

[MAA 2.6] TRANSFORMATIONS

SOLUTIONS

O. Practice questions

1.

$y = f(x) + 5$	(1, 5.5)	$y = f(x + 5)$	(-4, 0.5)
$y = f(x) - 5$	(1, -4.5)	$y = f(x - 5)$	(6, 0.5)
$y = 5f(x)$	(1, 2.5)	$y = f(5x)$	(0.2, 0.5)
$y = f(x)/5$	(1, 0.1)	$y = f(x/5)$	(5, 0.5)
$y = -f(x)$	(1, -0.5)	$y = f(-x)$	(-1, 0.5)

2. (a)

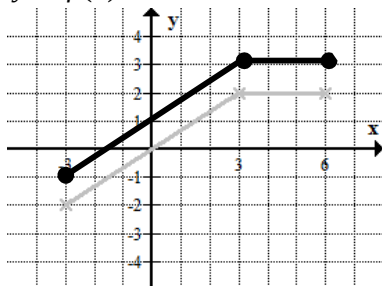
$y = f(x) + 3$	(-1, 6)	$y = f(x + 3)$	(-4, 3)
$y = f(x) - 3$	(-1, 0)	$y = f(x - 3)$	(2, 3)
$y = 3f(x)$	(-1, 9)	$y = f(3x)$	(-1/3, 3)
$y = f(x)/3$	(-1, 1)	$y = f(x/3)$	(-3, 3)
$y = -f(x)$	(-1, -3)	$y = f(-x)$	(1, 3)

(e) $f(x)$ (-1, 3)
 $2f(x)$ (-1, 6)
 $2f(x-3)$ (2, 6)
 $2f(x-3)+4$ (2, 10)

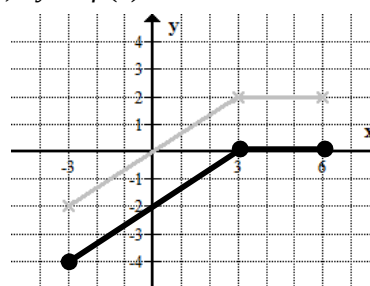
Hence the corresponding point is (2,10)

3. (a)

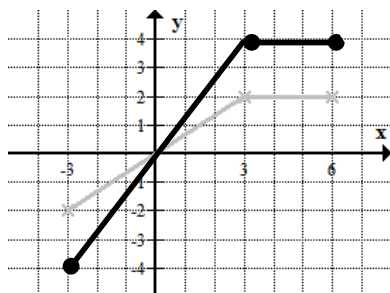
$$y = f(x) + 1$$



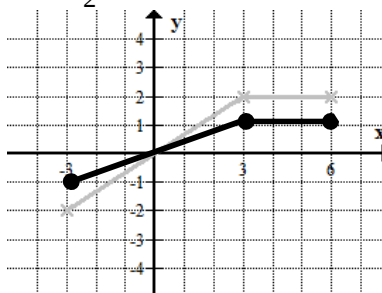
(b) $y = f(x) - 2$



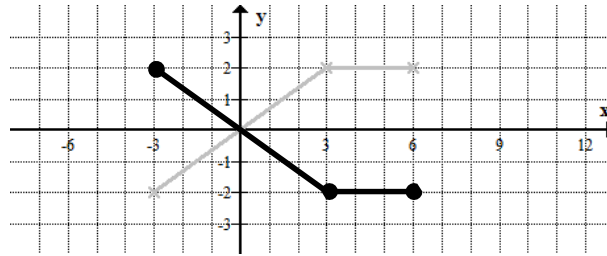
(c) $y = 2f(x)$



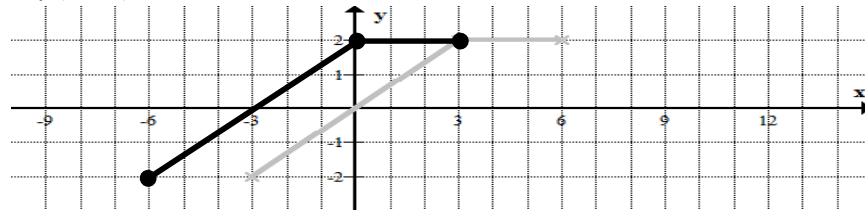
(d) $y = \frac{1}{2}f(x)$



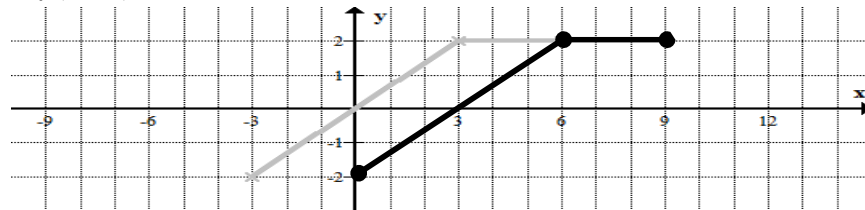
(e) $y = -f(x)$



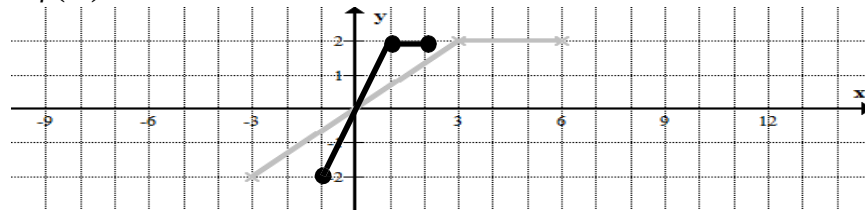
(f) $y = f(x+3)$



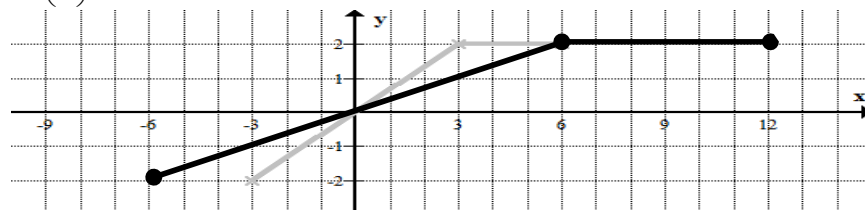
(g) $y = f(x-3)$



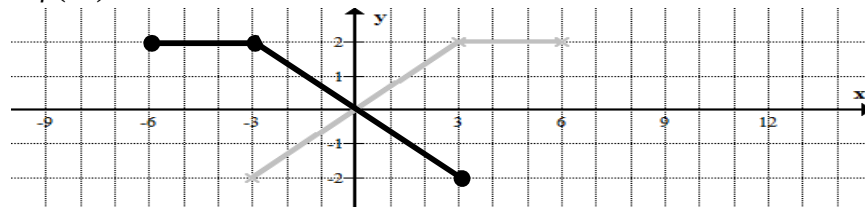
(h) $y = f(3x)$



(i) $y = f\left(\frac{x}{2}\right)$



(j) $y = f(-x)$



4. (a) $-f(x-2)+5$

$f(x)$	original
$-f(x)$	reflection in x -axis
$-f(x-2)$	horizontal translation 2 units to the right
$-f(x-2)+5$	vertical translation 5 units up

(b) $-3f(x+2)-1$

$f(x)$	original
$-f(x)$	reflection in x -axis
$-3f(x)$	vertical stretch with s.f. 3
$-3f(x+2)$	horizontal translation 2 units to the left
$-3f(x+2)-1$	vertical translation 1 unit down

(c) $f(2x-10)$

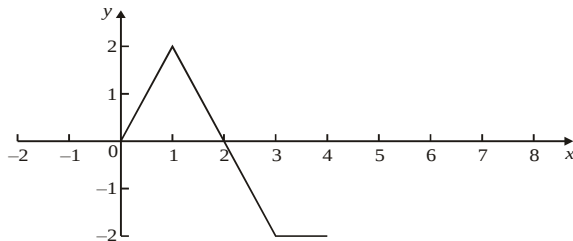
$f(x)$	original
$f(x-10)$	horizontal translation 10 units to the right
$f(2x-10)$	horizontal stretch with s.f. $1/2$ (i.e. shrink)

(d) $f(2(x-5))$

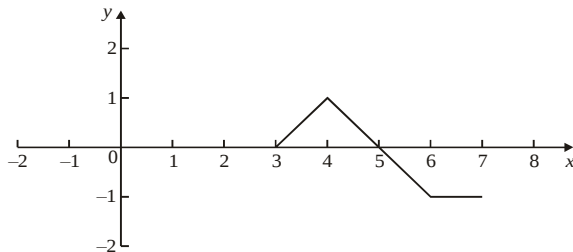
$f(x)$	original
$f(2x)$	horizontal stretch with s.f. $1/2$ (i.e. shrink)
$f(2(x-5))$	horizontal translation 5 units to the right

A. Exam style questions (SHORT)

5. (a) (i)



(ii)

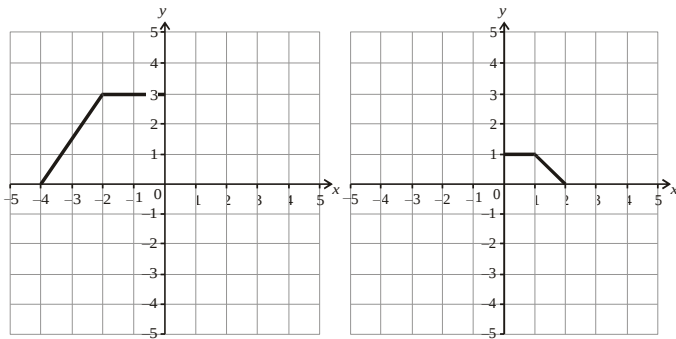


(b) $A'(3, 2)$ (Accept $x=3, y=2$)

6. (a) (I) D (ii) C (iii) A

(b) B: $f(x)+2$ E: $f(x-2)$

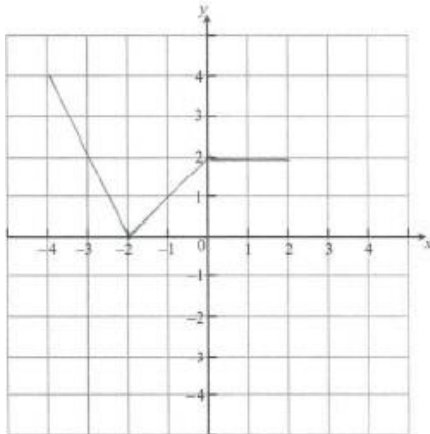
7. (a) (i) 1 (ii) 0.5
(b)



(c)

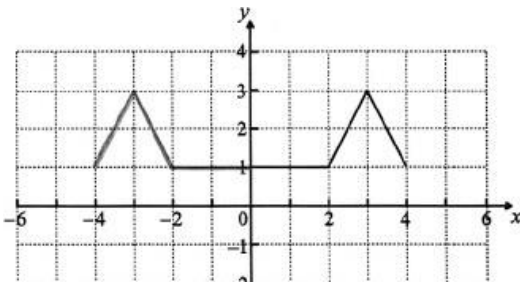
	$y = f(x)$	$y = 3f(-x)$	$y = f(2x)$
Domain	$0 \leq x \leq 4$	$-4 \leq x \leq 0$	$0 \leq x \leq 2$
Range	$0 \leq y \leq 1$	$0 \leq y \leq 3$	$0 \leq y \leq 1$

8. (a)



- (b) $x = 3 + 1, y = \frac{1}{2} \times 2$ P is (4, 1)

9. (a)



(b)

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	C
Maps f to $f(x) + 1$	D

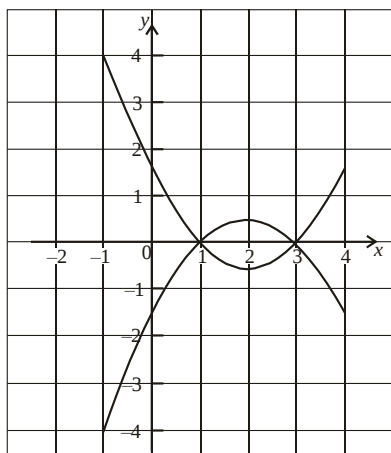
- (c) translation (move/shift/slide *etc.*) 6 units to the left and 2 units down

10. (a) By GDC the coordinates are $(-1, 1.66)$ [or $\left(-1, \frac{5}{3}\right)$]

[Notice: it can also be found by using derivatives later on]

- (b) (i) $(-3, -9)$
 (ii) $(1, -4)$
 (iii) reflection gives $(3, 9)$
 stretch gives $\left(\frac{3}{2}, 9\right)$

11. (a)



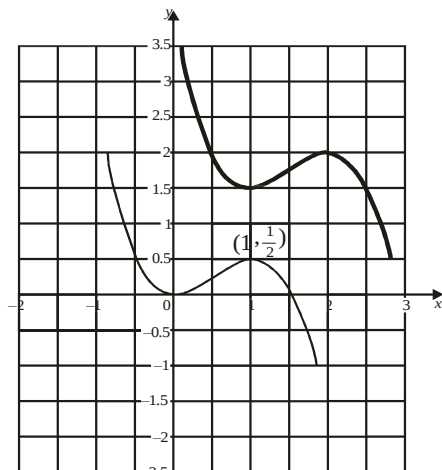
Note: reflection in x -axis, correct vertex and all intercepts approximately correct.

- (b) (i) $g(-3) = f(0)$ $f(0) = -1.5$
 (ii) translation (accept shift, slide, etc.) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

- (c)

	$y = f(x)$	$y = -f(x)$	$y = f(x+3)$
Domain	$-1 \leq x \leq 4$	$-1 \leq x \leq 4$	$-4 \leq x \leq 1$
Range	$-4 \leq y \leq 0.5$	$0.5 \leq y \leq 4$	$-4 \leq y \leq 0.5$

12. (a)



- (b) Minimum: $\left(1, \frac{3}{2}\right)$ Maximum: $(2, 2)$

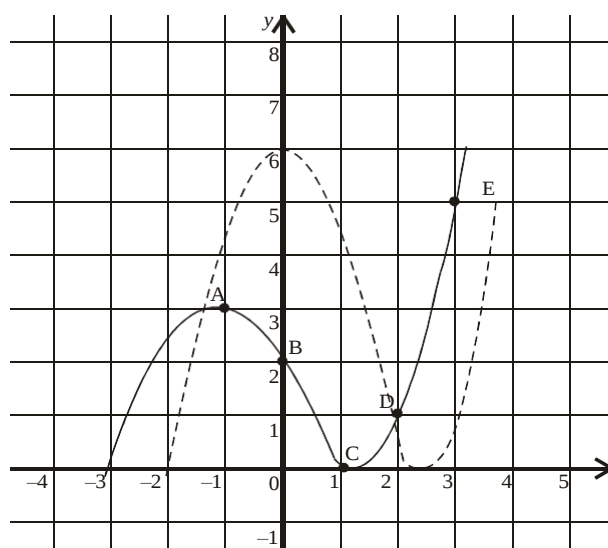
13. (a) $g(x) = 2f(x-1)$

x	0	1	2	3
$x-1$	-1	0	1	2
$f(x-1)$	3	2	0	1

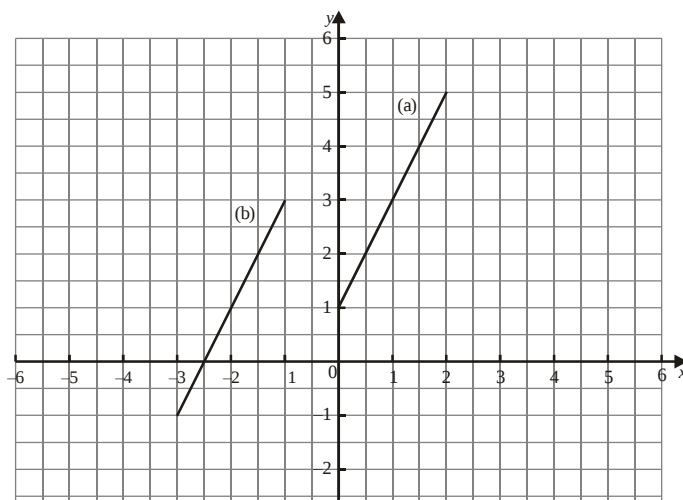
$g(0) = 2f(-1) = 6$ $g(1) = 2f(0) = 4$

$g(2) = 2f(1) = 0$ $g(3) = 2f(2) = 2$

(b) Graph passing through (0, 6), (1, 4), (2, 0), (3, 2)



14. (a)



(b) $g(-3) = -1$ $g(-1) = 3$ Line joining $g(-3)$ and $g(-1)$ drawn

(c)

	$y = f(x)$	$y = g(x)$
Domain	$0 \leq x \leq 2$	$-3 \leq x \leq -1$
Range	$1 \leq y \leq 5$	$-1 \leq y \leq 3$

15. (a) $y = (x-1)^2$
 $y = 4(x-1)^2$
 $y = 4(x-1)^2 + 3$

(b)

$y = x^2$	(0,0)
$y = (x-1)^2$	(1,0)
$y = 4(x-1)^2$	(1,0)
$y = 4(x-1)^2 + 3$	(1,1)

16. (a) in any order
translated 1 unit to the right
stretched vertically by factor 2

(b) **METHOD 1**

Finding coordinates of image on g

$$(-1, 1) \rightarrow (-1 + 1, 2 \times 1) = (0, 2)$$

P is (3, 0)

METHOD 2

$$h(x) = 2(x-4)^2 - 2$$

P is (3, 0)

17. (a) $(1, -2)$
(b) $g(x) = 3(x-1)^2 - 2$ (accept $p=1, q=-2$)
(c) $(1, 2)$

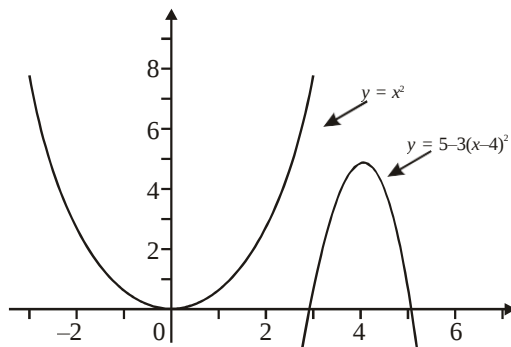
18. (a) $g(x) = -(x-3)^2 - 4$, therefore the maximum point is (3, -4)
(b) $f(x)$ is mapped onto $g(x)$ by a reflection in the x -axis followed by the translation $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

19. (a) the vertex is at (3, 5)

$$y = 2(x-3)^2 + 5$$

- (b) (i) $k = 2$
(ii) $p = 3$
(iii) $q = 5$

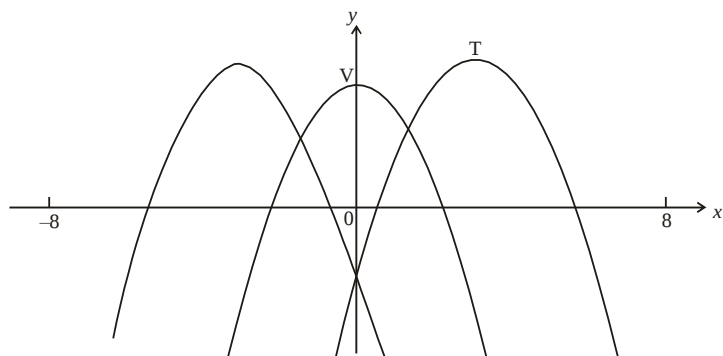
20.



$$q = 5$$

$$k = 3, p = 4$$

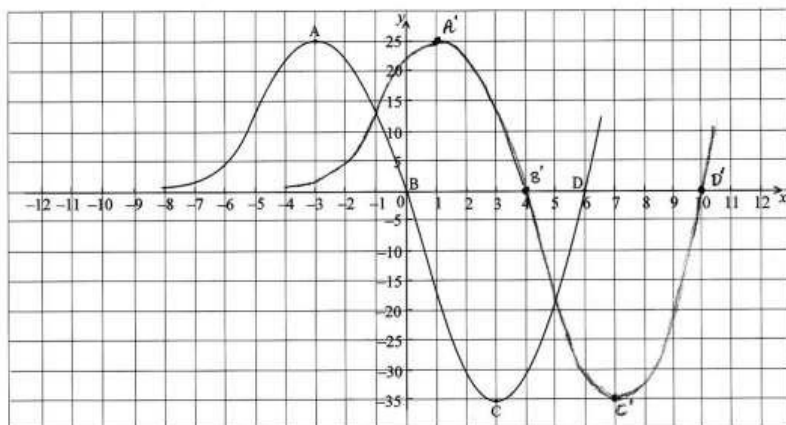
21. (a) (i) $h = 3$ (ii) $k = 1$
 (b) $g(x) = f(x-3) + 1, 5 - (x-3)^2 + 1, 6 - (x-3)^2, -x^2 + 6x - 3$
 (c)



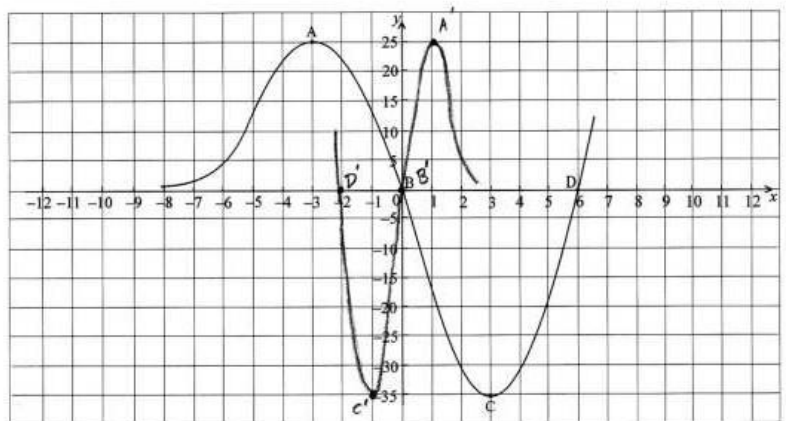
22. (a) By completing the square or finding the vertex (2,1)
 $3(x-2)^2 - 1$
 (b) **METHOD 1**
 Vertex shifted to (5, 4), $g(x) = 3(x-5)^2 + 4$
METHOD 2
 $g(x) = 3((x-3)-2)^2 - 1 + 5 = 3(x-5)^2 + 4$

23.

(a)



(b)



24.

$$\begin{aligned}
 y &= 2(x-2)^2 + 4(x-2) + 7 - 1 \\
 &= 2(x^2 - 4x + 4) + 4x - 8 + 6 \\
 &= 2x^2 - 8x + 8 + 4x - 2 \\
 \Rightarrow y &= 2x^2 - 4x + 6
 \end{aligned}$$

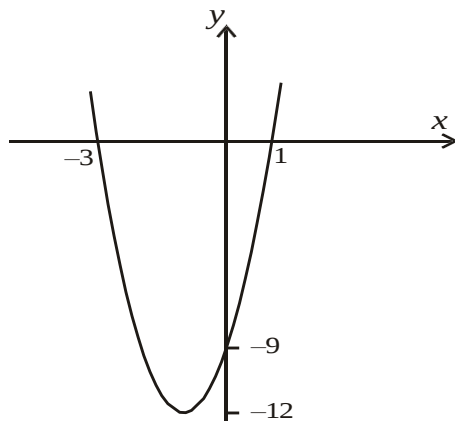
25. $g(x) = f(x-1) - 1 = 2(x-1)^3 - 3(x-1)^2 + (x-1) + 1 - 1 = \dots = 2x^3 - 9x^2 + 13x - 6$

B. Exam style questions (LONG)

26. (a) $f(x) = 3(x^2 + 2x + 1) - 12 = 3x^2 + 6x + 3 - 12 = 3x^2 + 6x - 9$

- (b) (i) vertex is $(-1, -12)$
 (ii) $x = -1$ (**must** be an equation)
 (iii) $(0, -9)$
 (iv) $3(x+3)(x-1) = 0$
 $(-3, 0), (1, 0)$

(c)



(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}, t = 3 \quad (p = -1, q = -12, t = 3)$

27. (a) $(f \circ g)(x) = (x-1)^2 + 4 \quad (x^2 - 2x + 5)$

(b) **METHOD 1**

vertex of $f \circ g$ at $(1, 4)$

adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates

vertex of h at $(4, 3)$

METHOD 2

find $h(x) = ((x-3)-1)^2 + 4 - 1, h(x) = (f \circ g)(x-3) - 1$

$$h(x) = (x-4)^2 + 3$$

vertex of h at $(4, 3)$

(c) $h(x) = (x-4)^2 + 3 = x^2 - 8x + 16 + 3 = x^2 - 8x + 19$

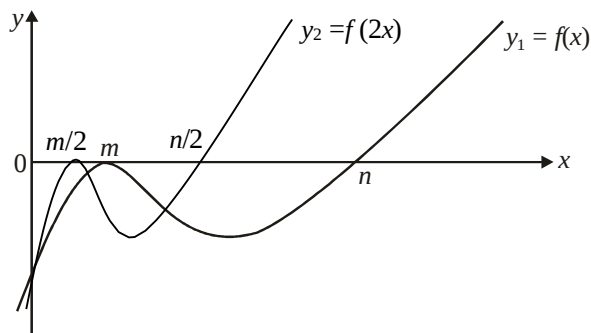
(d) $x^2 - 8x + 19 = 2x - 6$

$$x^2 - 10x + 25 = 0 \Leftrightarrow (x-5)^2 = 0$$

$$x = 5 \quad (p = 5)$$

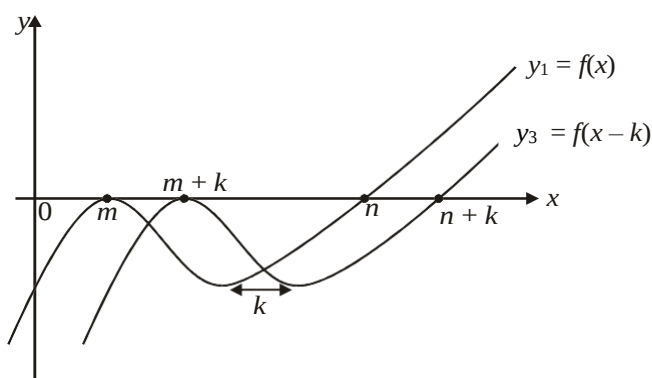
[there is also an alternative method using derivatives which we will study later on]

28. (a)



Notes: The graph of y_2 is y_1 stretched horizontally by s.f. $1/2$ (shrink) points of intersection with the x -axis $(m/2, 0)$ and $(n/2, 0)$

(b)



Notes: The graph of y_3 is y_1 shifted k units to the right. points of intersection with the x -axis $(m + k, 0)$ and $(n + k, 0)$

(c) (i) $m/2, n/2$

(ii) $m + 3, n + 3$

(iii) $\frac{m+3}{2}, \frac{n+3}{2}$

EITHER by using transformations

OR by substitution

For example, for (iii), the solutions are obtained by letting $2x - 3 = m$ and $2x - 3 = n$

so that $x = \frac{m+3}{2}, x = \frac{n+3}{2}$