INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 2.13-2.15]

RATIONAL AND MODULUS FUNCTIONS – INEQUALITIES

O. Practice questions

1. [Maximum mark: 18] ***[without GDC]***

For the following functions write down the roots, the y-intercept, the vertical and horizontal asymptotes (if there exist), the domain and the range.

	$f(x)=\frac{3x+8}{2x+7}$	$f(x)=\frac{8}{2x+7}$	$f(x)=\frac{3x+8}{7}$
Roots			
y **-intercept**			
V.A.			
H.A.			
Domain			
Range			

2. [Maximum mark: 15] ***[without GDC]***

For the following functions write down the roots, the y-intercept, the vertical and horizontal asymptotes and the domain.

	$f(x)=\frac{(x-3)(x-4)}{(x+1)(x-2)}$	$f(x)=\frac{(2x-3)(x-4)}{(x+1)(x-2)}$	$f(x)=\frac{2x-3}{(x+1)(x-2)}$
Roots			
y **-intercept**			
V.A.			
H.A.			
Domain			

3. [Maximum mark: 10] ***[without GDC]***

For the following functions, find all the asymptotes (horizontal, vertical, or oblique).

(a) $f(x) = \dfrac{4x^2 + 4x + 1}{2x + 3}$ [4]

(b) $f(x) = \dfrac{2x^3 + 5x^2 + 4x + 1}{x^2 - x - 2}$ [6]

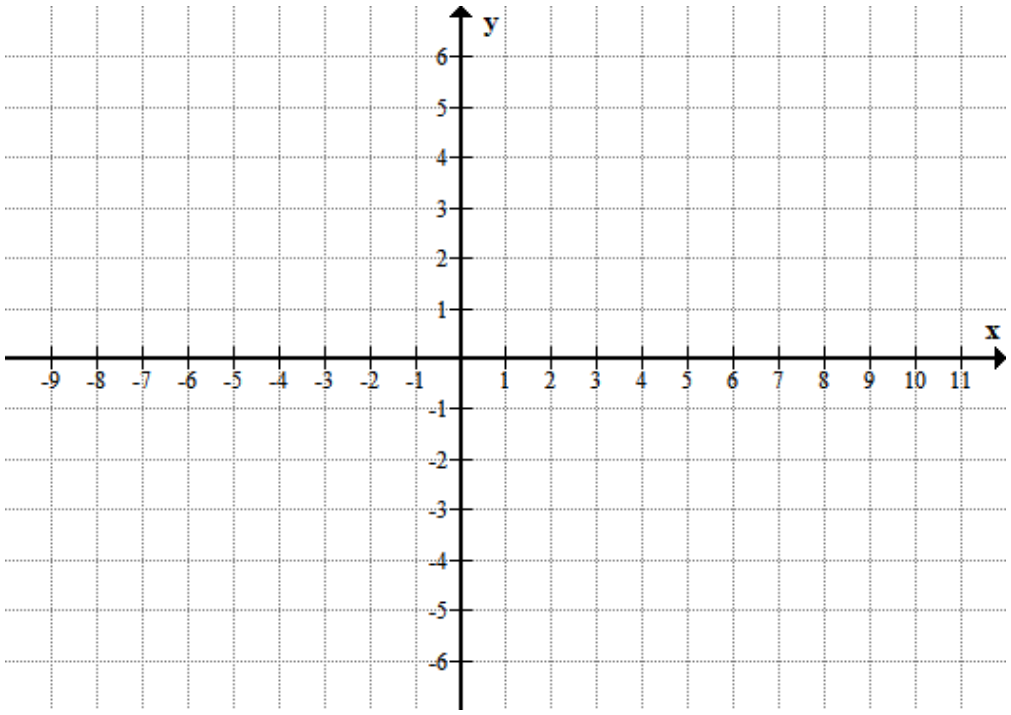
4. [Maximum mark: 10] ***[without GDC]***

Let $f(x) = \dfrac{(x-1)(x-6)}{(x+2)(x-3)}$.

(a) Solve the equation $f(x) = 1$. [3]

(b) On the diagram below, sketch the graph of $y = f(x)$. Indicate all the asymptotes, the $x-$ and the $y-$intercepts, and the information found in (a). [5]

(c) Hence, write down the domain and the range of f. [2]



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5. [Maximum mark: 12] ***[without GDC]***

Let $f(x) = \dfrac{(x+2)(x-3)}{3x-6}$

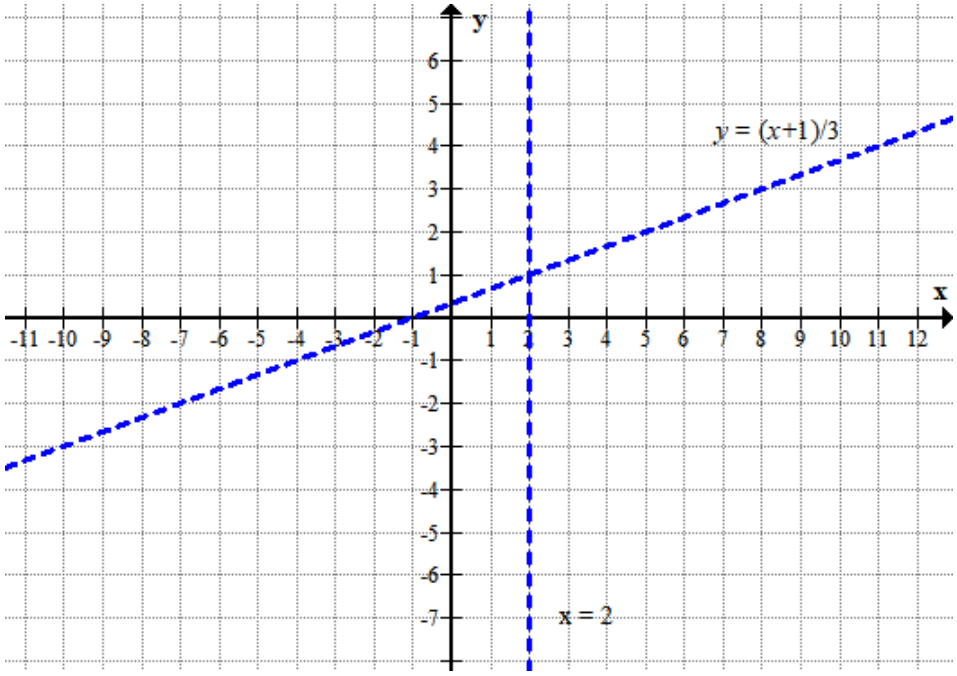
(a)_ Show that graph $y = f(x)$ has oblique asymptote the line $y = \dfrac{x+1}{3}$ [3]

(b) Solve the equation $f(x) = 1$. [3]

(c) On the diagram below, sketch the graph of $y = f(x)$. The asymptotes are shown.

Indicate the $x-$ and the $y-$ intercepts and the information found in (b). [4]

(d) Hence, write down the domain and the range of f. [2]



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6. [Maximum mark: 12] ***[with / without GDC]***

Express the following rational functions in partial fractions.

(a) $f(x) = \dfrac{6}{x^2 - x - 2}$ [4]

(b) $f(x) = \dfrac{3x + 6}{x^2 - x - 2}$ [4]

(c) $f(x) = \dfrac{x}{2x^2 - 2x - 4}$ [4]

7. [Maximum mark: 10] ***[without GDC]***

Consider the polynomial

$$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$$

(a) Complete the sign table below (Indicate + or – in each interval)

x	$-\infty$	1	2	3	4	5	$+\infty$
$f(x)$							

[2]

(b) Solve the inequality $f(x) > 0$. [2]

(c) Solve the inequality $f(x) \geq 0$. [2]

(d) Solve the inequality $\dfrac{(x-1)(x-3)(x-5)}{(x-2)(x-4)} > 0$. [2]

(e) Solve the inequality $\dfrac{(x-1)(x-3)(x-5)}{(x-2)(x-4)} \geq 0$. [2]

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8. [Maximum mark: 13] ***[without GDC]***

Consider the following functions.

$$g(x) = (x-1)(x-2)(x-3)(x-4)^2(x-5)$$

(a) Complete the sign table below (Indicate + or – in each interval)

x	$-\infty$	1	2	3	4	5	$+\infty$
$g(x)$							

[2]

(b) Solve the inequality $g(x) > 0$. [2]

(c) Solve the inequality $g(x) \geq 0$. [2]

(d) Solve the inequality $g(x) < 0$. [2]

(e) Solve the inequality $g(x) \leq 0$. [2]

(f) Solve the inequality $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)^2} \geq 0$. [3]

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9. [Maximum mark: 8] ***[without GDC]***

Solve the inequalities (notice that they all involve the same factors).

(i) $\dfrac{5(x-1)(x-2)^2}{(x-3)^3} \geq 0$ (ii) $\dfrac{(x-1)(x-3)^3}{5(x-2)^2} \geq 0$ (iii) $\dfrac{(x-2)^2(x-3)^3}{5(x-1)} \geq 0$

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10*. [Maximum mark: 6] ***[with / without GDC]***

Solve the inequality $x + \dfrac{2}{x} \geq 3$

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11. [Maximum mark: 6] ***[with / without GDC]***

(a) Solve the equation $|x-5|=3$ [3]

(b) Solve the inequality $|x-5|<3$ [3]

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12. [Maximum mark: 8] ***[with / without GDC]***

(a) Solve the equation $|x-5|=|x-3|$ [4]

(b) Solve the inequality $|x-5|<|x-3|$ [4]

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13*. [Maximum mark: 10] ***[with / without GDC]***

(a) Solve the equation $|x-5| = x-3$ [5]

(b) Solve the inequality $|x-5| < x-3$ [5]

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14*. [Maximum mark: 5] ***[with / without GDC]***

Solve the inequality $x^2 - 3|x| > 2$

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15*. [Maximum mark: 12] ***[with / without GDC]***

(a) Solve the equation $|x-5|=|x|-3$ [6]

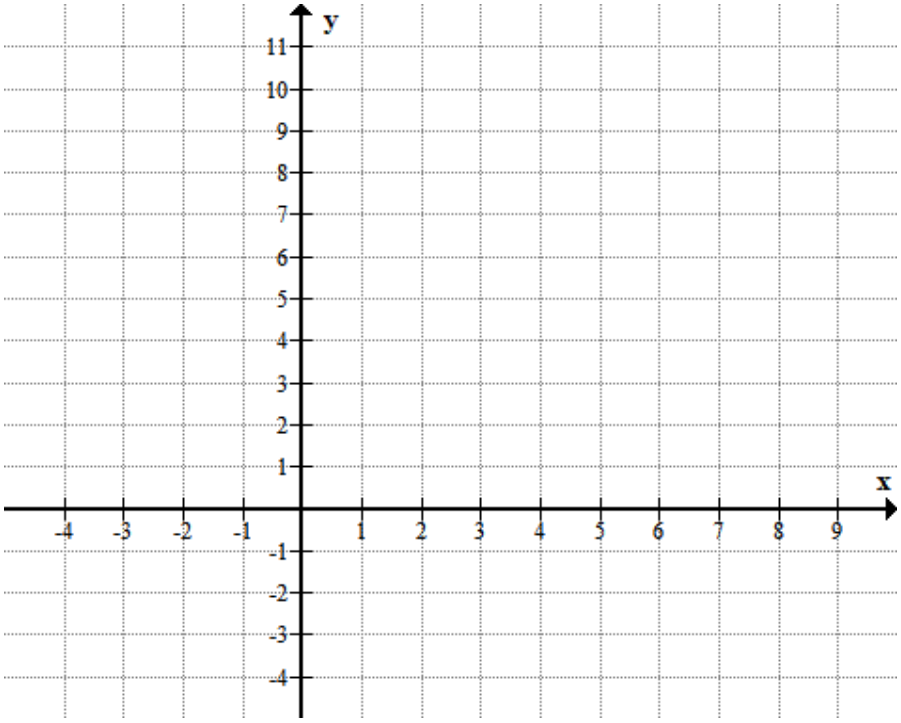
(b) Solve the inequality $|x-5|<|x|-3$ [6]

16*. [Maximum mark: 8] ***[without GDC]***

(a) Draw the graph of $f(x) = |x-5| - |x| + 3$ [4]

(b) **Hence**, solve the inequalities

(i) $f(x) > 0$ (ii) $f(x) > 4$ (iii) $f(x) > -4$ (iv) $f(x) > 10$ [4]



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A. Exam style questions (SHORT)

17. [Maximum mark: 5] ***[without GDC]***

Find all the asymptotes (horizontal, vertical, or oblique) of $f(x) = \dfrac{3x^2 - x + 1}{2x^2 - 14x + 24}$

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18. [Maximum mark: 5] ***[without GDC]***

Find all the asymptotes (horizontal, vertical, or oblique) of $f(x) = \dfrac{3x^2 - x + 1}{x^2 - x + 1}$

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19. [Maximum mark: 6] ***[without GDC]***

Find all the asymptotes (horizontal, vertical, or oblique) of $f(x) = \dfrac{6x^2 + 5x + 1}{3x + 7}$

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20. [Maximum mark: 7] ***[without GDC]***

Find all the asymptotes (horizontal, vertical, or oblique) of $f(x) = \dfrac{6x^3 + 1}{2x^2 - 14x + 24}$

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21. [Maximum mark: 7] ***[without GDC]***

Express in partial fractions the function $f(x) = \dfrac{5}{2x^2 - 14x + 24}$

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22. [Maximum mark: 7] ***[without GDC]***

Express in partial fractions the function $f(x) = \dfrac{5x + 1}{2x^2 - 14x + 24}$

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23. [Maximum mark: 8] ***[without GDC]***

Solve the inequalities (notice that they all involve the same factors).

(i) $(x-1)^2(x+1)(x-2) \geq 0$ (ii) $\dfrac{(x-1)^2(x+1)}{x-2} > 0$ (iii) $\dfrac{(x-1)^2(x+1)}{x-2} \geq 0$

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24. [Maximum mark: 5] ***[without GDC]***

Solve the inequalities (i) $\dfrac{(2x-1)^2(x+1)}{2-x} \geq 0$ (ii) $\dfrac{(2x-1)^2(x+1)}{2-x} \leq 0$

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25. [Maximum mark: 8] ***[with / without GDC]***

Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$.

(a) Solve the equation $f(x) = g(x)$ [3]

(b) Find the set of values of x such that $f(x) \leq g(x)$. [5]

26. [Maximum mark: 5] ***[with GDC]***

Solve the inequality $x^2 - 4 + \frac{3}{x} < 0$.

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27*. [Maximum mark: 8] ***[without GDC]***

Solve the inequality $x^2 - 4 + \frac{3}{x} < 0$ given that one of the zeroes of the function is $x = 1$.

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28. [Maximum mark: 5] ***[with / without GDC]***

Find the values of x for which $|5-3x| \leq |x+1|$.

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29. [Maximum mark: 5] ***[with / without GDC]***

Solve the inequality $|x-2| \geq |2x+1|$.

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30*. [Maximum mark: 6] ***[with / without GDC]***

Solve the inequality $\left|\frac{x+9}{x-9}\right| \leq 2$.

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31. [Maximum mark: 6] ***[with / without GDC]***

Find the largest set of values of x such that the function f given by $f(x) = \sqrt{\frac{8x-4}{x-3}}$ takes real values.

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32. [Maximum mark: 7] ***[with / without GDC]***

The functions f and g are defined by

$$f(x) = 2x - 1, \qquad g(x) = \frac{x}{x+1},\ x \neq -1$$

Find the values of x for which $(f \circ g)(x) \leq (g \circ f)(x)$.

B. Exam style questions (LONG)

33. [Maximum mark: 24] ***[with / without GDC]***

Let $f(x) = \dfrac{6}{x^2 - 5x + 4}$, $g(x) = \dfrac{x^2 - 5x + 10}{x^2 - 5x + 4}$, $h(x) = \dfrac{x^3 - 5x^2 + 4x + 12}{x^2 - 5x + 4}$

(a) Find the equations of all the asymptotes for the graphs of

(i) $y = f(x)$ (ii) $y = g(x)$ (iii) $y = h(x)$ [8]

(b) Express $f(x)$ in partial fractions [5]

(c) Express $g(x)$ in the form $g(x) = q + \dfrac{p}{x^2 - 5x + 4}$, where $p, q \in$, [2]

(d) **Hence** express

(i) $g(x)$ in the form $g(x) = C + \dfrac{A}{(x-a)} + \dfrac{B}{(x-b)}$

(ii) $h(x)$ in the form $h(x) = Ex + \dfrac{C}{(x-c)} + \dfrac{D}{(x-d)}$ [4]

(e) Solve the inequality $5f(x) \leq 3g(x)$ [5]

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34. [Maximum mark: 13] ***[with / without GDC]***

Let $f(x) = \dfrac{x^3 + 50}{x^2 - 6x + 8}, \quad x \neq 2, x \neq 4$

(a) Find the quotient and the remainder of the long division of $x^3 + 50$ by $x^2 - 6x + 8$ [3]

(b) **Hence,** express $f(x)$ in the form $f(x) = q(x) + \dfrac{ax + b}{x^2 - 6x + 8}$ [2]

(c) Write down all the asymptotes of $y = f(x)$ [3]

(d) Express $f(x)$ in the form $f(x) = q(x) + \dfrac{A}{x-2} + \dfrac{B}{x-4}$ [5]