INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 2.3]

FUNCTIONS – DOMAIN – RANGE – GRAPH

Note: In this assignment you may find functions that you have not studied yet, e.g.

$y = 2^x$, $y = \mathrm{e}^x$, $y = \sin x$, $y = \cos x$, $y = \log x$, $y = \ln x$

However, you may use a GDC to obtain the corresponding graphs (this is the only we need)

O. Practice questions

1. [Maximum mark: 7] ***[with / without GDC]***

Consider the function $f(x) = 3x + 7$

(a) Find $f(0)$ and $f(2)$ [2]

(b) Write down $f(a)$ in terms a. [1]

(c) Find a, given that $f(a) = 10$ [1]

(d) Find b, given that $f(b) = 22$ [1]

(e) Hence, complete the following table of values [2]

x	0			4	5
$f(x)$		10	13	19	

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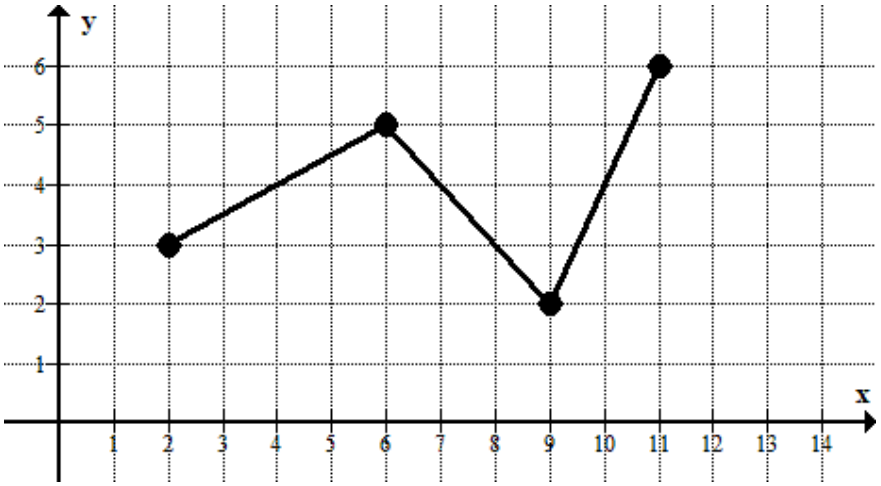
2. [Maximum mark: 10] ***[without GDC]***

The graph of the function f is shown below.



(a) Write down the domain of the function. [1]

(b) Write down the range of the function. [1]

(c) Write down the minimum value and the maximum value of the function. [2]

(d) Complete the following table of values [4]

x	2	4	6		
$f(x)$				2	6

(e) Given that $f(x) = 4$, there are three possible values for x. Write down the possible values of x. [2]

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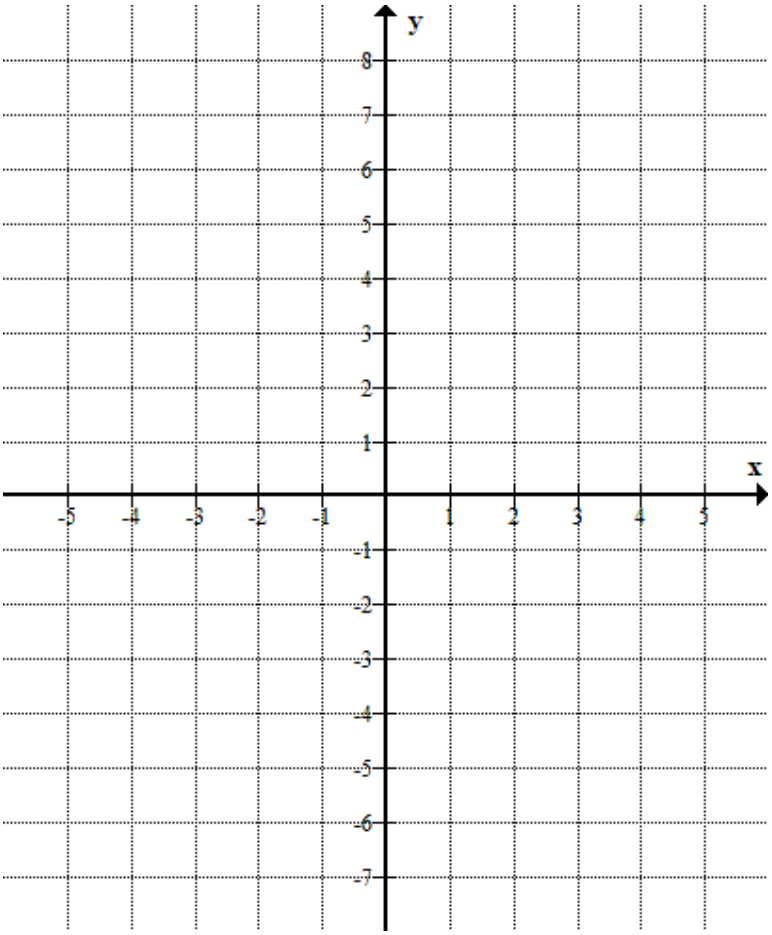
3. [Maximum mark: 8] ***[with / without GDC]***

(a) On the same set of axes, sketch the graphs of the following two functions

$f(x) = 2x$, $\quad -3 \le x \le 3$

$g(x) = 2x + 2$, $\quad -3 \le x \le 3$



[4]

(b) Write down the range of each function (for the given domain).

Function	$f(x) = 2x$	$g(x) = 2x + 2$
Range		

[2]

(c) The point points $\mathrm{P}(a,4)$ and $\mathrm{Q}(2,b)$, lie on the graph of g. Write down the values of a and b. [2]

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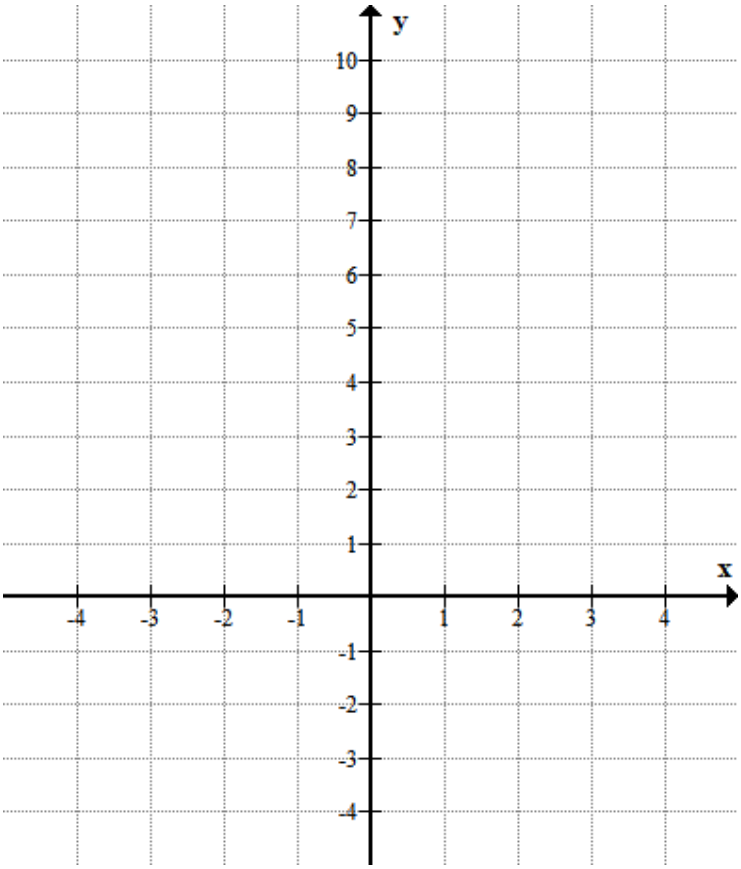
4. [Maximum mark: 10] ***[with GDC]***

(a) On the same set of axes, sketch the graphs of the following two functions

$$f(x) = x^2, \qquad -3 \le x \le 3$$

$$g(x) = x^2 - 2, \qquad -3 \le x \le 3$$

[4]



(b) For each function, write down

(i) the range (for the given domain);

(ii) the intersection points with the line $y = x$.

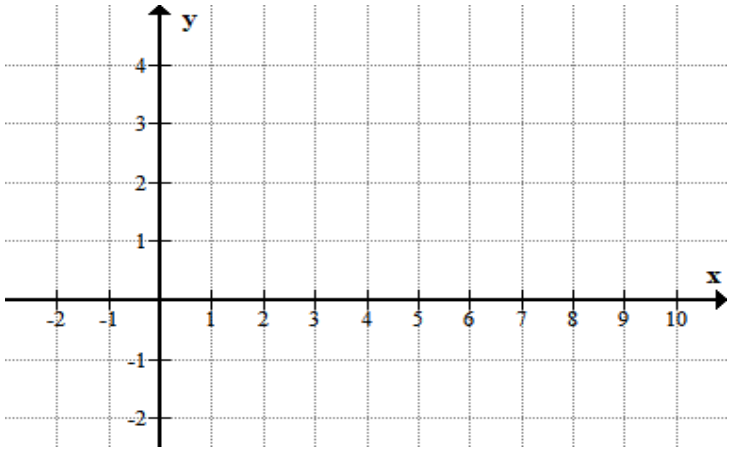
(iii) the intersection points with the horizontal line $y = 1$.

Function	Range	Points of intersection with line $y = x$	Points of intersection with line $y = 1$
$y = x^2$			
$y = x^2 - 2$			

[6]

5. [Maximum mark: 5] ***[with GDC]***

(a) Sketch the graph of the function $f(x) = \sqrt{x}$, $0 \le x \le 9$.



[2]

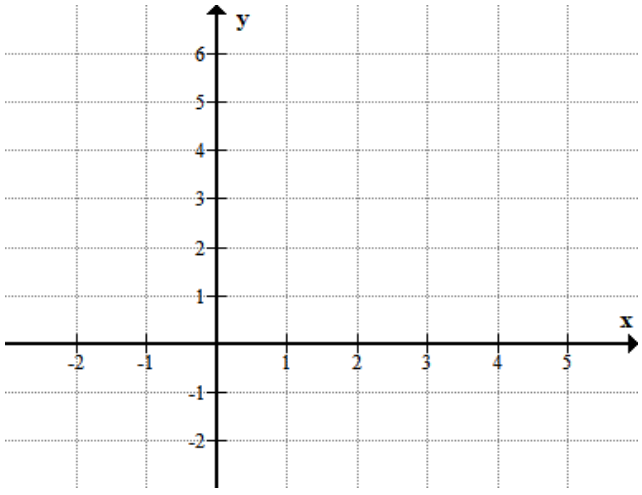
(b) Complete the following table

Range of f	
Point of intersection with the line $y = 1$	
Point of Intersection with the line $y = 2$	

[3]

6. [Maximum mark: 6] ***[with GDC]***

(a) Sketch the graph of the function $f(x) = 3x - 3\sqrt{x}$, $0 \le x \le 4$.



[3]

(b) Complete the following table

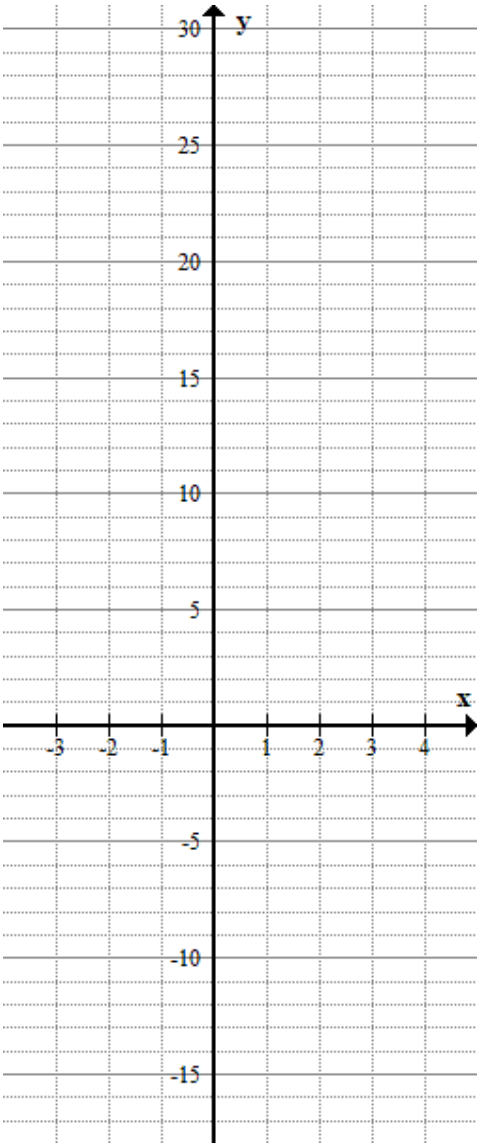
Coordinates of end points	
Coordinates of minimum point	
Range	

[3]

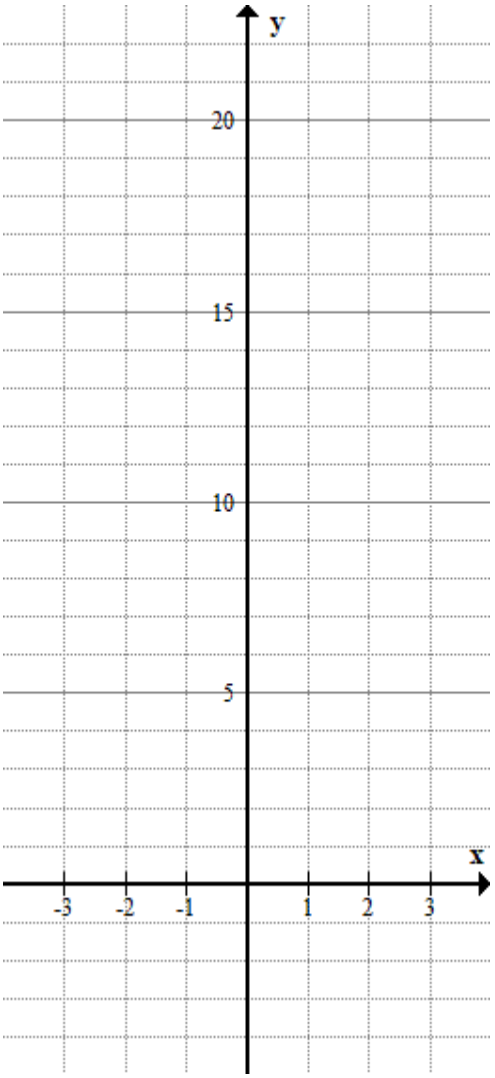
7. [Maximum mark: 10] ***[with / without GDC]***

(a) Sketch the graphs of the following two functions.

$f(x) = x^3$



$g(x) = x^4$



(b) Write down the largest possible domain and the corresponding range.

D_f:	R_f:	D_g:	R_g:

[6]

(c) Write down the range of these functions under the following restrictions

$f(x) = x^3$ $\qquad$ $g(x) = x^4$

D_f: $1 \le x \le 2$	R_f:	D_g: $1 \le x \le 2$	R_g:
D_f: $-1 \le x \le 2$	R_f:	D_g: $-1 \le x \le 2$	R_g:

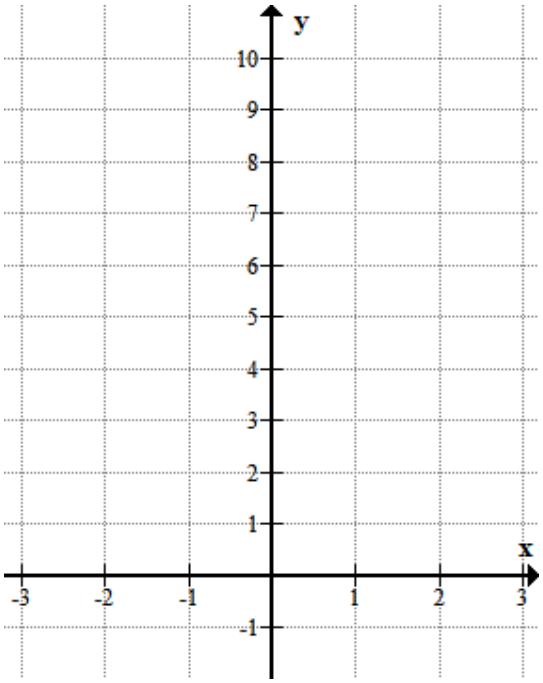
[4]

8. [Maximum mark: 14] ***[with GDC]***

(a) On the same set of axes below, sketch the graphs of the following two functions

$f(x) = 2^x, \quad -2 \le x \le 2$

$g(x) = 3^x, \quad -2 \le x \le 2$ [4]



(b) Complete the table

Function	$y = 2^x$	$y = 3^x$
Domain		
y-intercept		
Range		

[6]

(c) Using the graphs above

Solve the equation $2^x = 2$	
Solve the equation $3^x = 2$	
Solve the inequality $2^x > 2$	
Solve the inequality $3^x > 2$	

[4]

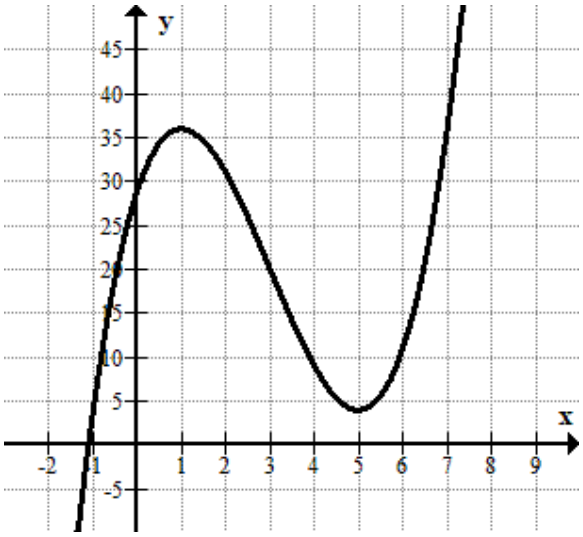
9. [Maximum mark: 8] ***[with GDC]***

Find the range for following functions in the given domain

$f(x) = 2x^2 - 8x + 9, \quad x \in R$	
$f(x) = 2x^2 - 8x + 9, \quad 0 \le x \le 2$	
$f(x) = 2x^2 - 8x + 9, \quad 0 \le x \le 5$	
$f(x) = 2x^2 - 8x + 9, \quad x \le 0$	

10*. [Maximum mark: 12] ***[with GDC]***

Part of the graph of the function $f(x) = x^3 - 9x^2 + 15x + 29$ is shown below



Find the range for following functions in the given domain

$f(x) = x^3 - 9x^2 + 15x + 29, \quad x \in R$	
$f(x) = x^3 - 9x^2 + 15x + 29, \quad x \ge 0$	
$f(x) = x^3 - 9x^2 + 15x + 29, \quad x \le 0$	
$f(x) = x^3 - 9x^2 + 15x + 29, \quad 0 \le x \le 6$	
$f(x) = x^3 - 9x^2 + 15x + 29, \quad 0 \le x \le 7$	
$f(x) = x^3 - 9x^2 + 15x + 29, \quad 0 \le x \le 8$	

Note

For the following question use the facts

Function of the form	Domain
$\frac{B}{A}$	$A \neq 0$
$\sqrt{A}$	$A \geq 0$
$\log A$ or $\ln A$	$A > 0$

11. [Maximum mark: 8] ***[without GDC]***

Find the largest possible domain for the following functions

$f(x) = 3 - 6x$ $\quad g(x) = \frac{7}{3-6x}$ $\quad h(x) = \sqrt{3-6x}$ $\quad k(x) = \log(3-6x)$

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A. Exam style questions (SHORT)

12. [Maximum mark: 8] ***[with / without GDC]***

Find the largest possible domain for the following functions

(a) $f(x) = x^2 - 4$ [1]

(b) $g(x) = \dfrac{7}{x^2 - 4}$ [2]

(c) $h(x) = \sqrt{x^2 - 4}$ [3]

(d) $k(x) = \log(x^2 - 4)$ [2]

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13. [Maximum mark: 5] ***[with GDC]***

The function f is given by $f(x) = \sqrt{x+7} + 5$

(a) Write down the domain of the function. [2]

(b) Sketch the graph of the function (by using the GDC) to deduce the range of f. [3]

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14. [Maximum mark: 6] ***[without GDC]***

The function f is given by $f(x) = \sqrt{x-4} + \frac{2}{x-5} + \log(10-x)$.

Find the largest possible domain of the function.

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15. [Maximum mark: 6] ***[with GDC]***

The function f is given by $f(x) = 2 - x^2 - e^x$.

(a) Sketch the graph of the function. Indicate the roots of the equation $f(x) = 0$. [4]

(b) Write down the maximum value of $f(x)$. [1]

(c) Write down the range of the function. [1]

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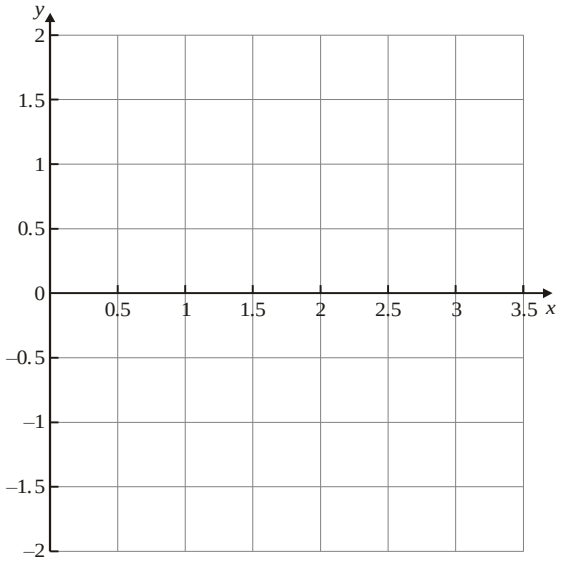
16. [Maximum mark: 6] ***[with GDC]***

Let $f(x) = \sin(2x + 1)$, $0 \leq x \leq \pi$. [x must be in **radians**].

(a) Sketch the curve of $y = f(x)$ on the grid below. [2]



(b) Find the x-coordinates of the maximum and minimum points of $f(x)$, giving your answers correct to one decimal place. [4]

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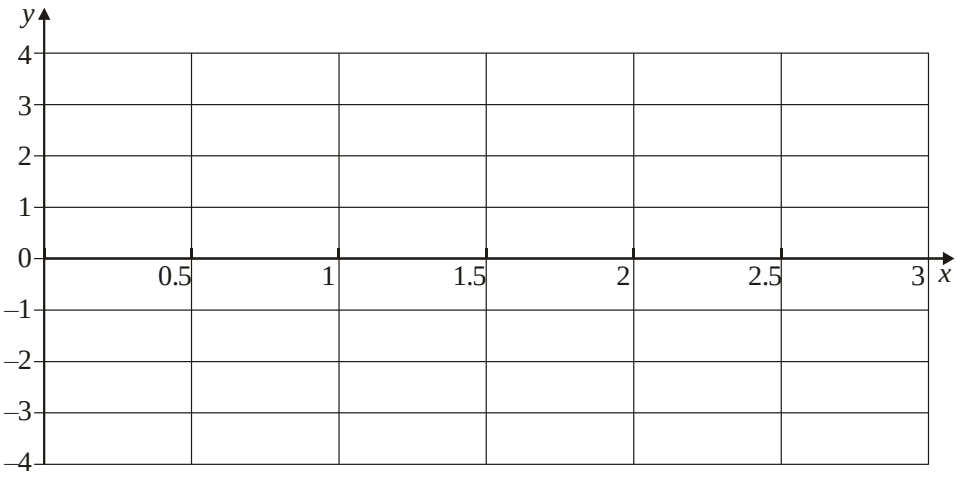
17. [Maximum mark: 8] ***[with GDC]***

Let $f(x) = 2 + \cos(2x) - 2\sin(0.5x)$ for $0 \le x \le 3$, [x must be in **radians**]

(a) On the grid below, sketch the curve of $y = f(x)$, indicating clearly the point P on the curve where the function has a max or min value. [4]



(b) Write down the solutions of $f(x) = 0$. [2]

(c) Write down the range of f. [2]

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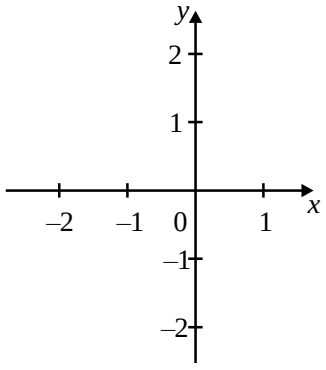
18. [Maximum mark: 6] ***[with GDC]***

(a) On the following diagram, sketch the graphs of $y = 2^x$ and $y = \cos x$ for $-2 \le x \le 1$.



[3]

(b) The equation $2^x = \cos x$ has a solution between -2 and -1. Find this solution. [2]

(c) Write down the x-intercept of the function $g(x)=2^x - \cos x$, between -2 and -1. [1]

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19. [Maximum mark: 8] ***[with GDC]***

Note: Radians are used throughout this question.

Let $f(x) = \sin(1+\sin x)$.

(a) Sketch the graph of $y = f(x)$, for $0 \le x \le 6$. [4]

(b) Write down the x-coordinates of all minimum and maximum points, for $0 \le x \le 6$. [2]

(c) Write down the range of $y = f(x)$, for $0 \le x \le 6$. [2]

Give your answers correct to **four** significant figures.

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20*. [Maximum mark: 5] ***[with GDC]***

(a) Solve the equation $x^3 = 2x^2 + 5x - 2$. [2]

(b) Solve the inequality $x^3 > 2x^2 + 5x - 2$. [3]

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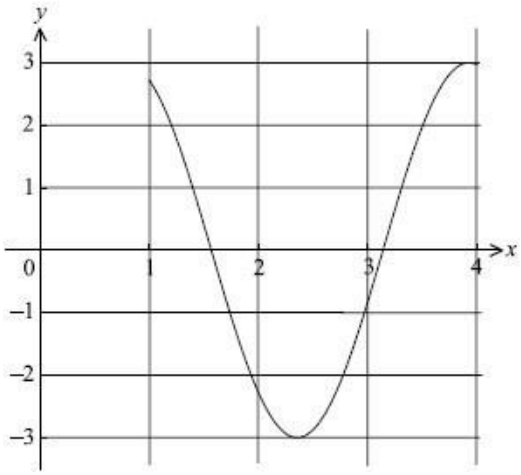
21. [Maximum mark: 6] ***[with GDC]***

Let $f(x) = 3\sin 2x$, for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$ for $1 \le x \le 4$.

The graph of f is shown below.



(a) On the same diagram, sketch the graph of g. [2]

(b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution. [2]

(c) Let $h(x) = g(x) - f(x)$. Given that $h(x) \le 0$ for $p \le x \le q$, write down the value of p and of q. [2]

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22*. [Maximum mark: 5] ***[with GDC]***

For $-3 \le x \le 3$, find the coordinates of the points of intersection of the curves

$y = x\sin x$ and $x + 3y = 1$. *[set up in **rad**]*

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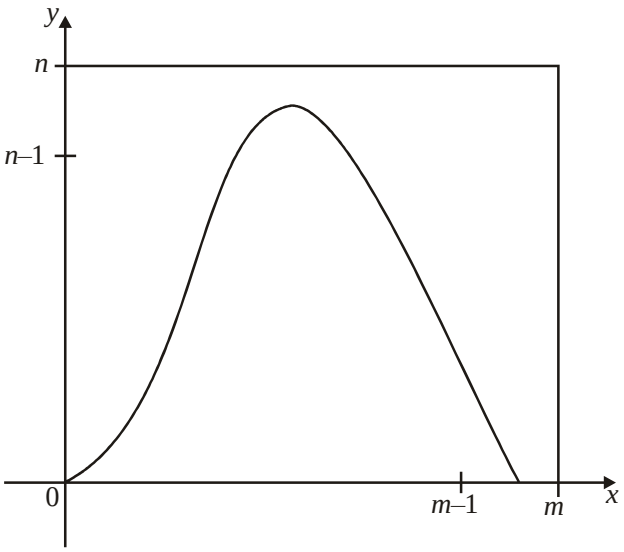
23*. [Maximum mark: 4] ***[with GDC]***

The diagram below shows the graph of $y = x \sin \frac{x}{3}$, for $0 \leq x < m$, and $0 \leq y < n$, where x is in radians and m and n are integers.



Find the value of m and the value of n.

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24.** [Maximum mark: 5] ***[with GDC]***

Given that $x > 0$, find the solution of the following system of equations:

$$\frac{8x^3}{y} = 3 \quad \text{and} \quad xy - y = x^2 + \frac{9}{4}$$

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B. Exam style questions (LONG)

25. [Maximum mark: 11] ***[without GDC]***

Let

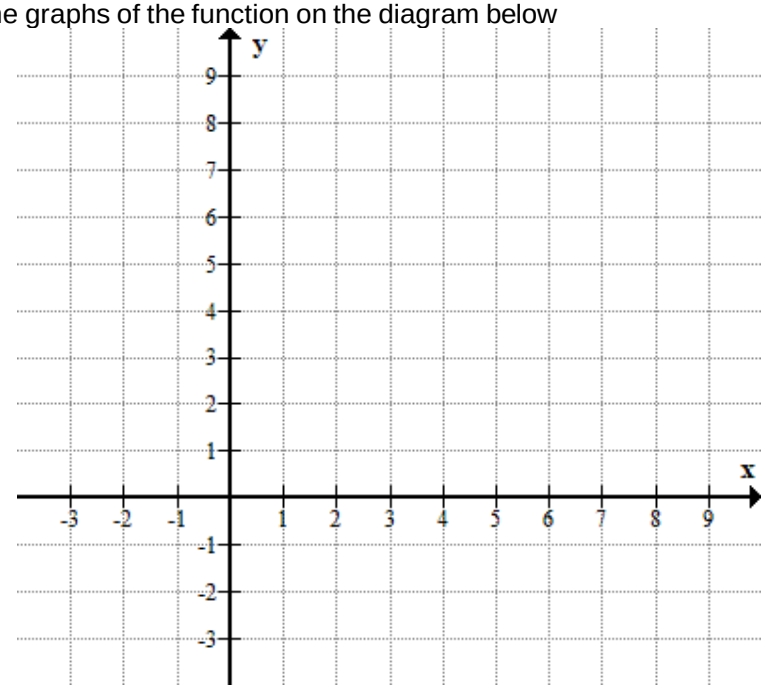
$$f(x) = \begin{cases} x-2 & 0 \le x < 5 \\ 5x-22 & 5 \le x \le 6 \end{cases}$$

(a) Complete the table of values

x	0	1	2	3	4	5	6
$f(x)$							

[4]

(b) Sketch the graphs of the function on the diagram below



[3]

(c) Write down the domain and the range of the function. [2]

(d) Write down the y - intercept and the x - intercept. [2]

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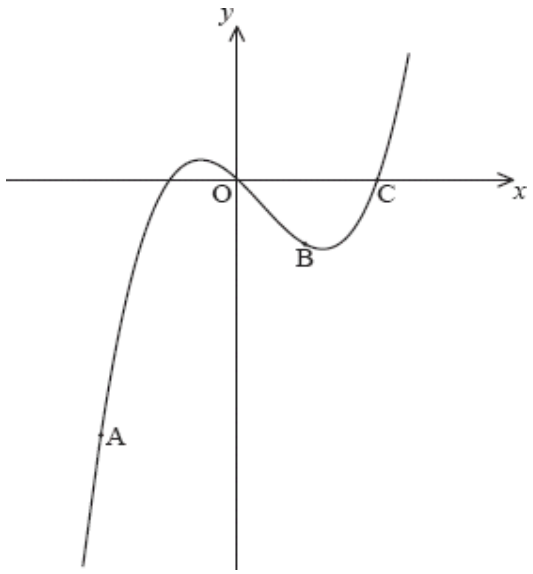
26*. [Maximum mark: 11] ***[with GDC]***

Consider the function $f(x) = px^3 + qx^2 + rx$. Part of the graph is shown below.



The graph passes through the origin O and the points A(–2, –8), B(1, –2) and C(2, 0).

(a) Find three linear equations in p, q and r. [3]

(b) Hence find the value of p, of q and of r. [2]

(c) Write down the largest possible domain and the range of the function. [2]

(d) Solve the inequality $f(x) \geq 0$. [4]

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