

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches
MAA

EXERCISES [MAA 1.11-1.12]
COMPLEX NUMBERS (CARTESIAN FORM)

CARTESIAN FORM

O. Practice questions

1. [Maximum mark: 6] **[without GDC]**

Let $f(z) = z^2 - 8z + 20$.

- (a) Find the discriminant Δ of the quadratic function f . [1]
(b) Find the complex roots of the equation $f(z) = 0$ in the form $z = x \pm yi$ [3]
(c) Use factorisation to express f in the form $f(z) = (z - h)^2 + k$. [2]

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4. [Maximum mark: 6] **[without GDC]**

(a) Find $(1 - i\sqrt{3})^2$ in the form $a + bi$, where $a, b \in \mathbb{Z}$. [3]

(b) Find $(1 - i\sqrt{3})^3$. [3]

[Confirm the results by your GDC]

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5. [Maximum mark: 4]

(a) Given that $(a - 2) + 3i = 7 + (b - 1)i$, find the value of a and of b , where $a, b \in \mathbb{Z}$. [2]

(b) Given that $(c - 2) + (d - 1)i = 0$, find the value of c and of d , where $c, d \in \mathbb{Z}$. [2]

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A. Exam style questions (SHORT)

7. [Maximum mark: 6] *[without GDC]*

Let the complex number z be given by $z = 1 + \frac{i}{i - \sqrt{3}}$.

Express z in the form $a + bi$, giving the **exact** values of the real constants a , b .

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8. [Maximum mark: 5] *[without GDC]*

Express $\frac{1}{(1 - i\sqrt{3})^3}$ in the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}$.

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9. [Maximum mark: 6] **[with / without GDC]**

Let $z = \frac{2}{1-i} + 1 - 4i$. Express z^2 in the form $x + yi$ where $x, y \in \mathbb{Z}$.

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10. [Maximum mark: 6] **[without GDC]**

Consider the equation $2(p + iq) = q - ip - 2(1 - i)$, where p and q are both real numbers. Find p and q .

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11. [Maximum mark: 6] **[without GDC]**

Given that $(a + i)(2 - bi) = (7 - i)$, find the value of a and of b , where $a, b \in \mathbb{Z}$.

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12. [Maximum mark: 4] **[with / without GDC]**

Find the values of a and b , where a and b are real, given that $(a + bi)(2 - i) = 5 - i$

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13. [Maximum mark: 4] **[with without GDC]**

Let $z = x + yi$. Find the values of x and y if $(1-i)z = 1-3i$.

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14. [Maximum mark: 5] **[without GDC]**

The complex number z satisfies $i(z+2) = 1-2z$, where $i = \sqrt{-1}$. Write z in the form $z = a + bi$, where a and b are real numbers.

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15. [Maximum mark: 6] **[without GDC]**

The two complex numbers $z_1 = \frac{a}{1+i}$ and $z_2 = \frac{b}{1-2i}$ where $a, b \in \mathbb{R}$, are such that

$z_1 + z_2 = 3$. Calculate the value of a and of b .

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16. [Maximum mark: 6] **[without GDC]**

Solve the following equation for z , where z is a complex number.

$$\frac{z}{3+4i} + \frac{z-1}{5i} = \frac{5}{3-4i}$$

Give your answer in the form $a+bi$, where $a, b \in \mathbb{Z}$

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18. [Maximum mark: 6] **[without GDC]**

Given that $|z| = 2\sqrt{5}$, find the complex number z that satisfies the equation

$$\frac{25}{z} - \frac{15}{z^*} = 1 - 8i.$$

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19. [Maximum mark: 6] **[with / without GDC]**

Let z_1 and z_2 be complex numbers. Solve the simultaneous equations

$$2z_1 + 3z_2 = 7$$

$$z_1 + iz_2 = 4 + 4i$$

Give your answers in the form $z = a + bi$, where $a, b \in \mathbb{Z}$.

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0. Practice questions

22. [Maximum mark: 7] *[without GDC]*

Consider the polynomial $f(z) = z^3 - 3z^2 + 7z - 5$

- (a) Confirm that 1 is a root of $f(z)$ [1]

It is given that $f(1+2i) = 0$

- (b) Write down the two complex roots of $f(z)$. [1]

- (c) Write down the three linear factors of $f(z)$ (with complex coefficients). [3]

- (d) Express $f(z)$ in the form $(z-a)(z^2+bz+c)$ where $a, b, c \in \mathbb{Z}$. [2]

[illegible]

A. Exam style questions (SHORT)

27. [Maximum mark: 5] *[without GDC]*

$(z + 2i)$ is a factor of $2z^3 - 3z^2 + 8z - 12$. Find the other two factors.

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28. [Maximum mark: 6] *[without GDC]*

Let $P(z) = z^3 + az^2 + bz + c$, where a, b and $c \in \mathbb{R}$. Two of the roots of $P(z) = 0$ are -2 and $(-3 + 2i)$. Find the value of a , of b and of c .

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29. [Maximum mark: 6] **[without GDC]**

The polynomial $P(z) = z^3 + mz^2 + nz - 8$ is divisible by $(z + 1 + i)$, where $z \in \mathbb{C}$ and $m, n \in \mathbb{R}$. Find the value of m and of n .

METHOD A: Substitute the solution $z = -1 - i$.

[not the ideal way but good for practice!]

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[Much quicker!]

[illegible]

(c) Hence or otherwise, find $(1+i)^{32}$. [2]

[illegible]

[illegible]

