

[MAA 2.16] SYMMETRIES OF FUNCTIONS – MORE TRANSFORMATIONS

SOLUTIONS

O. Practice questions

1. (a) $f(-x) = 3(-x)^4 - 5(-x)^2 + 1 = 3x^4 - 5x^2 + 1 = f(x)$ even
- (b) $f(-x) = 3(-x)^5 - 5(-x)^3 + 1 = -3x^5 + 5x^3 + 1$ neither
- (c) $f(-x) = 3(-x)^5 - 5(-x)^3 + 7(-x) = -3x^5 + 5x^3 - 7x = -f(x)$ odd
- (d) $f(-x) = \frac{5(-x)^6 + 3|-x| - 1}{(-x)^3 + (-x)} = \frac{5x^6 + 3|x| - 1}{-x^3 - x} = -\frac{5x^6 + 3|x| - 1}{x^3 + x} = -f(x)$ odd

2. (a) $y = f(x) - 2$

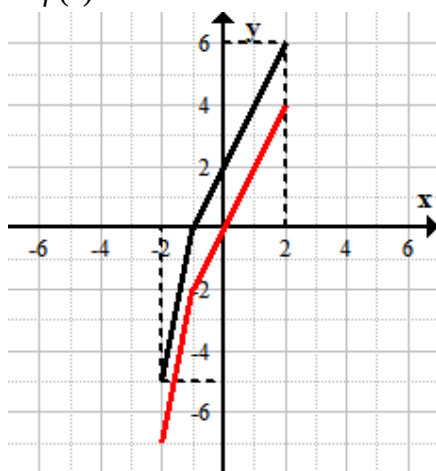


Image of $A(0, 2)$: $A'(0, 0)$

- (b) $y = f(x - 2)$

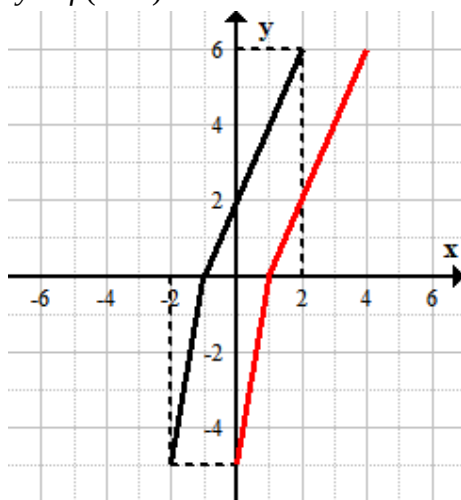


Image of $A(0, 2)$: $A'(2, 2)$

(c) $y = \frac{f(x)}{2}$

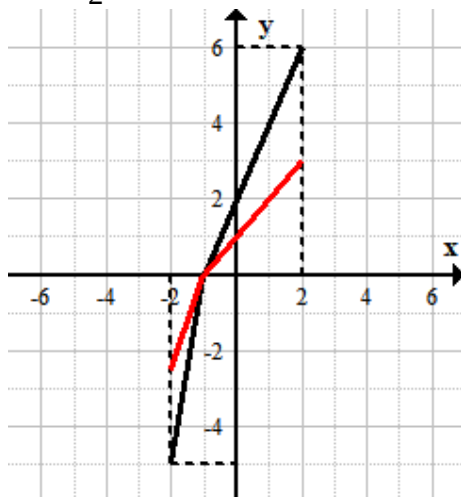


Image of $A(0,2)$: $A'(0,1)$

(d) $y = f\left(\frac{x}{2}\right)$

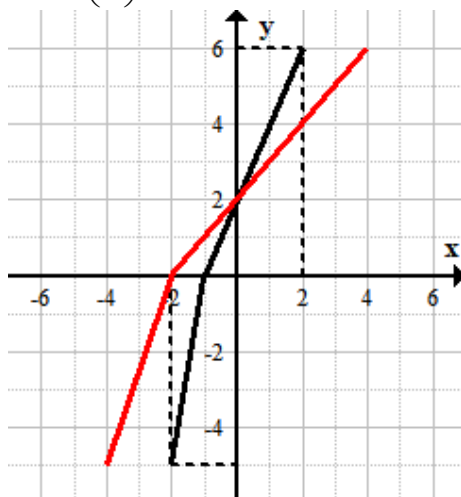


Image of $A(0,2)$: $A'(0,2)$

(e) $y = f(2x)$

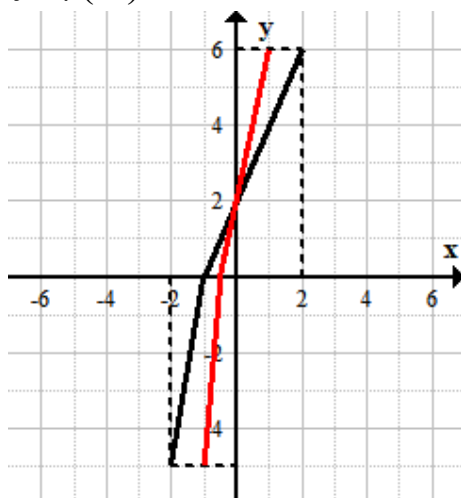


Image of $A(0,2)$: $A'(0,2)$

(f) $y = -f(x)$

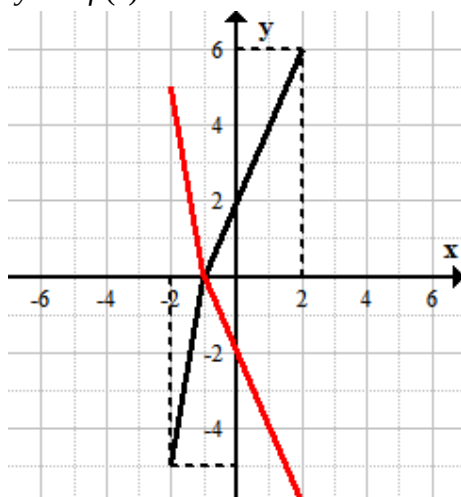


Image of $A(0,2)$: $A'(0,-2)$

(g) $y = f(-x)$

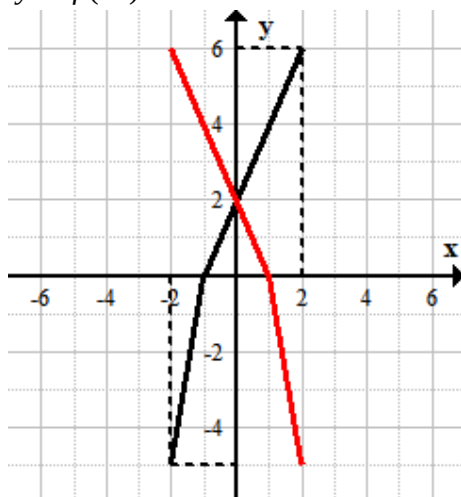


Image of $A(0,2)$: $A'(0,2)$

(h) $y = |f(x)|$

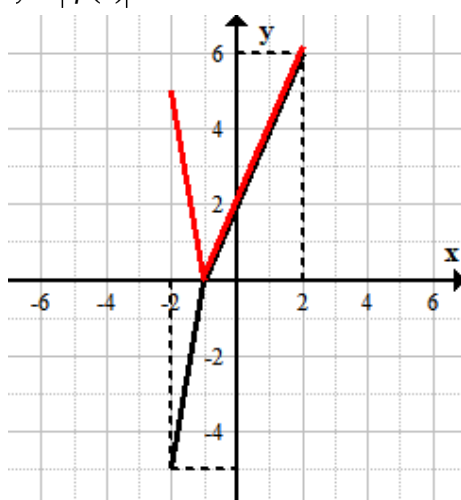


Image of $A(0,2)$: $A'(0,2)$

(i) $y = f(|x|)$

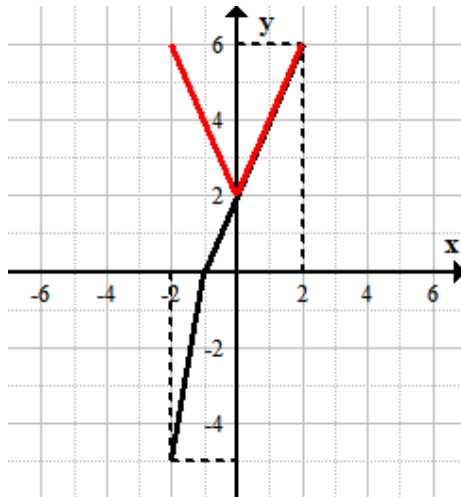
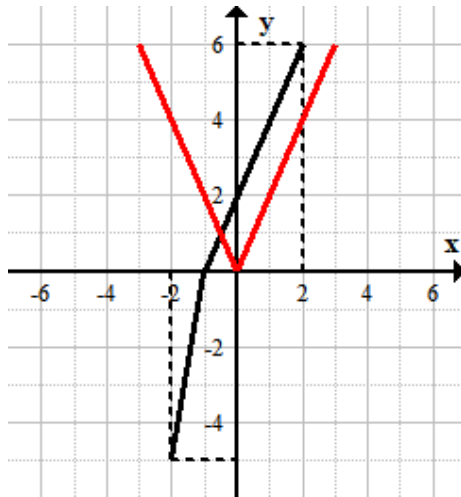


Image of A(0,2): A'(0,2)

(j) $y = f(|x|-1)$



Images of A(0,2): A'(1,2) and A''(-1,2)

(k) $y = f(|x-1|)$

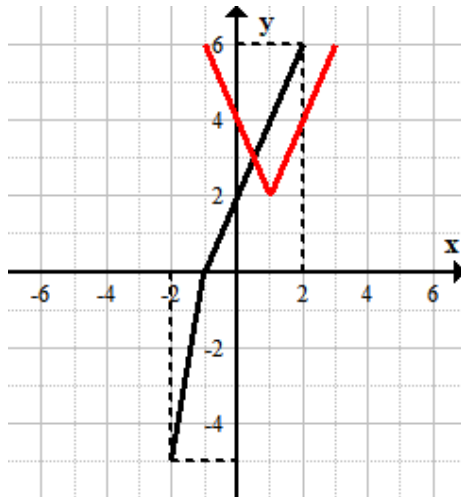


Image of A(0,2): A'(1,2)

3. (a)

Function	$y = f(x)$	$y = \frac{1}{f(x)}$	$y = f^{-1}(x)$
Domain	$x \neq -2$	$x \neq 2$	$x \neq 2$
Range	$y \neq 2$	$y \neq \frac{1}{2}$	$y \neq -2$

(b)

Transformation	$2f(x)$	$f(x)+2$	$f(x-7)$	$-f(x)$	$\frac{1}{f(x)}$
Horizontal asymptote	$y = 4$	$y = 4$	$y = 2$	$y = -2$	$y = \frac{1}{2}$

(c) The image of point A(3, 0.4) under the transformation $2f(3x)+5$ is A'(1, 5.8).

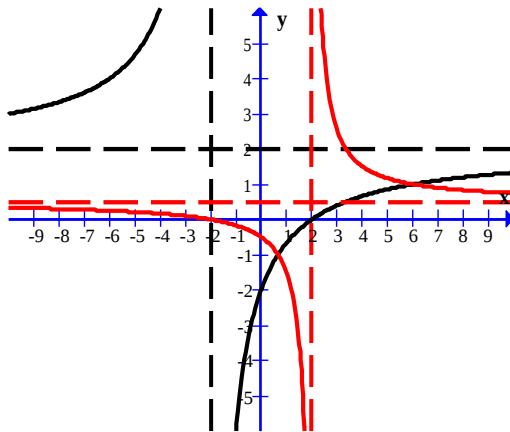
Step by step:

$$\begin{array}{ll} f(x) & (3, 0.4) \\ f(3x) & (1, 0.4) \\ 2f(3x) & (1, 0.8) \\ 2f(3x)+5 & (1, 5.8) \end{array}$$

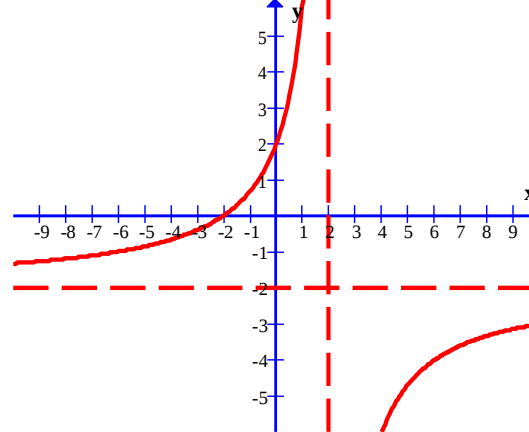
(d), (e), (f) $f(x) = \frac{2x-4}{x+2}$ $\frac{1}{f(x)} = \frac{x+2}{2x-4}$ $f^{-1}(x) = \frac{-2x-4}{x-2}$

[you may sketch the last two graphs, (e) and (f), directly from the graph of $f(x)$]

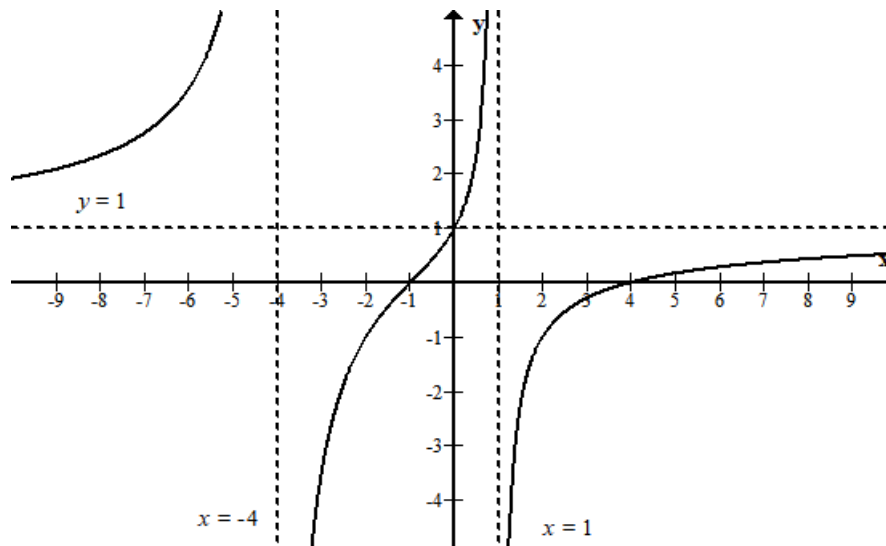
(d) and (e)



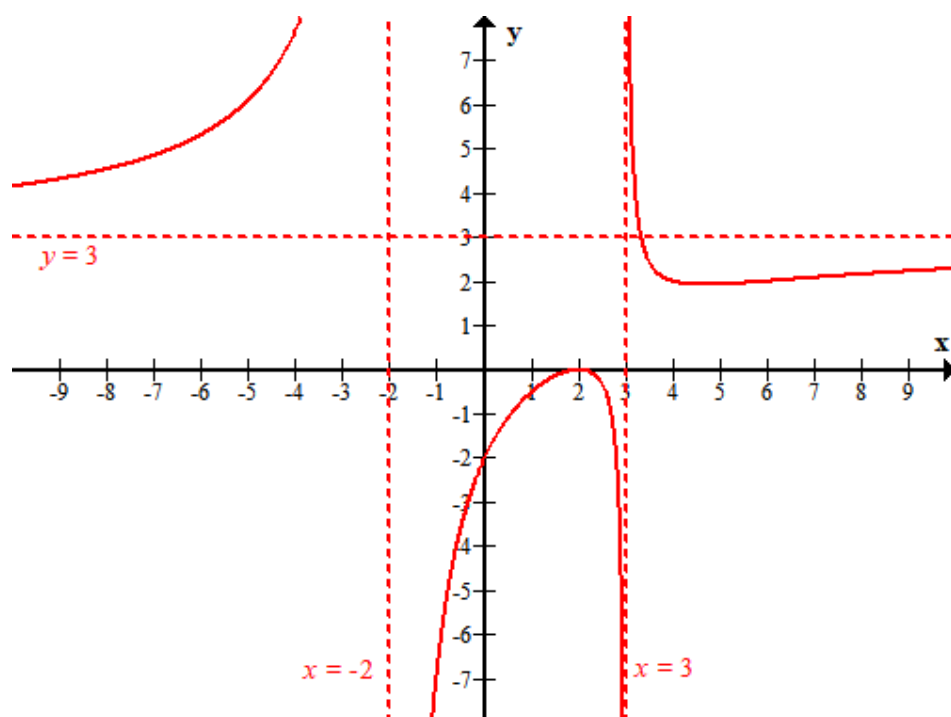
(f)



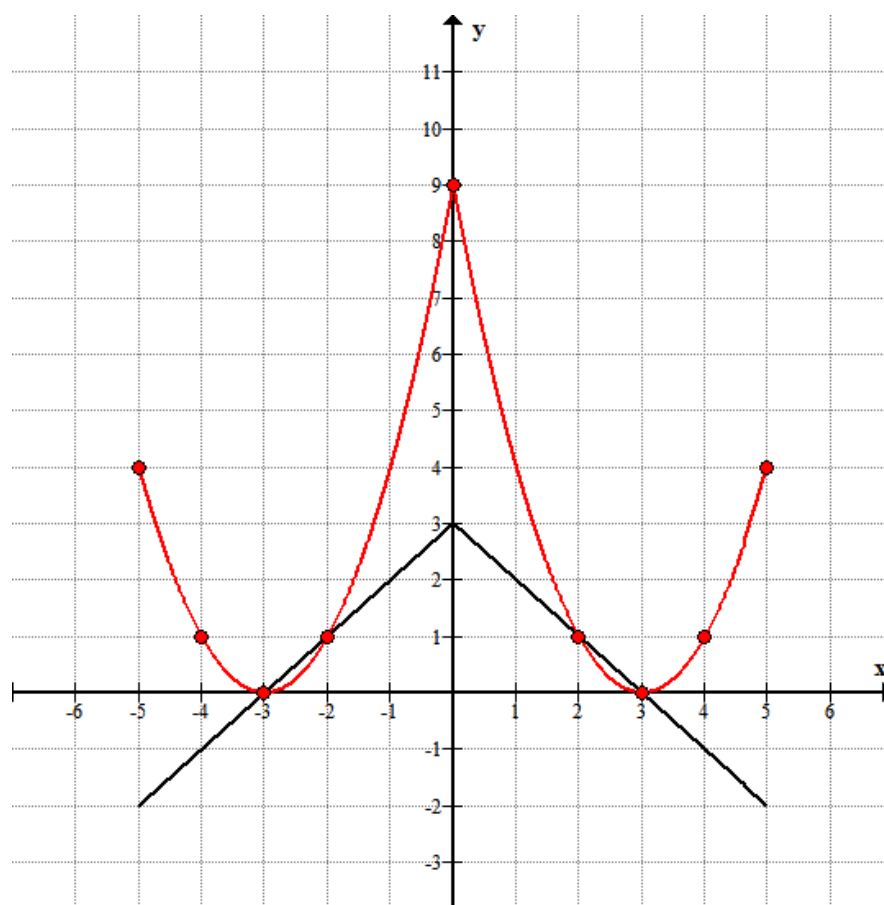
4.



5.



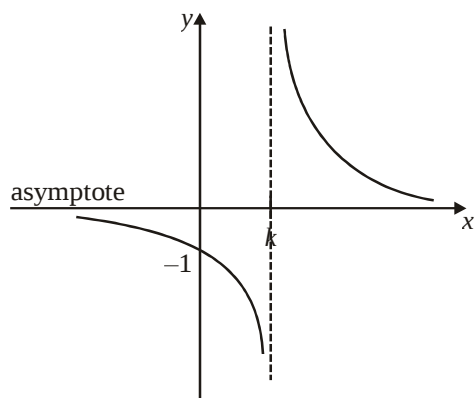
6.



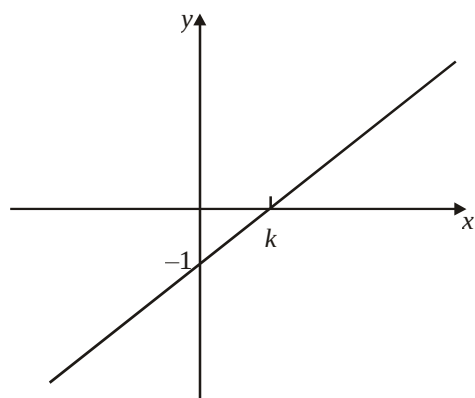
A. Exam style questions (SHORT)

7. (a) $f(-x) = |-x| - (-x) = |x| + x$ neither
 (b) $f(-x) = |-x| - 3 = |x| - 3 = f(x)$ even
 (c) $f(-x) = |-x - 3| = |x + 3|$ neither
 (d) $f(-x) = 3(-x)|-x| + \frac{1}{-x} = -3x|x| - \frac{1}{x} = -f(x)$ odd

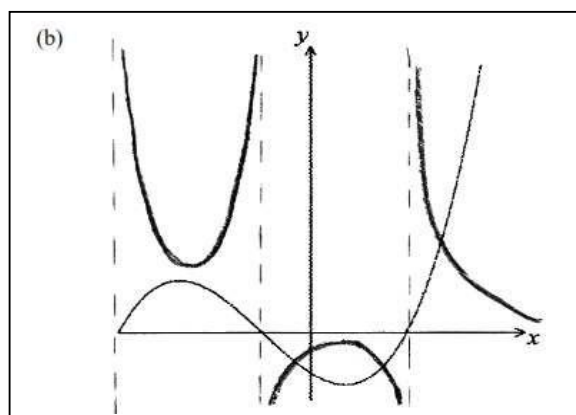
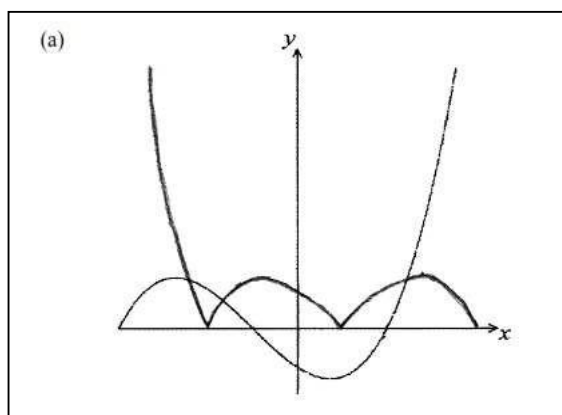
8. (a)



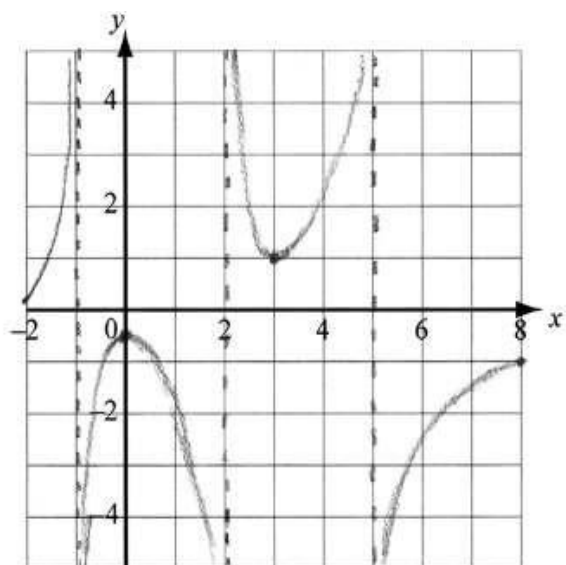
- (b)



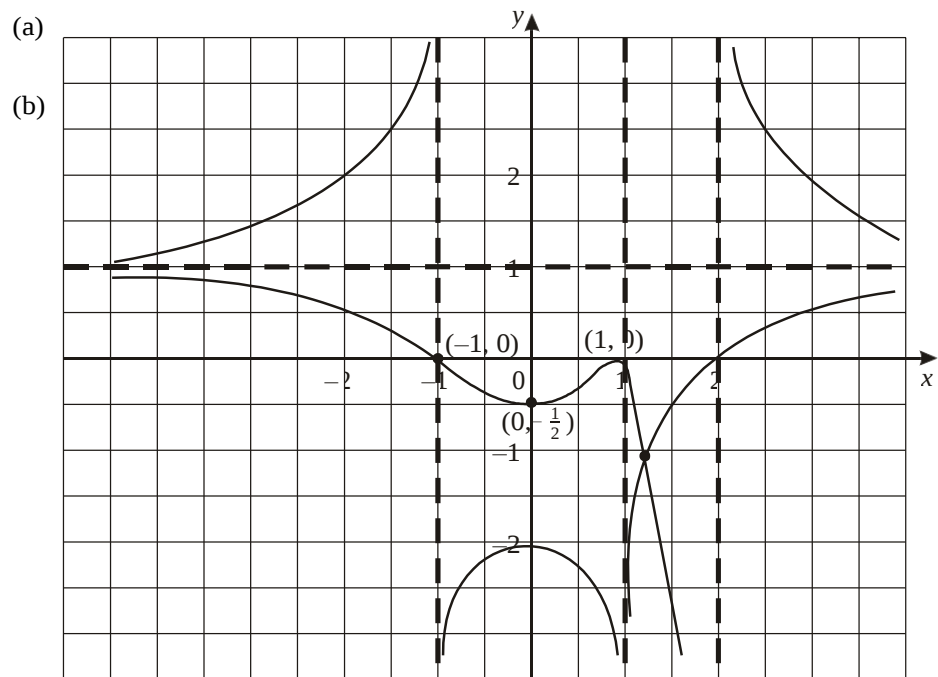
- 9.



10.

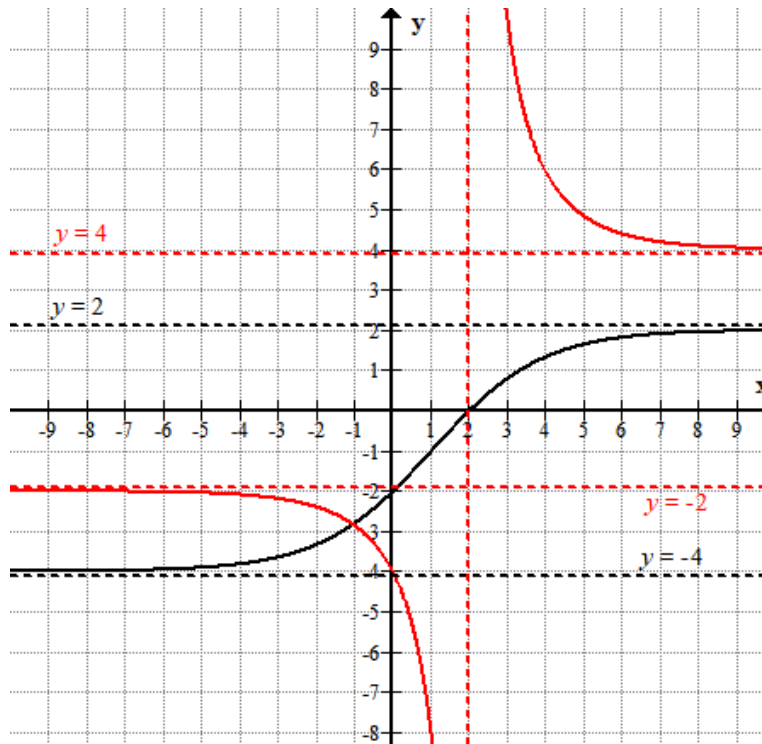


11.



(c) $-2 \leq k < 1, k > 1$

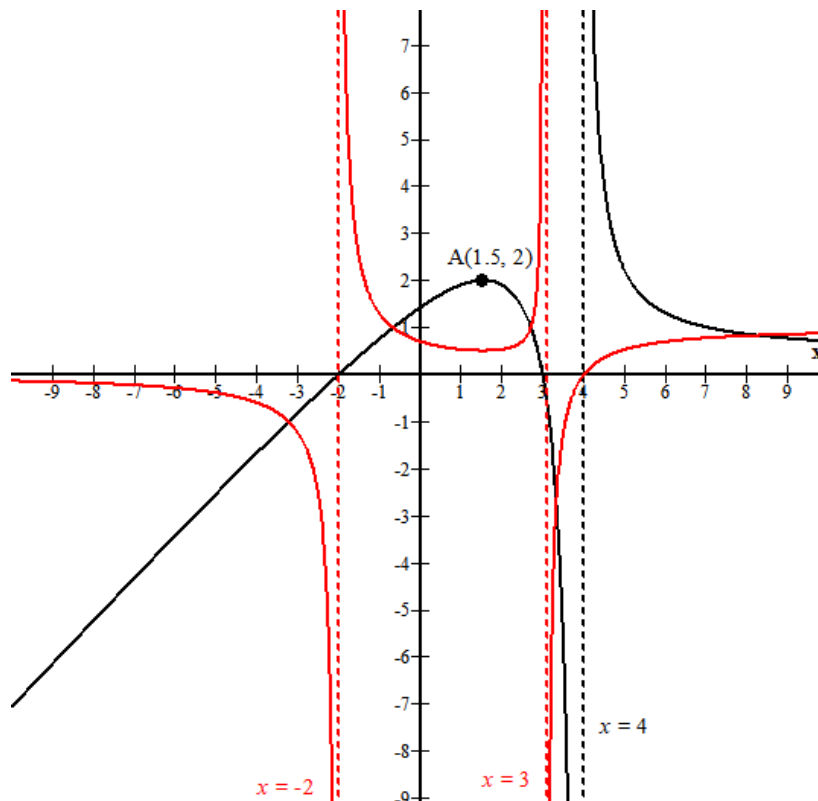
12.



- (b) (i) Domain $x \in \mathbb{R}$, Range $-4 < y < 2$
(ii) Domain $x \neq 2$, Range $y < -2$ or $y > 4$

13. (a) (i) $(0, \frac{2}{3})$ (ii) $(\frac{3}{2}, \frac{1}{1})$

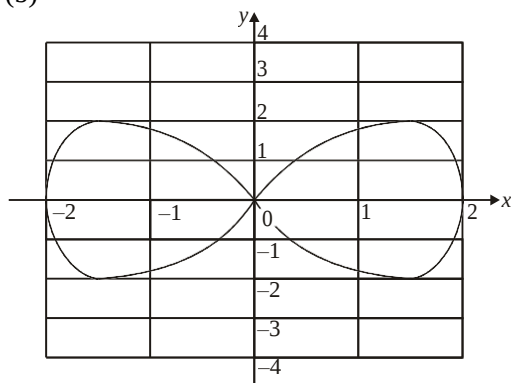
(b)



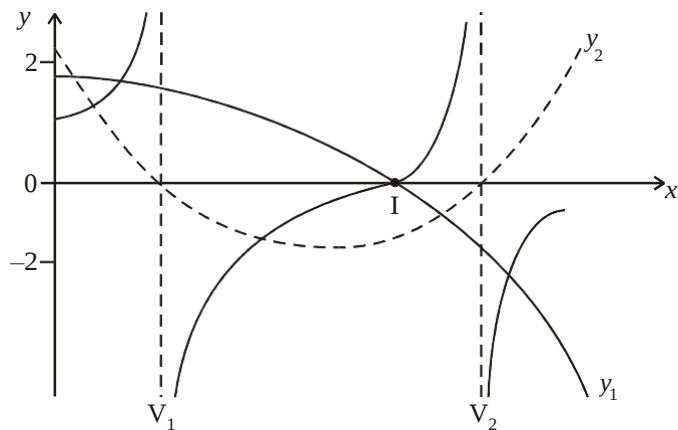
14. (a)

Function	Range
$y = f(x)$	$0 \leq y \leq 4$
$y = f(x)^2$	$0 \leq y \leq 16$
$y = \sqrt{f(x)}$	$0 \leq y \leq 2$

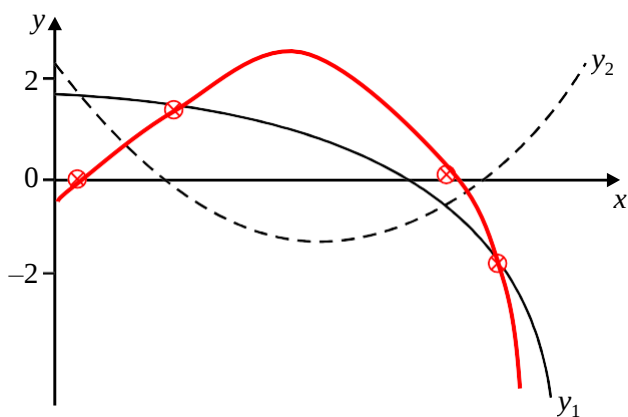
(b)



15. (a)



(b)



16. (a) $x = 1.43$

(b) (i) $(3, -131)$

(ii) $-131 \leq y \leq 208.2$

(c) $x = 4, x = 1.43$

(d) $x = 3$