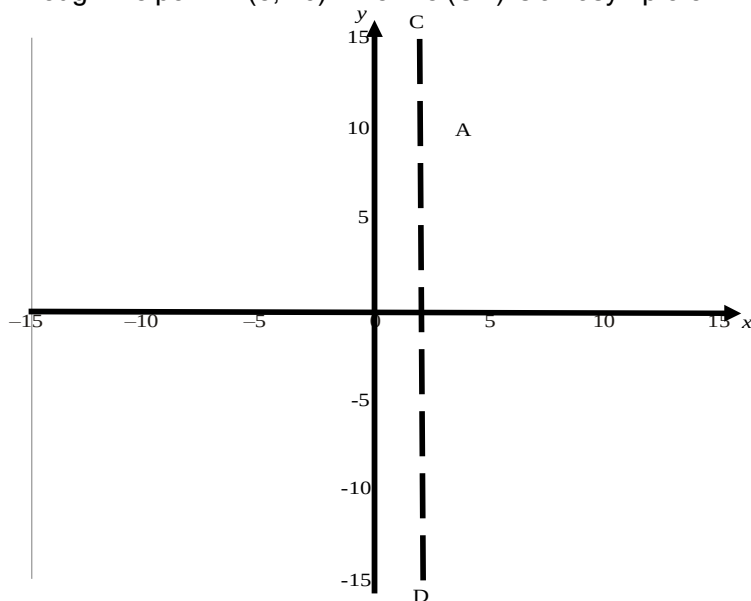
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14. [Maximum mark: 6] [without GDC]

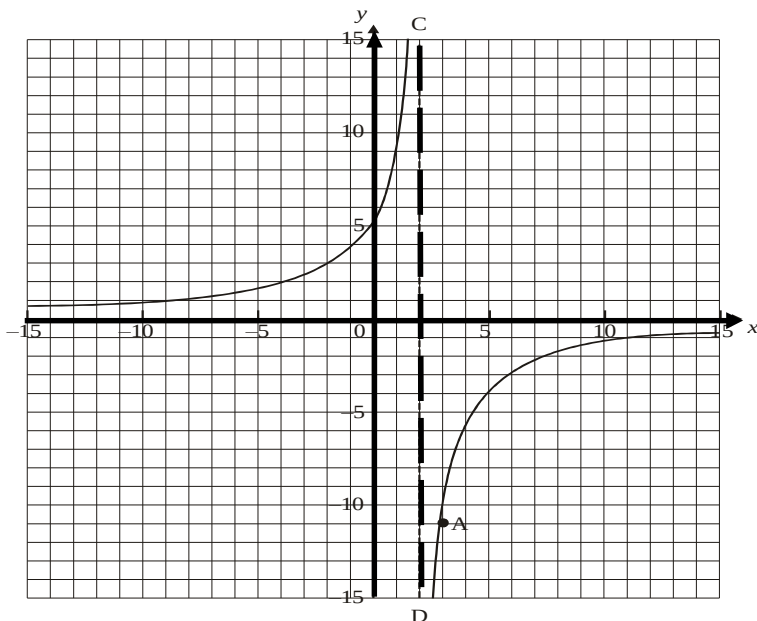
- (a) The diagram shows part of the graph of the function $f(x) = \frac{q}{x-p}$. The curve passes through the point A (3, 10). The line (CD) is an asymptote.



Find the value of (i) p (ii) q

[4]

- (b) The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to A' (3, -10).



Give a full geometric description of the transformation.

[2]

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15. [Maximum mark: 7] **[with / without GDC]**

The function $f(x)$ is defined as $f(x) = 3 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$.

(a) Sketch the curve of f for $-5 \leq x \leq 5$, showing the asymptotes. [3]

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x -intercept;

(iii) the value of the y -intercept. [4]

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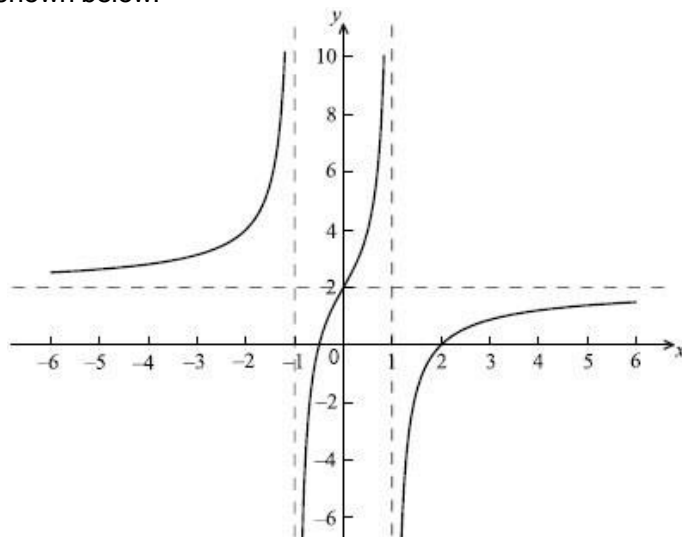
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16. [Maximum mark: 4] **[without GDC]**

Let $f(x) = p - \frac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$. Part of the graph of f , including the asymptotes, is shown below.



The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$.

Write down the value of (i) p (ii) q

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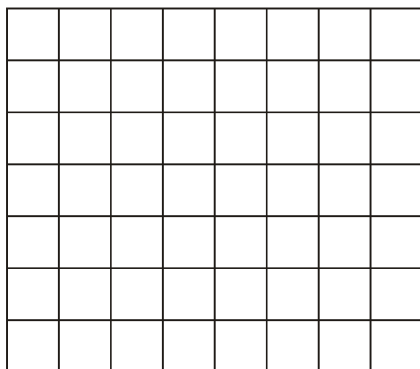
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17*. [Maximum mark: 6] **[with GDC]**

The function f is defined by $f(x) = \frac{3}{\sqrt{9 - x^2}}$, for $-3 < x < 3$.

(a) On the grid below, sketch the graph of f .

[2]



(b) Write down the equation of each vertical asymptote.

[2]

(c) Write down the range of the function f .

[2]

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