INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 1.2-1.3]

ARITHMETIC SEQUENCES

O. Practice questions

SEQUENCES (IN GENERAL)

1. [Maximum mark: 5] ***[without GDC]***

Consider the sequence u_n : 2, 5, 10, 3, 7, 4, ….

(notice that there is no particular pattern in this sequence)

(a) Write down the values of u_1 and S_1. [2]

(b) Write down the values of u_2 and S_2. [2]

(c) Find S_5. [1]

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2. [Maximum mark: 5] ***[without GDC]***

Consider the (simplest) sequence u_n : 1, 2, 3, 4, 5, 6, ….

(a) Write down the values of u_2 and u_{20}. [2]

(b) Find the value of S_5. [2]

(c) Express the value of u_n in terms of n. [1]

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3. [Maximum mark: 6] ***[without GDC]***

Consider the sequence u_n given by $u_n = 10n$

(a) Find the first three terms . [2]

(b) Find u_{10} [1]

(c) Find S_1, S_2 and S_3. [3]

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4. [Maximum mark: 6] ***[without GDC]***

Consider the sequence u_n given by $u_n = 10n^2$

(a) Find the first three terms . [2]

(b) Find u_{10}. [1]

(c) Find S_1, S_2 and S_3. [3]

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5. [Maximum mark: 6] ***[with / without GDC]***

Consider the sequence u_n given by $u_n = 10^n$

(a) Find the first three terms . [2]

(b) Find u_{10}. [1]

(c) Find S_1, S_2 and S_3. [3]

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6. [Maximum mark: 6] ***[with / without GDC]***

Consider the sequence u_n given by the recursive definition

$$u_1 = 10\,, \quad u_{n+1} = u_n + 10$$

(a) Find the first three terms . [3]

(b) Find S_1, S_2 and S_3. [3]

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7. [Maximum mark: 6] ***[with / without GDC]***

Consider the sequence u_n given by the recursive definition

$$u_1 = 10\,, \quad u_{n+1} = 2u_n + 10$$

(a) Find the first three terms . [3]

(b) Find S_1, S_2 and S_3. [3]

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8. [Maximum mark: 6] ***[with / without GDC]***

(a) Express $\sum_{r=1}^{3}(2r)$ as a sum of three terms and find the result. [2]

(b) Express $\sum_{r=1}^{3} r^2$ as a sum of three terms and find the result. [2]

(c) Express $\sum_{r=1}^{3} 2^r$ as a sum of three terms and find the result. [2]

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9. [Maximum mark: 5] ***[with GDC]***

(a) Find $A = \sum_{r=1}^{10}(2r^2+1)$, $B = \sum_{r=1}^{20}(2r^2+1)$, $C = \sum_{r=11}^{20}(2r^2+1)$. [3]

(b) Find a relation between A, B and C. [2]

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ARITHMETIC SEQUENCES

10. [Maximum mark: 10] ***[with / without GDC]***

Consider the arithmetic sequence 11, 15, 19, 23, …

(a) Find u_{101} and S_{101} [4]

(b) Find the sum of the first 20 terms. [2]

(c) Express the general term u_n in the form $u_n = an + b$. [2]

(d) **Hence** find the value of n given that $u_n = 51$. [2]

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11. [Maximum mark: 8] ***[without GDC]***

In an arithmetic sequence, the first term is –5, while the fifth term is 27.

(a) Find the common difference d. [2]

(b) Find the eleventh term of the sequence. [2]

(c) Find the sum of the first eleven terms. [2]

(d) If the n-th term is 155, find the value of n. [2]

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12. [Maximum mark: 7] ***[with / without GDC]***

The 5th term of an arithmetic sequence is 30, while the 13th term is 70.

(a) Find the first term u_1 and the common difference d. [3]

(b) Find the general term u_n in terms of n. [1]

(c) Find the sum of the first n terms in terms of n. [1]

(d) Find the 20th term of the sequence and the sum of the first 20 terms. [2]

13. [Maximum mark: 6] ***[without GDC]***

In an arithmetic sequence, let $S_1 = 10$, and $S_2 = 25$

(a) Write down u_1 and u_2 . [2]

(b) Find the common difference d . [1]

(c) Find S_3 and S_4 . [2]

(d) Express S_n in terms of n . [1]

14* [Maximum mark: 8] ***[with / without GDC]***

Calculate the following sums

(a) (i) $\sum_{r=1}^{3}(2r+1)$ (ii) $\sum_{r=1}^{200}(2r+1)$ (iii) $\sum_{r=4}^{200}(2r+1)$ [5]

(b) $\sum_{r=101}^{200}(2r+1)$ [3]

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15*. [Maximum mark: 7] ***[with / without GDC]***

Consider the arithmetic sequence 11, 15, 19, 23, …

(a) Find the number of terms which are less than 100. [3]

(b) Find the last term which is less than 100. [2]

(c) Find the sum of all terms which are less than 100. [2]

A. Exam style questions (SHORT)

16. [Maximum mark: 4] ***[with / without GDC]***

The first three terms of an arithmetic sequence are 7, 9.5, 12.

(a) What is the 41st term of the sequence? [2]

(b) What is the sum of the first 101 terms of the sequence? [2]

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17. [Maximum mark: 4] ***[without GDC]***

An arithmetic series has five terms. The first term is 2 and the last term is 32. Find the sum of the series.

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18. [Maximum mark: 6] ***[without GDC]***

In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

(a) Find d. [2]

(b) Find u_{20}. [2]

(c) Find S_{20}. [2]

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19. [Maximum mark: 4] ***[without GDC]***

In an arithmetic sequence, the first term is 5 and the fourth term is 40. Find the second term.

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20. [Maximum mark: 6] ***[with GDC]***

Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 m more than the previous week. He continues for one year (52 weeks).

(a) How far does Arturo swim in the final week? [3]

(b) How far does he swim altogether? [3]

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21. [Maximum mark: 6] ***[with GDC]***

A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

(a) Calculate the number of seats in the 20th row. [4]

(b) Calculate the **total** number of seats. [2]

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22. [Maximum mark: 4] ***[with GDC]***

Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.

(a) On which day does she run a distance of 10 km in training? [2]

(b) What is the total distance she will have run in training by the end of that day? Give your answer exactly. [2]

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23. [Maximum mark: 4] ***[with GDC]***

A woman deposits $100 into her son's savings account on his first birthday. On his second birthday she deposits $125, $150 on his third birthday, and so on.

(a) How much money would she deposit into her son's account on his 17th birthday? [2]

(b) How much in total would she have deposited after her son's 17th birthday? [2]

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24. [Maximum mark: 6] ***[with GDC]***

A teacher earns an annual salary of 45 000 USD for the first year of her employment

Her annual salary increases by 1750 USD each year.

(a) Calculate the annual salary for the fifth year of her employment. [3]

She remains in this employment for 10 years.

(b) Calculate the **total** salary she earns in this employment during these 10 years. [3]

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25. [Maximum mark: 6] ***[with GDC]***

A concert choir is arranged, per row, according to an arithmetic sequence. There are 20 singers in the fourth row and 32 singers in the eighth row.

(a) Find the common difference of this arithmetic sequence. [3]

There are 10 rows in the choir and 11 singers in the first row.

(b) Find the **total** number of singers in the choir. [3]

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26. [Maximum mark: 4] ***[with GDC]***

A tree begins losing its leaves in October. The number of leaves that the tree loses each day increases by the same number on each successive day.

Date in October	1	2	3	4	.....
Number of leaves lost	24	40	56	72	.....

(a) Calculate the number of leaves that the tree loses on the 21st October. [2]

(b) Find the total number of leaves that the tree loses in the 31 days of the month of October. [2]

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27. [Maximum mark: 5] ***[with GDC]***

In an arithmetic sequence $u_1 = 7$, $u_{20} = 64$ and $u_n = 3709$.

(a) Find the value of the common difference. [3]

(b) Find the value of n. [2]

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28. [Maximum mark: 4] ***[without GDC]***

The second term of an arithmetic sequence is 7. The sum of the first four terms of the arithmetic sequence is 12. Find the first term, u_1, and the common difference, d.

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29. [Maximum mark: 4] ***[without GDC]***

In an arithmetic sequence the second term is 7 and the sum of the first five terms is 50. Find the common difference of this arithmetic sequence.

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30. [Maximum mark: 6] ***[with GDC]***

An arithmetic sequence, u_1, u_2, u_3, ..., has $d = 11$ and $u_{27} = 263$.

(a) Find u_1. [2]

(b) (i) Given that $u_n = 516$, find the value of n.

(ii) For this value of n, find S_n. [4]

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31. [Maximum mark: 6] ***[with / without GDC]***

Consider the arithmetic sequence 2, 5, 8, 11, …

(a) Find u_{101}. [3]

(b) Find the value of n so that $u_n = 152$. [3]

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32. [Maximum mark: 6] ***[with GDC]***

Consider the arithmetic series $2 + 5 + 8 + ...$

(a) Find an expression for S_n, the sum of the first n terms.

(b) Find the value of n for which $S_n = 1365$.

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33. [Maximum mark: 6] ***[without GDC]***

In an arithmetic sequence, the first term is –2, the fourth term is 16, and the nth term is 11 998.

(a) Find the common difference d. [3]

(b) Find the value of n. [3]

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34. [Maximum mark: 7] ***[with / without GDC]***

In an arithmetic sequence $u_{21} = -37$ and $u_4 = -3$.

(a) Find (i) the common difference; (ii) the first term. [4]

(b) Find S_{10}. [3]

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35. [Maximum mark: 5] ***[with GDC]***

In an arithmetic series, the first term is –7 and the sum of the first 20 terms is 620.

(a) Find the common difference. [3]

(b) Find the value of the 78th term. [2]

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36. [Maximum mark: 6] ***[without GDC]***

(a) Write down the first three terms of the sequence $u_n = 3n$, for $n \geq 1$. [1]

(b) Find (i) $\sum_{n=1}^{20} 3n$; (ii) $\sum_{n=21}^{100} 3n$. [5]

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<u>Notice</u>

There are two methods for question b(ii). Practice on both of them.

Method 1: By expressing $\sum_{n=21}^{100} 3n$ as a difference of two sums

$$\sum_{n=21}^{100} 3n = \sum_{n=1}^{100} 3n - \sum_{n=1}^{20} 3n = S_{100} - S_{20} \quad \text{(where } u_1 = 3 \text{ and } d = 3\text{)}$$

Method 2: By expanding $\sum_{n=21}^{100} 3n$

$$\sum_{n=21}^{100} 3n = 63 + 66 + 69 + \cdots + 300 = S_{80} \quad \text{(where } u_1 = 63 \text{ and } d = 3\text{)}$$

37. [Maximum mark: 6] ***[without GDC]***

Let $u_n = 3 - 2n$.

(a) Write down the value of u_1 , u_2 , and u_3 . [3]

(b) Find $\sum_{n=1}^{20} (3 - 2n)$. [3]

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38. [Maximum mark: 6] ***[with / without GDC]***

The n th term of an arithmetic sequence is given by $u_n = 5 + 2n$.

(a) Write down the common difference. [1]

(b) (i) Given that the n th term of this sequence is 115, find the value of n .

(ii) For this value of n , find the sum of the sequence. [5]

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39. [Maximum mark: 6] ***[with GDC]***

Consider the arithmetic sequence 3, 9, 15, ..., 1353.

(a) Write down the common difference. [1]

(b) Find the number of terms in the sequence. [3]

(c) Find the sum of the sequence. [2]

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40. [Maximum mark: 6] ***[with GDC]***

In an arithmetic sequence, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of u_1 and of d.

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41. [Maximum mark: 5] ***[with GDC]***

In an arithmetic sequence $u_1 = 7$, $u_{20} = 64$ and $u_n = 3709$.

(a) Find the value of the common difference. [3]

(b) Find the value of n. [2]

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42. [Maximum mark: 7] ***[with / without GDC]***

In an arithmetic sequence, the 5th term is 27, the 16th term is 115, while the n-th term is 155. Find

(a) The value of n. [5]

(b) The sum of the first n terms. [2]

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43. [Maximum mark: 5] ***[with GDC]***

An arithmetic sequence has 5 and 13 as its first two terms respectively.

(a) Write down, in terms of n, an expression for the nth term, u_n. [2]

(b) Find the number of terms of the sequence which are less than 400. [3]

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44. [Maximum mark: 6] ***[without GDC]***

Let S_n be the sum of the first n terms of an arithmetic sequence, whose first three terms are u_1, u_2 and u_3. It is known that $S_1 = 7$, and $S_2 = 18$.

(a) Write down u_1. [1]

(b) Calculate the common difference. [3]

(c) Calculate u_4. [2]

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45. [Maximum mark: 6] ***[with GDC]***

Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \ldots + 3750 = s.$$

Calculate s .

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46*. [Maximum mark: 6] ***[with / without GDC]***

An 81 metre rope is cut into n pieces of increasing lengths that form an arithmetic sequence with a common difference of d metres. Given that the lengths of the shortest and longest pieces are 1.5m and 7.5m respectively, find the values of n and d

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47. [Maximum mark: 4] ***[with GDC]***

Find the sum of the arithmetic series $17 + 27 + 37 + ... + 417$.

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48. [Maximum mark: 6] ***[with / without GDC]***

Calculate the sums (a) $7 + 10 + 13 + 16 + \cdots + 157$ (b) $\sum_{r=1}^{51}(3r+4)$

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49*. [Maximum mark: 6] ***[with / without GDC]***

Consider the sum of the arithmetic series $5+9+13+17+\cdots+85$

(a) Find the general term u_n in the form $an+b$ [2]

(b) **Hence**, express the sum of the series in sigma notation. [4]

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50*. [Maximum mark: 5] ***[with / without GDC]***

Find the sum of the positive terms of the arithmetic sequence $85, 78, 71, ...$

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51*. [Maximum mark: 6] ***[without GDC]***

Mr Jones decides to increase the amount of money he spends on food by d GBP every year. In the first year he spends a GBP. In the 8th year he spends twice as much as in the 4th year. In the 20th year he spends 4000 GBP. Find the value of d.

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52*. [Maximum mark: 5] ***[with GDC]***

Consider the arithmetic series $-6+1+8+15+\cdots$

Find the least number of terms so that the sum of the series is greater than 10 000.

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53. [Maximum mark: 5] ***[without GDC]***

The first three terms of an arithmetic sequence are $a+3$, $2a+4$ and $a+9$.

(a) Find the value of a. [3]

(b) Confirm the result. [2]

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54*. [Maximum mark: 6] ***[with GDC]***

The first four terms of an arithmetic sequence are 2, $a-b$, $2a+b+7$ and $a-3b$, where a and b are constants. Find a and b.

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55.** [Maximum mark: 7] ***[without GDC]***

The ratio of the fifth term to the twelfth term of a sequence in an arithmetic progression is $\frac{6}{13}$. If each term of this sequence is positive, and the product of the first term and the third term is 32, find the sum of the first 100 terms of this sequence.

56*. [Maximum mark: 5] ***[without GDC]***

The sum of the first n terms of an arithmetic sequence is $S_n = 3n^2 - 2n$.

(a) Find u_1 and u_2. [2]

(b) Express the nth term u_n in terms of n. [3]

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57.** [Maximum mark: 6] ***[without GDC]***

The sum of the first n terms of a series is given by $S_n = 2n^2 - n$, where $n \in \mathbb{Z}^+$.

(a) Find the first three terms of the series. [3]

(b) Find an expression for the nth term of the series, giving your answer in terms of n. [3]

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Notice:

Mind the difference between the two questions. The second one does not mention that the sequence is arithmetic, thus for u_n you have to use the formula $u_n = S_n - S_{n-1}$.

58.** [Maximum mark: 8] ***[without GDC]***

The sum of the first n terms of a sequence is given by

$$S_n = 4n^2 + n \text{, where } n \in \mathbb{Z}^+.$$

(a) Find S_4, S_5 and hence u_5. [3]

(b) Find an expression for u_n, the nth term of the sequence. [3]

(c) Show that the sequence is arithmetic by checking the difference $u_n - u_{n-1}$. [2]

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B. Exam style questions (LONG)

59*. [Maximum mark: 10] ***[with GDC]***

Consider the sequence

$$5, 9, 13, 17, \cdots$$

Find

(a) the number of terms which are less than 100. [3]

(b) the greatest term which is less than 100. [2]

(c) the sum of the terms which are less than 100. [2]

(d) the greatest value of n such that $S_n < 1000$ [3]

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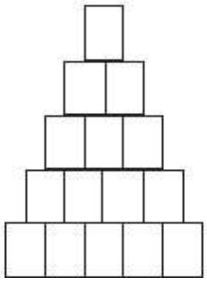
60. [Maximum mark: 14] ***[with / without GDC]***

Clara organizes cans in triangular piles, where each row has one less can than the row below. For example, the pile of 15 cans shown has 5 cans in the bottom row and 4 cans in the row above it.



(a) A pile has 20 cans in the bottom row. Show that the pile contains 210 cans. [4]

(b) There are 3240 cans in a pile. How many cans are in the bottom row? [4]

(c) (i) There are S cans and they are organized in a triangular pile with n cans in the bottom row. Show that $n^2 + n - 2S = 0$.

(ii) Clara has 2100 cans. Explain why she cannot organize them in a triangular pile. [6]

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61. [Maximum mark: 10] ***[with GDC]***

Consider the arithmetic sequence 1, 4, 7, 10, 13, …

(a) Find the value of the eleventh term. [2]

(b) Show that the sum of the first n terms of this sequence is $S_n = \frac{n}{2}(3n-1)$. [2]

(c) **Hence** find the sum of the first 100 terms in this arithmetic sequence. [2]

(d) The sum of the first n terms is 477.

(i) Show that $3n^2 - n - 954 = 0$.

(ii) **Hence** find the number of terms, n. [4]

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62. [Maximum mark: 10] ***[without GDC]***

The first three terms of an arithmetic sequence are

$$2k + 3,\ 5k - 2 \text{ and } 10k - 15.$$

(a) Show that $k = 4$. [3]

(b) Find the values of the first three terms of the sequence. [2]

(c) Write down the value of the common difference. [1]

(d) Calculate the 20th term of the sequence. [2]

(e) Find the sum of the first 15 terms of the sequence. [2]

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63*. [Maximum mark: 12] ***[without GDC]***

The first three terms of an arithmetic sequence are $x^2 - 3$, x^2 and $4x$

(a) When $x = 3$, the sequence is arithmetic.

(i) Write down the first three terms.

(ii) Find the common difference. [3]

(c) Find the other value of x for which the sequence is arithmetic. [4]

(d) For this value of x (found in question (c))

(i) write down the first three terms.

(ii) find the common difference.

(iii) find the sum of the first 4 terms [5]

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64.** [Maximum mark: 15] ***[with GDC]***

Find

(a) the sum of all multiples of 8 between 1 and 900. [3]

(b) the sum of all multiples of 8 between 100 and 900. [3]

(c) the sum of all numbers which are multiples **of both 8 and 6**, between 1 and 900. [3]

(d) the sum of all numbers which are multiples **of 8 but not of 6**, between 1 and 900. [3]

(e) the sum of all numbers which are multiples **of 8 or of 6**, between 1 and 900. [3]

65.** [Maximum mark: 15] ***[with GDC]***

Consider the positive integers grouped in the following way:

[1], [2,3,4,5], [6,7,8,9,10,11,12], [13,14,…,22], ...

i.e. the groups contain 1, 4, 7, 10, … consecutive integers respectively.

(each subsequent group contains 3 more integers)

(a) Find

(i) the number of elements in the 20th group.

(ii) the last term of the 20th group. [5]

(b) Find in terms of n

(i) the number of elements in the nth group.

(ii) the last term of the nth group. [5]

(c) Hence find the first term of the 20th group. [2]

(d) Show that the sum of the terms in the 20th group is 32567. [3]

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