

[MAA 2.1] LINES

SOLUTIONS

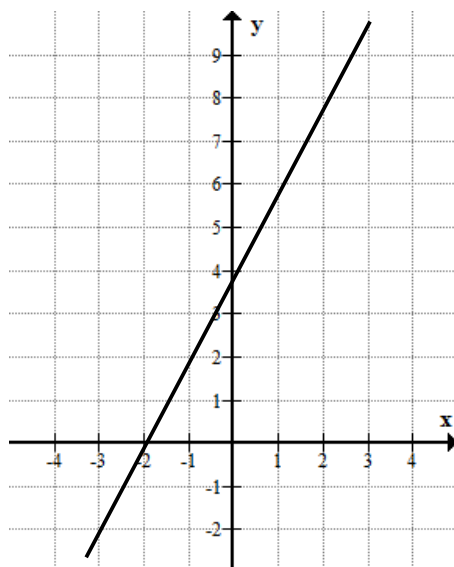
O. Practice questions

1. (a) $m_{AB} = \frac{4}{3}$ (b) $m_{AB} = -\frac{3}{4}$ (c) $M(\frac{7}{2}, 9)$ (d) $d = 5$ (e) $(8, 15)$

2. (a) $a = 1$ (b) $a = 11$ (c) $a = 5$ (d) $a = 0$ or $a = 6$

3. (a) (i) $m = 2$ (ii) $y = 4$ (iii) $x = -2$

(b)



(c) A does not lie on the line since $2 \times 7 + 4 = 18 \neq 19$ while B lies on the line since $2 \times 8 + 4 = 20$

4. (a) $m_{AB} = \frac{3}{2}$, $y - 4 = \frac{3}{2}(x - 3)$ (b) $y = \frac{3}{2}x - \frac{1}{2}$ (c) $3x - 2y = 1$

5. (i) $3x + 2y = 18$, For $x=0$ $2y = 18 \Rightarrow y = 9$ therefore A $(0, 9)$

(ii) For $y = 0$, $3x = 18 \Rightarrow x = 6$ therefore B $(6, 0)$

(iii) midpoint between $(0, 9)$ and $(6, 0)$: $(\frac{0+6}{2}, \frac{9+0}{2}) = (3, 4.5)$

6. (a) $x = 2$ (b) $y = 5$ (c) $P(2, 5)$

7. (a) $y = 3$ (b) $x = 2$ (c) $y = \frac{3}{2}x$

8. (a) (i) $m = -2$ (ii) $y = 5$ (iii) $x = 2.5$ ($= \frac{5}{2}$)

(b) $y = -2x + 5$

(c) $y = 3$, $x = 2$ (d) $a = 5$, $b = 15$

A. Exam style questions (SHORT)

9. (a) $y = -2x + 3$ gradient of line $L_1 = -2$

(b) **METHOD 1**

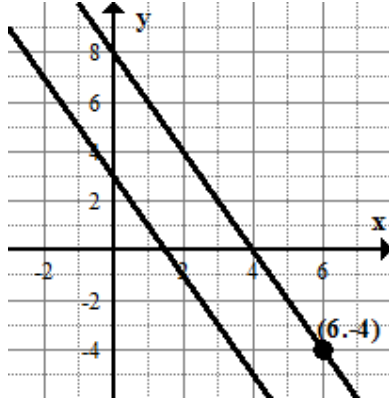
$$(y - y_1) = m(x - x_1) \Rightarrow (y - (-4)) = -2(x - 6)$$

$$y + 4 = -2x + 12 \Rightarrow y = -2x + 8$$

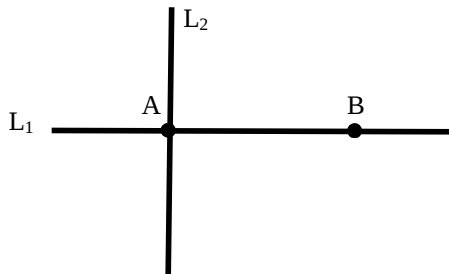
METHOD 2

Substituting the point $(6, -4)$ in $y = mx + c \Rightarrow c = 8$ so $y = -2x + 8$

- (c) when line L_1 cuts the x -axis, $y = 0 \Rightarrow y = -2x + 8 \Rightarrow x = 4$



10. (a) (i) $m_{AB} = 3$ (ii) $y = 3x - 1$ (b) $y = -\frac{1}{3}x + \frac{17}{3}$ (c) $3x - y = 1$ and $x + 3y = 17$
(d) $A(2, 5)$. The solution is in fact the point of intersection which is A as expected.



11. (a) $m_{AB} = \frac{2}{3}$ (b) $M(1, 7)$ (c) $y = -\frac{3}{2}x + \frac{17}{2}$ (d) $d = \sqrt{13}$
12. (a) $m_{AB} = \frac{2}{4} = \frac{1}{2}$, $m_{\perp} = -2$, Midpoint $(4, 2)$, $y - 2 = -2(x - 4)$ or $y = -2x + 10$
(b) $y = 10$

13. $m_{AB} = \frac{28}{12} = \frac{7}{3}$, $m_{\perp} = -\frac{3}{7}$,

Midpoint $(14, 27)$,

$$y - 27 = -\frac{3}{7}(x - 14)$$

$$\Leftrightarrow 7y - 189 = -3x + 42$$

$$\Leftrightarrow 3x + 7y = 231$$

14. Let $P(x, x+1)$ be on L_1 .

$$\sqrt{x^2 + (x+1)^2} = 5 \Leftrightarrow x^2 + (x+1)^2 = 25 \Leftrightarrow x = -4 \text{ or } x = 3,$$

Therefore, the point is $P(-4, -3)$ or $P(3, 4)$

15. Let $A(a, a+1)$ and $B(b, 2b+1)$ be the points on L_1 and L_2 respectively. Then

$$\frac{a+b}{2} = 5 \text{ and } \frac{a+1+2b+1}{2} = 8,$$

that is

$$a+b=10$$

$$a+2b=14$$

hence $a=6, b=4$

and finally $A(6,7), B(4,9)$

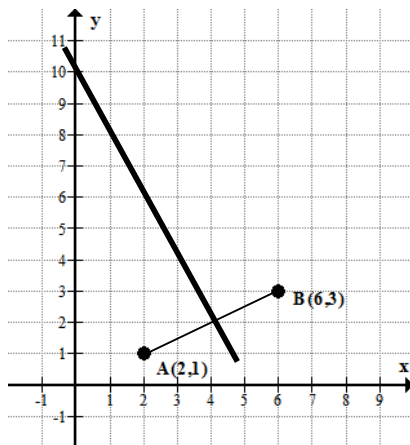
16. Let R be $(x_1, 5-x_1)$ and Q be $(\frac{1}{2}x_1 - 2, \frac{1}{2}x_1 - 2)$

Since P is the mid-point of $[QR]$

$$\frac{x_1 + x_2}{2} = 1 \text{ and } \frac{5-x_1 + \frac{1}{2}x_1 - 2}{2} = 1$$

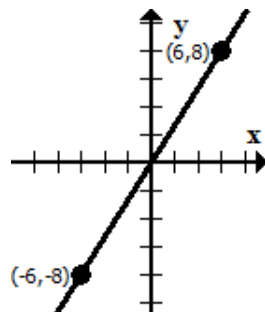
$$\Rightarrow x_1 + x_2 = 2 \text{ and } -2x_1 + x_2 = -2$$

$$\Rightarrow x_1 = \frac{4}{3} \text{ and } x_2 = \frac{2}{3} \Rightarrow R \text{ is } \left(\frac{4}{3}, \frac{11}{3} \right) \text{ and } Q \text{ is } \left(\frac{2}{3}, \frac{5}{3} \right)$$

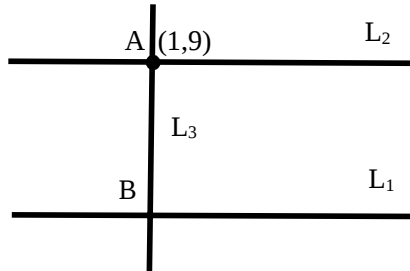


B. Exam style questions (LONG)

17. (a) For $x = 3k$, we obtain $y = 4k$
 (b) $\sqrt{(3k)^2 + (4k)^2} = 10 \Leftrightarrow 25k^2 = 100 \Leftrightarrow k^2 = 4 \Leftrightarrow k = \pm 2$ (or by GDC)
 (c) $(6, 8)$ and $(-6, -8)$
 (d)



18. (a) $L_2 : y = 2x + 7$ $L_2 : y = -\frac{1}{2}x + \frac{19}{2}$
 (b) B(5,7)
 (c) $d = \sqrt{20} = 2\sqrt{5}$
 (d) In fact the distance from A to the line L_1 is $2\sqrt{5}$



19. (a) $x=5$ (b) $y=5$ (c) $P(5,5)$ $\frac{3+7}{2}=5$ $\frac{2+8}{2}=5$
 (d) (i) 12 (ii) 6 (iii) 6
 (e) $m_{BC} = -\frac{3}{2}$, $m_{\perp} = \frac{2}{3}$, Midpoint (5,5), $y-5 = \frac{2}{3}(x-5) \Rightarrow 2x-3y = -5$
 (f) For $x=3$, $y = \frac{11}{3} \neq 2$
20. (a) A(6,-1) (b) (i) B(-2,7) (ii) C(-2,-5) (c) Area = $\frac{12 \times 8}{2} = 48$