

HL Functions and Graphs IB Exam Questions Worksheet

1. Find the equations of all the asymptotes of the graph of $y = \frac{x^2 - 5x - 4}{x^2 - 5x + 4}$. N02 P1 510 Q4

2. The one-one function f is defined on the domain $x > 0$ by $f(x) = \frac{2x-1}{x+2}$.
 - a) State the range, A , of f .
 - b) Obtain an expression for $f^{-1}(x)$, for $x \in A$. M02 P1 510 Q15

3. The functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + x$. The function $(f \circ g)(x)$ is defined for $x \in \mathbb{R}$, **except** for the interval $]a, b[$.
 - a) Calculate the value of a and b .
 - b) Find the range of $f \circ g$. N02 P1 510 Q7

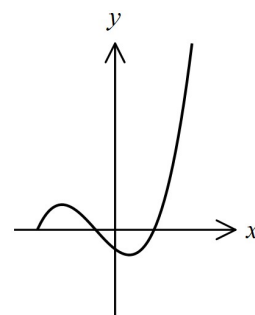
4.
 - a) Find the largest set S of values of x such that the function $f(x) = \frac{1}{\sqrt{3-x^2}}$ takes real values.
 - b) Find the range of the function f defined on the domain S . N04 P1 TZ0 Q7

5. Let f and g be two functions. Given that $(f \circ g)(x) = \frac{x+1}{2}$ and $g(x) = 2x-1$, find $f(x-3)$. N05 P1 TZ0 Q13

6. The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x-2) - \ln(x^2-4)$.
 - a) Express $f(x)$ in the form $\ln\left(\frac{x}{x+a}\right)$.
 - b) Find an expression for $f^{-1}(x)$. M05 P1 TZ2 Q19

7. Find the largest set of values of x such that the function f given by $f(x) = \sqrt{\frac{8x-4}{x-3}}$ takes real values. N05 P1 TZ0 Q19

8. The graph below shows the graph of a function f . Make two copies of this function.
 - a) On your first copy of the function, sketch the graph of $|f(-x)|$.
 - b) On your second copy of the function, sketch the graph of $\frac{1}{f(x)}$.



01 TZ0 Q20

9. For $0 \leq x \leq 6$, find the coordinates of the points of intersection of the curves $y = x^2 \cos x$ and $x + 2y = 1$.

M04 P1 511 Q3

10. a) Sketch the graph of $y = \frac{x-12}{\sqrt{x^2-4}}$.

b) i) Write down the x -intercept.

ii) Write down the equations of all asymptotes.

M05 P1 TZ1 Q16

11. The function f is defined by $f: x \rightarrow x^3$. Find an expression for $g(x)$ in terms of x in each of the following cases.

a) $(f \circ g)(x) = x + 1$.

b) $(g \circ f)(x) = x + 1$.

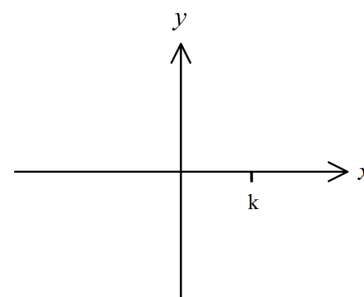
M04 P1 512 Q11

12. Let $f(x) = \frac{k}{x-k}$, $x \neq k$, $k > 0$.

Make two copies of the set of axes on the right.

- a) On your first set of axes, sketch the graph of f . Label clearly any points of intersection with the axes and any asymptotes.

- b) On your second set of axes, sketch the graph of $\frac{1}{f}$.
Label clearly any points of intersection with the axes.



M04 P1 512 Q10

13. Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$.

Find the set of values of x such that $f(x) \leq g(x)$.

N03 P1 510 Q17

14. The function f is defined for $x \leq 0$ by $f(x) = \frac{x^2-1}{x^2+1}$.

Find an expression for $f^{-1}(x)$.

M03 P1 512 Q17

15. a) On the same axes, sketch the graphs of the functions of $f(x)$ and $g(x)$,
where: $f(x) = 4 - (1-x)^2$, for $-2 \leq x \leq 4$,
and $g(x) = \ln(x+3) - 2$, for $-3 \leq x \leq 5$.

- b) i) Write down the equation of any vertical asymptotes.

ii) State the x -intercept and the y -intercept of $g(x)$.

- c) Find the values of x for which $f(x) = g(x)$.

N02 P2 510 Q3

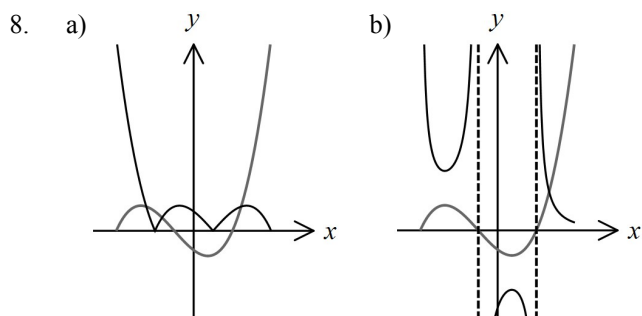
Solutions

1. Asymptotes are $y = 1$, $x = 4$, $x = 1$ 2. a) $A \geq]\frac{-1}{2}, 2[$ or $\frac{-1}{2} < y < 2$ b) $f^{-1}(x) = \frac{1+2x}{2-x}$

3. a) $a = -2$, $b = 1$ b) Range: $y \geq 0$ 4. a) Domain: $-\sqrt{3} < x < \sqrt{3}$ b) Range: $f(x) \geq \frac{1}{\sqrt{3}}$

5. $f(x-3) = \frac{x}{4}$ 6. a) $f(x) = \ln\left(\frac{x}{x+2}\right)$ b) $f^{-1}(x) = -\frac{2e^x}{e^x-1}$

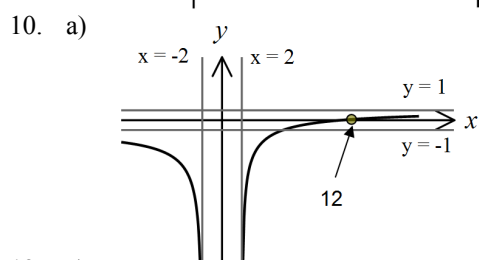
7. $-\infty < x \leq \frac{1}{2}$ and $3 < x \leq \infty$



9. $x = 0.524$, $y = 0.238$

$x = 1.69$, $y = -0.346$

$x = 4.63$, $y = -1.81$

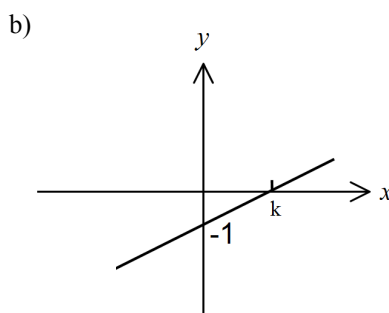
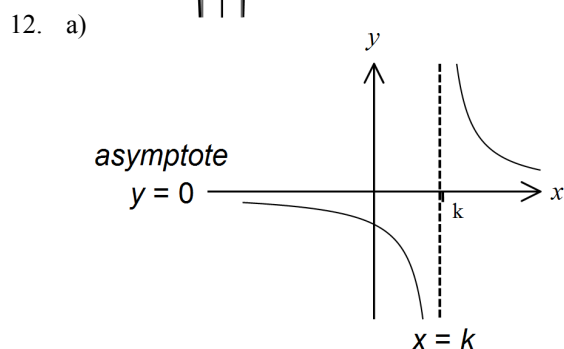


b) i) x -intercept = 12

ii) Asymptotes are:

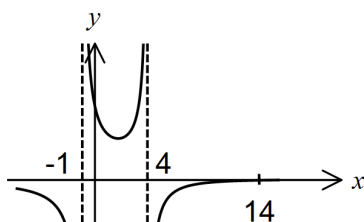
$x = -2$, $x = 2$ and $y = -1$, $y = 1$

11. a) $g(x) = \sqrt[3]{x+1}$ b) $g(x) = \sqrt[3]{x} + 1$

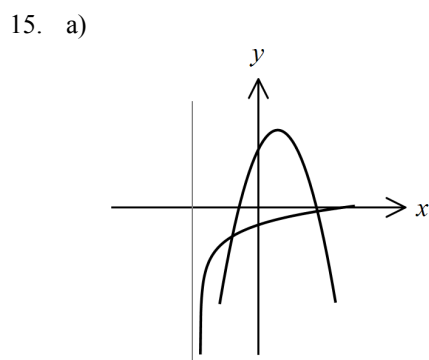


13. $f(x) - g(x) = \frac{x+4}{x+1} - \frac{x-2}{x-4}$

Therefore, $f(x) - g(x) \leq 0$
when $x < -1$ and $4 < x \leq 14$



14. $f^{-1}(x) = -\sqrt{\frac{1+x}{1-x}}$



b) i) Vertical asymptote is $x = -3$

ii) x -intercept = 4.39 or $e^2 - 3$

y -intercept = -0.901 or $\ln 3 - 2$

c) $x = -1.34$ or $x = 3.05$