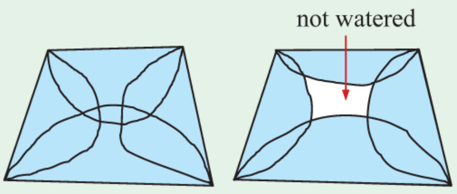
Chapter 19

Deductive geometry

Contents:

OPENING PROBLEM

Market gardener Joe has four long straight pipes of different lengths. He places the pipes on the ground, and joins them with rubber hose to form a garden bed in the shape of a quadrilateral. A sprinkler which casts water in semi-circles of diameter equal to the length of a pipe is placed at the midpoint of each pipe.



Joe's sketch Clinton's sketch

Joe draws a rough sketch of the watering system, and decides that his sprinklers will water the whole of the garden. His son Clinton is sceptical of his father's idea, and draws his own sketch which suggests that there will be an unwatered patch in the centre.

Things to think about:

- **a** By drawing an accurate diagram of this situation, can you determine whether Joe or Clinton is correct?
- **b** Can you prove that your answer is true using geometric theorems?

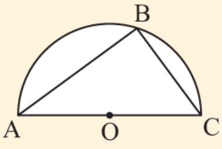
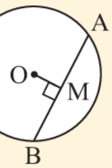
The geometry of triangles, quadrilaterals, and circles has been used for at least 3000 years in art, design, and architecture. Many amazing discoveries have been made by mathematicians and non-mathematicians who were simply drawing figures with rulers and compasses.

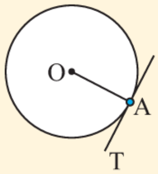
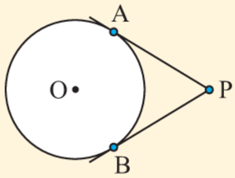
A CIRCLE THEOREMS

In geometry, we use logical reasoning to prove that certain observations about geometrical figures are true. We do this using special results called **theorems**.

CIRCLE THEOREMS

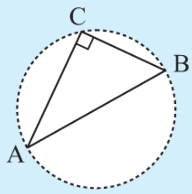
You should be familiar with the following **circle theorems**:

Name of theorem	*Statement*	*Diagram*
Angle in a semi-circle	The angle in a semi-circle is a right angle.	$A\widehat{B}C = 90^\circ$ 
Chords of a circle	The perpendicular from the centre of a circle to a chord bisects the chord.	$AM = BM$ 

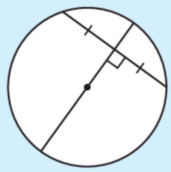
Name of theorem	*Statement*	*Diagram*
Radius-tangent	The tangent to a circle is perpendicular to the radius at the point of contact.	$O\widehat{A}T = 90°$ 
Tangents from an external point	Tangents from an external point are equal in length.	$AP = BP$ 

Two useful **converses** are:

- If line segment [AB] subtends a right angle at C, then the circle through A, B, and C has diameter [AB].

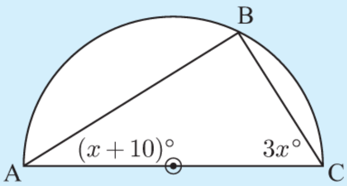


- The perpendicular bisector of a chord of a circle passes through its centre.



Example 1 **Self Tutor**

Find x, giving brief reasons for your answer.



$A\widehat{B}C$ measures $90°$ {angle in a semi-circle}

$\therefore\ (x+10)+3x+90=180$ {angles in a triangle}

$\therefore\ 4x+100=180$

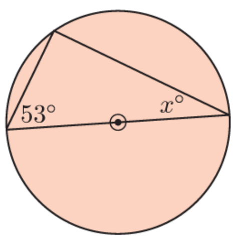
$\therefore\ 4x=80$

$\therefore\ x=20$

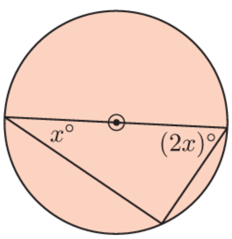
EXERCISE 19A

1 Find x, giving brief reasons for your answers:

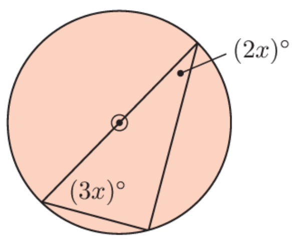
a



b

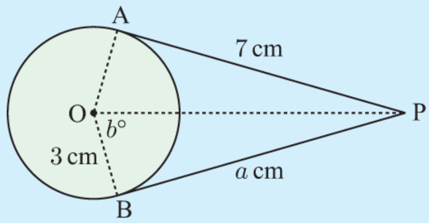


c



Example 2 **Self Tutor**

In the figure alongside, find a and b.



$\text{AP} = \text{BP}$ {tangents from an external point}

$\therefore \; a = 7$

Now, $\text{O}\widehat{\text{B}}\text{P} = 90°$ {radius-tangent theorem}

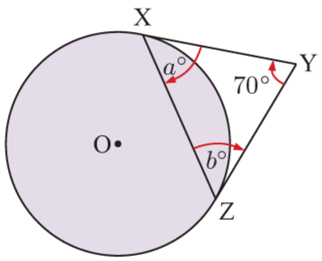
$\therefore$ in $\triangle\text{OBP}$, $\tan b° = \frac{7}{3}$

$\therefore \; b = \tan^{-1}(\frac{7}{3})$

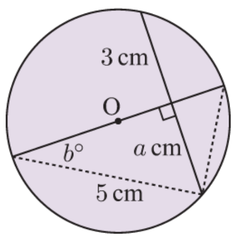
$\therefore \; b \approx 66.8$

2 Find a and b in the following figures:

a



b



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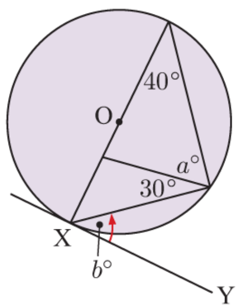
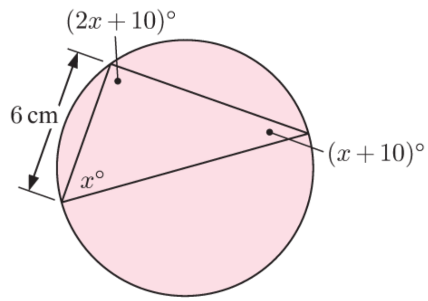
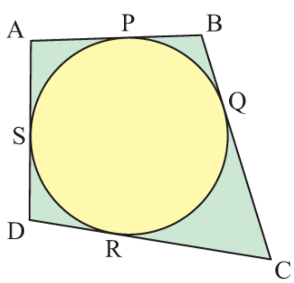


3 **a** Find x.

b Hence, find the diameter of the circle.



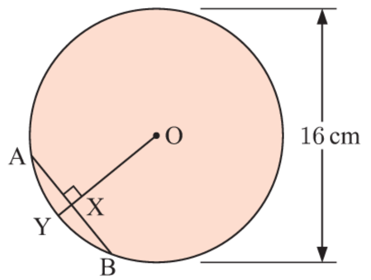
4



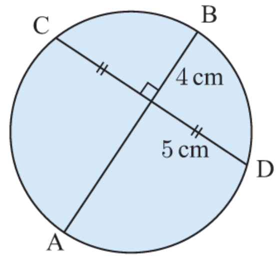
A circle is drawn, and four tangents to it are constructed as shown.

Deduce that $\text{AB} + \text{CD} = \text{BC} + \text{AD}$.

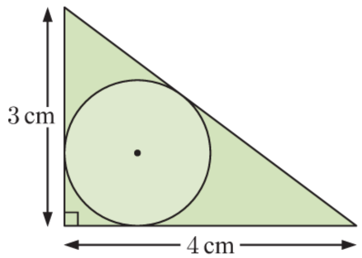
5 The chord [AB] is equal in length to the radius of the circle. Find the length of [XY].



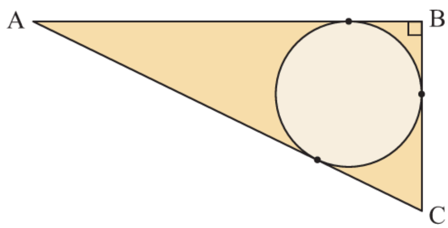
6 [AB] is the perpendicular bisector of the chord [CD].
Find the diameter of the circle.



7 A circle touches the three sides of the triangle as shown. Find the radius of the circle.



8 A circle is inscribed in a right angled triangle. The radius of the circle is 3 cm, and [BC] has length 8 cm.
Find the perimeter of the triangle ABC.



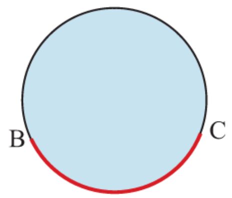
B FURTHER CIRCLE THEOREMS

Before we can explore additional circle theorems, we need some more terminology for describing the parts of a circle.

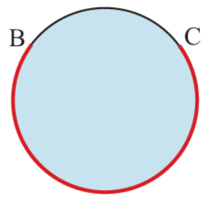
Any continuous part of a circle is called an **arc**.

If the arc is less than half the circle, it is called a **minor arc**.

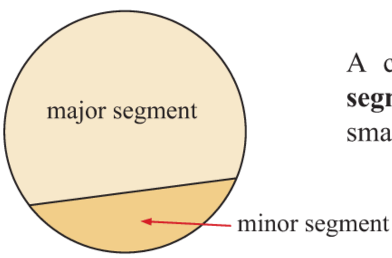
If the arc is greater than half the circle, it is called a **major arc**.



a minor arc BC



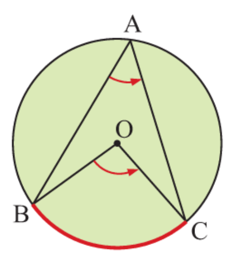
a major arc BC



A chord divides the interior of a circle into two regions called **segments**. The larger region is called a **major segment**, and the smaller region is called a **minor segment**.

In the diagram opposite:

- the minor arc BC **subtends** the angle BAC, where A is on the circle
- the minor arc BC also subtends angle BOC at the centre of the circle.

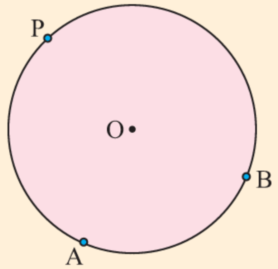


INVESTIGATION 1 FURTHER CIRCLE THEOREMS

The use of the **geometry package** is recommended, but this Investigation can also be done using a ruler, compass, and protractor.

Part 1: Angle at the centre theorem

GEOMETRY PACKAGE



1 Draw a large circle with centre O.
Mark on it points A, B, and P.

2 Join [AO], [BO], [AP], and [BP].

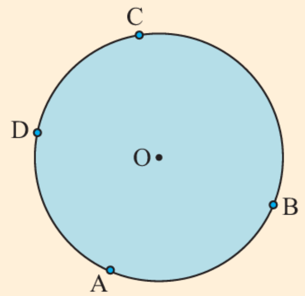
3 Measure angles AOB and APB.
What do you notice?

4 Repeat the above steps with another circle.

5 Copy and complete:
"*The angle at the centre of a circle is ...... the angle on the circle subtended by the same arc.*"

Part 2: Angles subtended by the same arc theorem

GEOMETRY PACKAGE



1 Draw a circle with centre O.
Mark on it points A, B, C, and D.

2 Join [AC], [BC], [AD], and [BD].

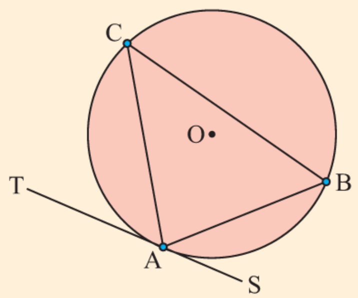
3 Measure angles ACB and ADB.
What do you notice?

4 Repeat the above steps with another circle.

5 Copy and complete:
"*Angles subtended by an arc on the circle are ...... in size.*"

Part 3: Angle between a tangent and a chord theorem

GEOMETRY PACKAGE



1 Draw a circle and mark on it points A, B, and C.

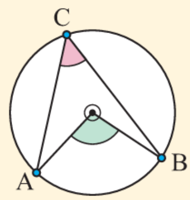
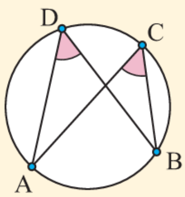
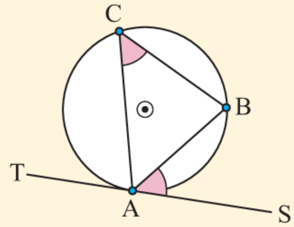
2 Draw tangent TAS at A, and join [AB], [BC], and [CA].

3 Measure angles BAS and ACB.
What do you notice?

4 Repeat the above steps with another circle.

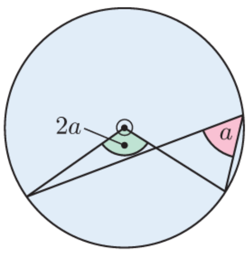
5 Copy and complete:
"*The angle between a tangent and a chord at the point of contact is ...... to the angle subtended by the chord in the alternate ......*"

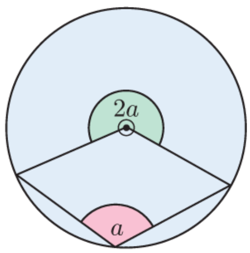
From the **Investigation** you should have discovered the following theorems:

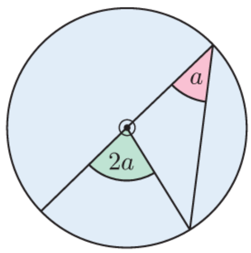
Name of theorem	*Statement*	*Diagram*
Angle at the centre	The angle at the centre of a circle is twice the angle on the circle subtended by the same arc.	$A\widehat{O}B = 2 \times A\widehat{C}B$ 
Angles subtended by the same arc	Angles subtended by an arc on the circle are equal in size.	$A\widehat{D}B = A\widehat{C}B$ 
Angle between a tangent and a chord	The angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment.	$B\widehat{A}S = A\widehat{C}B$ 

Note:

- The following diagrams show other cases of **the angle at the centre theorem**. These cases can be shown using the **geometry package**.

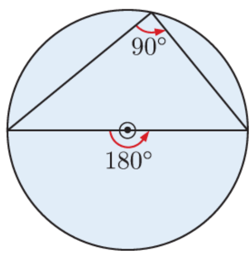






GEOMETRY PACKAGE

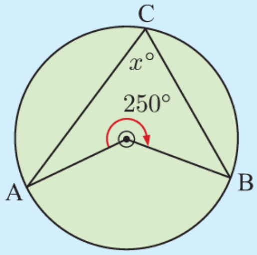
- The **angle in a semi-circle theorem** is a special case of the angle at the centre theorem.



Example 3 **Self Tutor**

Find x:

a



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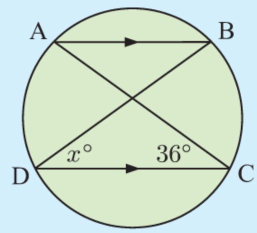


a Obtuse $\widehat{AOB} = 360° - 250°$ {angles at a point}

$\therefore\ \widehat{AOB} = 110°$

$\therefore\ 2x = 110$ {angle at the centre}

$\therefore\ x = 55$

b $\widehat{ABD} = 36°$ {angles on the same arc}

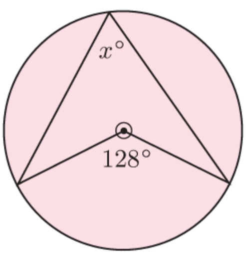
and $\widehat{BDC} = \widehat{ABD}$ {equal alternate angles}

$\therefore\ x = 36$

EXERCISE 19B

1 Find, giving reasons, the value of x:

a



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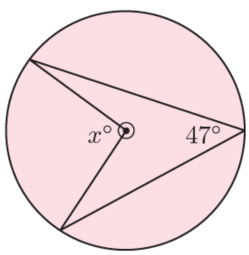

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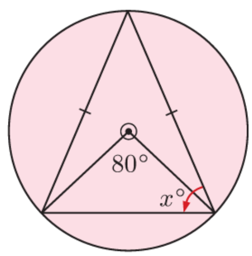

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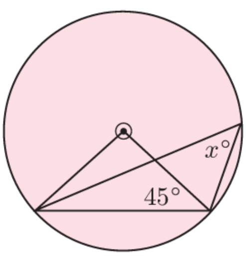

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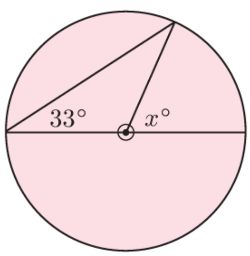

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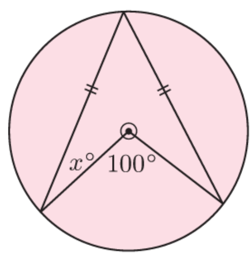
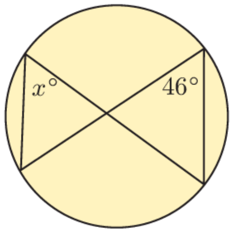


2 Find, giving reasons, the value of each pronumeral:

a



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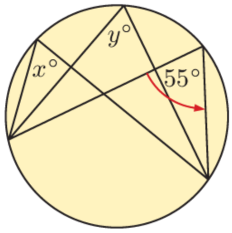

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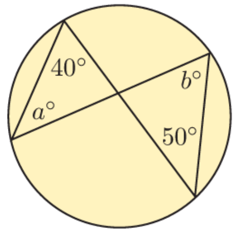

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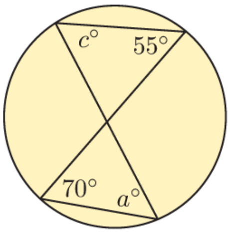

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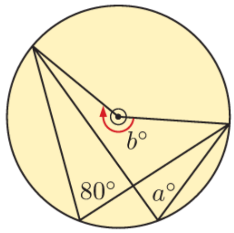

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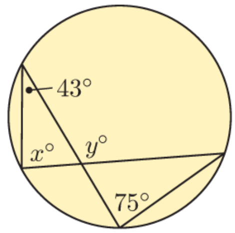

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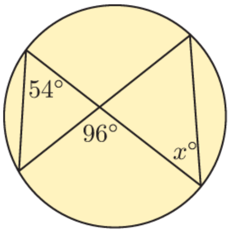

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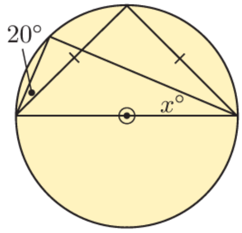

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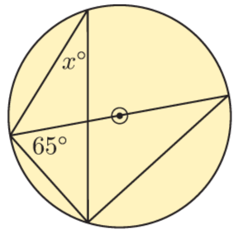
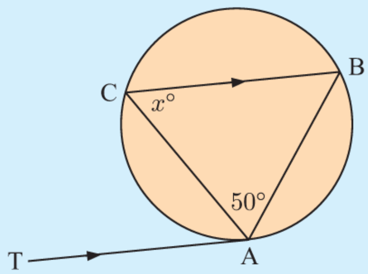
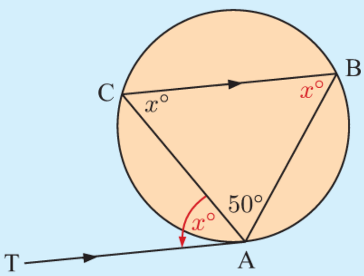


Example 4 **Self Tutor**

[AT] is a tangent to the circle. Find x.





$\widehat{CAT} = x°$ {equal alternate angles}

$\therefore\ \widehat{ABC} = \widehat{CAT} = x°$ {angle between a tangent and a chord}

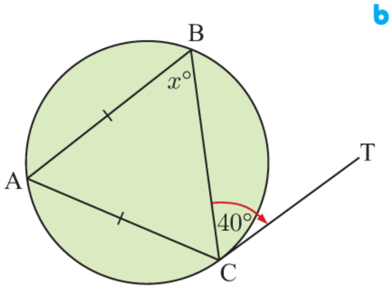
$\therefore\ x + x + 50 = 180$ {angles in a $\triangle$}

$\therefore\ 2x = 130$

$\therefore\ x = 65$

3 In each diagram, C is the point of contact of tangent [CT]. Find x, giving reasons:

a



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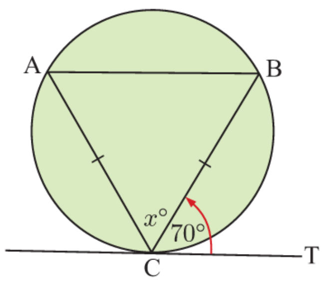

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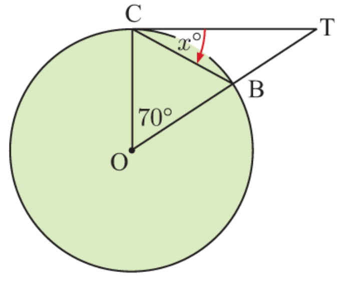
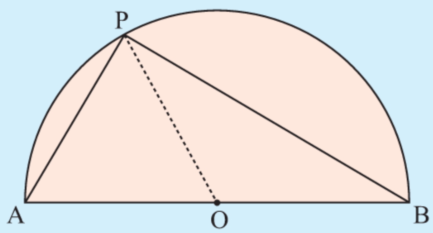


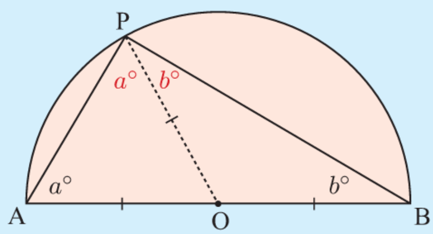
C GEOMETRIC PROOF

The circle theorems and other geometric facts can be formally **proven** using mathematical tools we already possess, such as the isosceles triangle theorem and congruence.

Example 5 — Self Tutor

Use the given figure to prove the *angle in a semi-circle theorem*.





Let $\widehat{PAO} = a°$ and $\widehat{PBO} = b°$.

Now $OA = OP = OB$ {equal radii}

$\therefore$ $\triangle$s OAP and OBP are isosceles.

$\therefore$ $\widehat{OPA} = a°$ and $\widehat{BPO} = b°$ {isosceles $\triangle$s}

Now $a + (a + b) + b = 180$ {angles of $\triangle$APB}

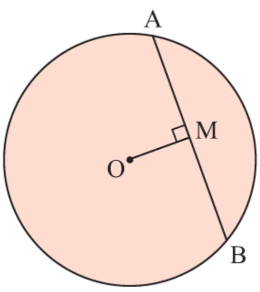
$\therefore$ $2a + 2b = 180$

$\therefore$ $a + b = 90$

So, $\widehat{APB}$ is a right angle.

EXERCISE 19C

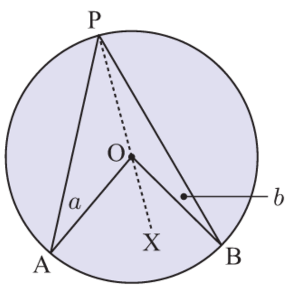
1 In this question we prove the *chords of a circle* theorem.



a For the given figure, join [OA] and [OB], and classify $\triangle$OAB.

b Apply the isosceles triangle theorem to triangle OAB. What geometrical fact results?

2



In this question we prove the *angle at the centre* theorem.

a Explain why $\triangle$s OAP and OBP are isosceles.

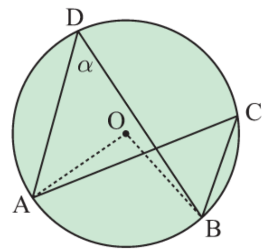
b Find the measure of the following angles in terms of a and b:

i $\widehat{APO}$ **ii** $\widehat{BPO}$ **iii** $\widehat{AOX}$

iv $\widehat{BOX}$ **v** $\widehat{APB}$ **vi** $\widehat{AOB}$

c What can be deduced from **b v** and **b vi**?

3 In this question we prove the *angles subtended by the same arc* theorem.

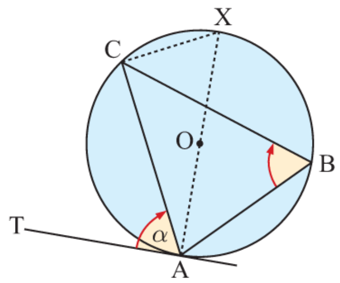


a Using the results of **2**, find the size of $\mathrm{A\widehat{O}B}$ in terms of α.

b Hence find the size of $\mathrm{A\widehat{C}B}$ in terms of α.

c State the relationship between $\mathrm{A\widehat{D}B}$ and $\mathrm{A\widehat{C}B}$.

4



In this question we prove the *angle between a tangent and a chord* theorem.

a We draw diameter [AX] and join [CX].

Find the size of: **i** $\mathrm{T\widehat{A}X}$ **ii** $\mathrm{A\widehat{C}X}$

b If $\mathrm{T\widehat{A}C} = \alpha$, find in terms of α:

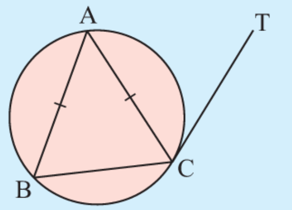
i $\mathrm{C\widehat{A}X}$ **ii** $\mathrm{C\widehat{X}A}$ **iii** $\mathrm{C\widehat{B}A}$

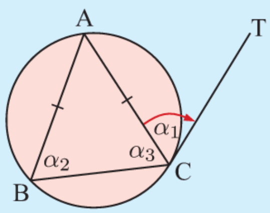
c State the relationship between $\mathrm{T\widehat{A}C}$ and $\mathrm{C\widehat{B}A}$.

Example 6 **Self Tutor**

$\triangle$ABC is isosceles and is inscribed in a circle.
[TC] is a tangent to the circle.

Prove that [AC] bisects $\mathrm{B\widehat{C}T}$.





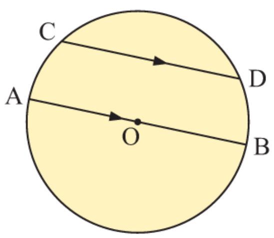
$\alpha_1 = \alpha_2$ {tangent and chord theorem}

and $\alpha_2 = \alpha_3$ {isosceles $\triangle$ theorem}

$\therefore \alpha_1 = \alpha_3$

So, [AC] bisects $\mathrm{B\widehat{C}T}$.

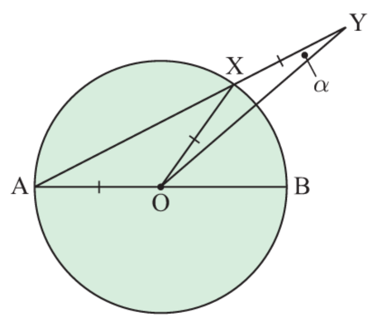
5



[AB] is a diameter of a circle with centre O.
[CD] is a chord parallel to [AB].

Prove that [BC] bisects $\mathrm{D\widehat{C}O}$.

6 [AB] is the diameter of a circle with centre O. X is a point on the circle, and [AX] is produced to Y such that OX = XY.



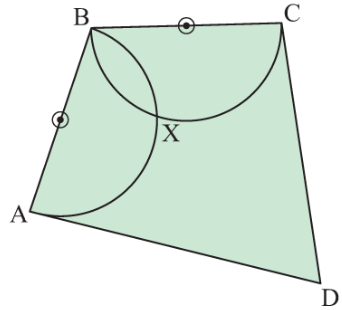
a If $\mathrm{X\widehat{Y}O} = \alpha$, find in terms of α:

i $\mathrm{X\widehat{O}Y}$ **ii** $\mathrm{A\widehat{X}O}$ **iii** $\mathrm{X\widehat{A}O}$

iv $\mathrm{X\widehat{O}B}$ **v** $\mathrm{B\widehat{O}Y}$

b What is the relationship between $\mathrm{B\widehat{O}Y}$ and $\mathrm{Y\widehat{O}X}$?

7 Revisit the **Opening Problem** on page **412**. Consider the two semi-circles in the figure alongside.



a Determine the measure of $B\widehat{X}A$ and $B\widehat{X}C$.

b What does **a** tell us about the points A, X, and C?

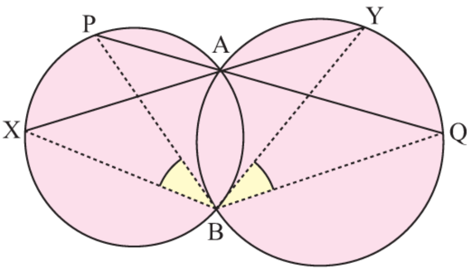
c Do the two illustrated sprinklers water all of the area on one side of the diagonal [AC]?

d Will Joe's four sprinklers water the whole garden? Explain your answer.

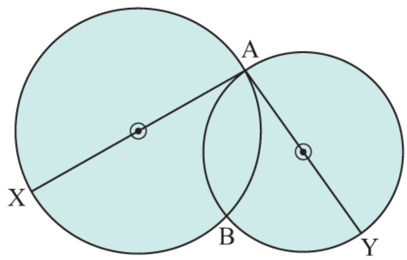
8 P is any point on a circle. [QR] is a chord of the circle parallel to the tangent at P. Prove that triangle PQR is isosceles.

9 Two circles intersect at A and B. Straight lines [PQ] and [XY] are drawn through A to meet the circles as shown.

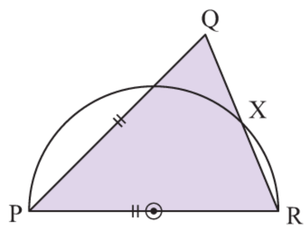
Show that $X\widehat{B}P = Y\widehat{B}Q$.



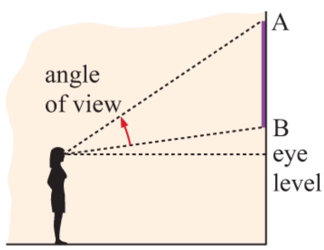
10 Two circles intersect at A and B. [AX] and [AY] are diameters as shown. Prove that X, B, and Y are collinear.

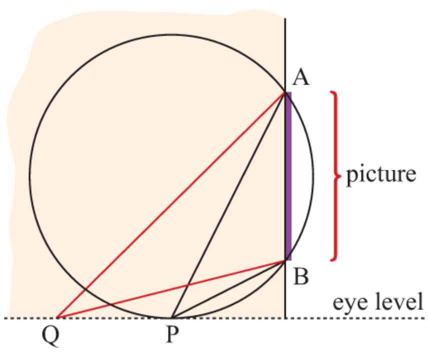


11 Triangle PQR is isosceles with $PQ = PR$.
A semi-circle with diameter [PR] is drawn to cut [QR] at X.
Prove that X is the midpoint of [QR].



12 Brigitta notices that her angle of view of a picture on a wall depends on how far she is standing from the wall. When she is very close to the wall, the angle of view is small. When she moves backwards so that she is a long way from the wall, the angle of view is also small.



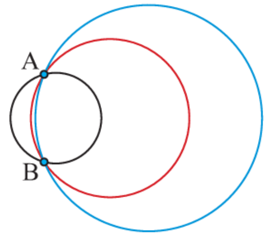


It becomes clear to Brigitta that there must be a point P in front of the picture at which her angle of view is greatest. The position of P can be found by drawing the circle through A and B which touches the 'eye level' line at P.

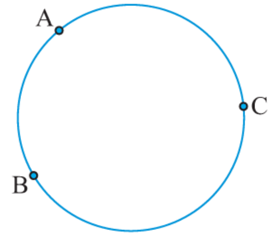
Prove this result by choosing any other point Q on the 'eye level' line, and showing that this angle must be less than $A\widehat{P}B$.

D CYCLIC QUADRILATERALS

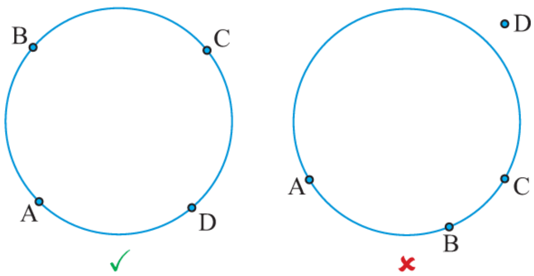
Suppose we are given two points A and B, and we must draw a circle passing through these points. There are infinitely many circles that we can draw.



If we are given three points A, B, and C which are not collinear, there is a unique circle which passes through the points.

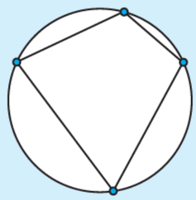


If we are given four points A, B, C, and D, no three of which are collinear, there may or may not be a circle which passes through the points. To see this, we draw the unique circle which passes through A, B, and C. The fourth point D may or may not lie on this circle.



Four points are said to be **concyclic** if a circle can be drawn through them.

If four concyclic points are joined to form a convex quadrilateral, then the quadrilateral is called a **cyclic quadrilateral**.



GEOMETRY PACKAGE

In the following **Investigation** we will discover an important property of cyclic quadrilaterals.

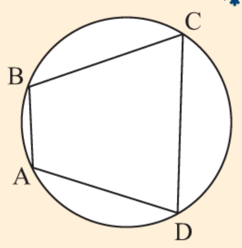
INVESTIGATION 2 CYCLIC QUADRILATERALS

This Investigation can be done using a compass, ruler, and protractor, or you can use the **geometry package** by clicking on the icon.

GEOMETRY PACKAGE

What to do:

1 Draw several circles, and on each circle draw a different cyclic quadrilateral with vertices A, B, C, and D. Make sure the quadrilaterals are large enough for you to measure the angles with a protractor.



2 Measure all angles to the nearest degree, and record your results in a table like the one following.

Figure	$\widehat{A}$	$\widehat{B}$	$\widehat{C}$	$\widehat{D}$	$\widehat{A}+\widehat{C}$	$\widehat{B}+\widehat{D}$
1						
2						
⋮						

PRINTABLE QUADRILATERALS

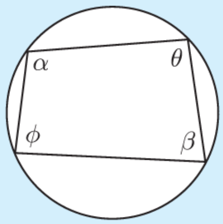
3 Write a sentence to summarise your results.

From the **Investigation** you should have discovered the following theorem:

OPPOSITE ANGLES OF A CYCLIC QUADRILATERAL THEOREM

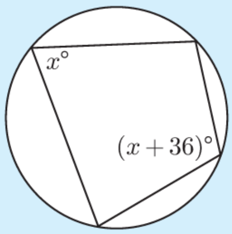
The opposite angles of a cyclic quadrilateral are **supplementary**.

$\alpha+\beta=180°$ and $\theta+\phi=180°$



Example 7 Self Tutor

Find x given:

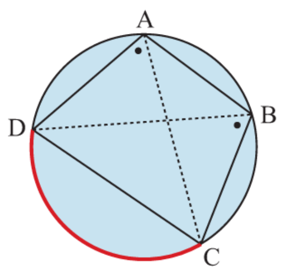


The angles given are opposite angles of a cyclic quadrilateral.

$$\therefore\ x+(x+36)=180$$
$$\therefore\ 2x+36=180$$
$$\therefore\ 2x=144$$
$$\therefore\ x=72$$

We can use the *angles subtended by the same arc* theorem to discover another property of cyclic quadrilaterals.

For the cyclic quadrilateral ABCD, notice that $D\widehat{A}C=D\widehat{B}C$.

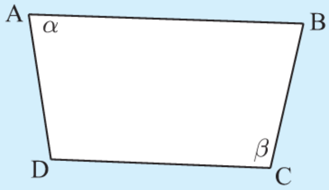


The converses of these two properties give us two useful tests for determining whether a quadrilateral is cyclic.

TESTS FOR CYCLIC QUADRILATERALS

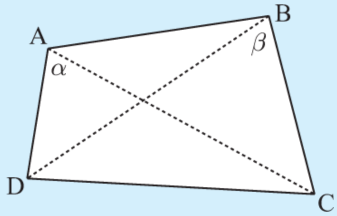
A quadrilateral is a **cyclic quadrilateral** if one of the following is true:

- one pair of opposite angles is supplementary



If $\alpha + \beta = 180°$ then ABCD is a cyclic quadrilateral.

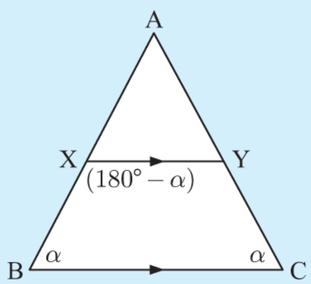
- one side subtends equal angles at the other two vertices



If $\alpha = \beta$ then ABCD is a cyclic quadrilateral.

Example 8 **Self Tutor**

Triangle ABC is isosceles with $AB = AC$. X and Y lie on [AB] and [AC] respectively such that [XY] is parallel to [BC]. Prove that XYCB is a cyclic quadrilateral.



$\triangle$ABC is isosceles with $AB = AC$.

Let $C\widehat{B}X = \alpha$

$\therefore\ B\widehat{C}Y = \alpha$ {equal base angles}

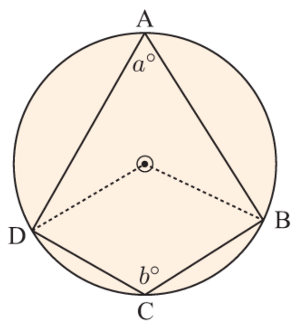
and $B\widehat{X}Y = 180° - \alpha$ {co-interior angles}

$\therefore\ B\widehat{X}Y + B\widehat{C}Y = 180°$

$\therefore$ XYCB is a cyclic quadrilateral {opposite angles are supplementary}

EXERCISE 19D

1 In this question we prove the *opposite angles of a cyclic quadrilateral* theorem.



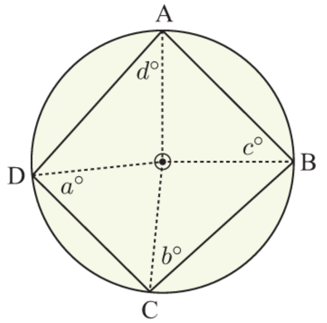
a For the given figure, find:

  i $D\widehat{O}B$ in terms of a

  ii reflex $D\widehat{O}B$ in terms of b.

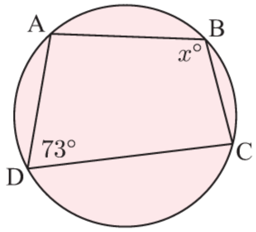
b Hence, show that $a + b = 180$.

2 An alternative method for establishing the *opposite angles of a cyclic quadrilateral* theorem is to use the figure alongside. Show how this can be done.

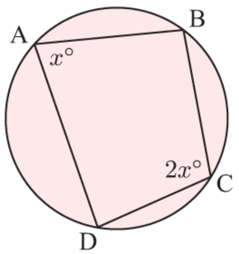


3 Find x, giving reasons:

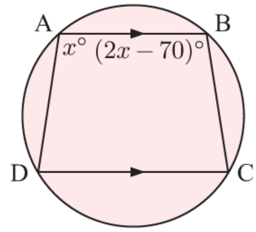
a



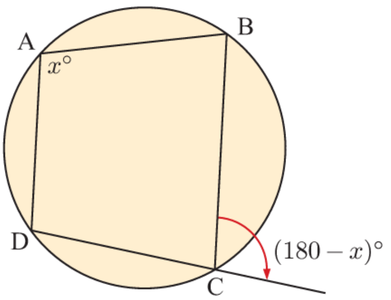
b



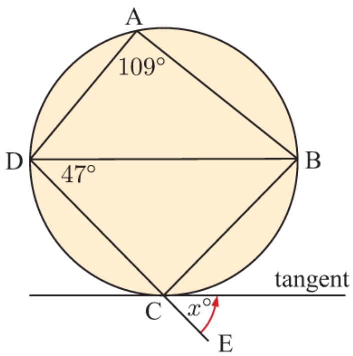
c



d

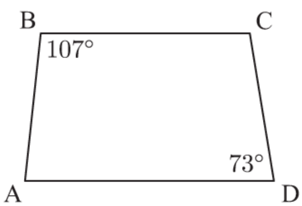


e



4 Is ABCD a cyclic quadrilateral? Explain your answer.

a



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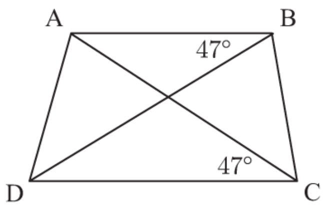

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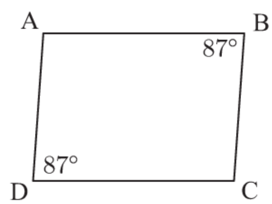
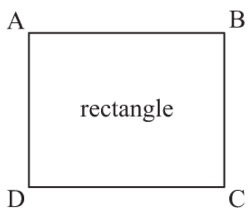


d



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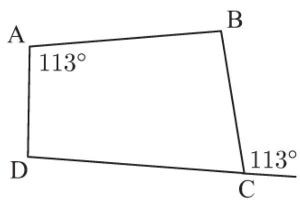

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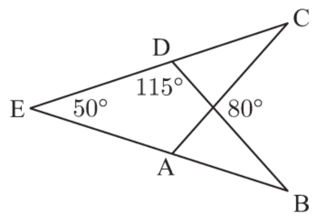
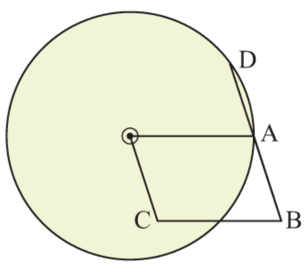
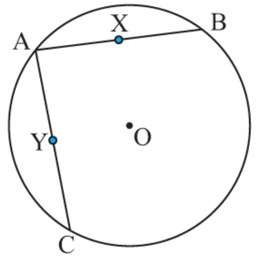


5 A parallelogram is inscribed in a circle. Prove that the parallelogram must be a rectangle.

6 OABC is a parallelogram.
A circle with centre O and radius [OA] is drawn.
[BA] produced meets the circle at D.
Prove that DOCB is a cyclic quadrilateral.



7 [AB] and [AC] are chords of a circle with centre O. X and Y are the midpoints of [AB] and [AC] respectively. Prove that OXAY is a cyclic quadrilateral.



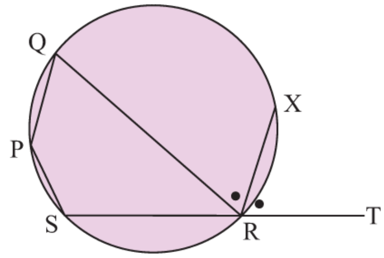
8 ABCD is a cyclic quadrilateral, and X is any point on diagonal [CA]. [XY] is drawn parallel to [CB] to meet [AB] at Y. [XZ] is drawn parallel to [CD] to meet [AD] at Z. Prove that XYAZ is a cyclic quadrilateral.

9 ABC is an isosceles triangle with $AB = AC$. The angle bisectors at B and C meet the sides [AC] and [AB] at X and Y respectively. Show that BCXY is a cyclic quadrilateral.

10 The non-parallel sides of a trapezium have equal length. Prove that the trapezium is a cyclic quadrilateral.

11 [AB] and [CD] are two parallel chords of a circle with centre O. [AD] and [BC] meet at E. Prove that AEOC is a cyclic quadrilateral.

12 [RX] is the bisector of angle QRT.
Prove that [PX] bisects angle QPS.

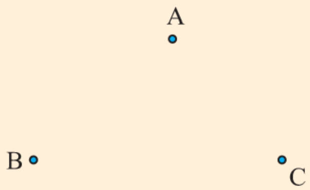


13 Two circles meet at points X and Y. Line segment [AXB] meets one circle at A and the other at B. Line segment [CYD] meets one circle at C and the other at D. Prove that [AC] is parallel to [BD].

INVESTIGATION 3 CIRCLES AND TRIANGLES

What to do:

1 Consider three points A, B, and C. Explain how to use a ruler and geometrical compass to locate the centre of the circle passing through A, B, and C. Clearly state any geometrical theorems you have used.
Check your construction by drawing the circle.



DEMO

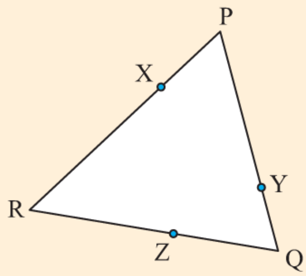
2 Draw any triangle PQR with sides between 6 cm and 10 cm. Label any points X, Y, and Z on [PR], [PQ], and [RQ] respectively.

a Use **1** to accurately construct circles through:

i P, X, and Y **ii** Q, Y, and Z

iii R, X, and Z.

b What do you notice about the three circles?

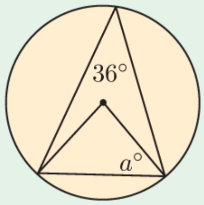


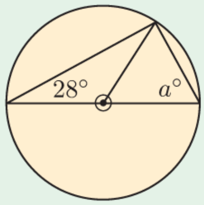
3 Repeat **2** with a different acute-angled triangle.

4 Use cyclic quadrilaterals to prove that the result you have found is always true.
Hint: Do **not** draw all three circles on your figure.

REVIEW SET 19A

1 Find the value of a:

a


b


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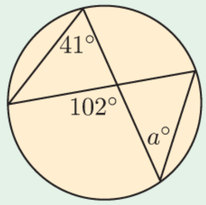

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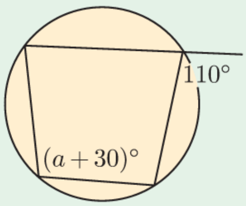
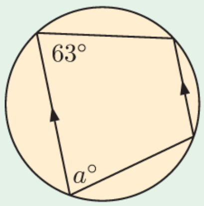
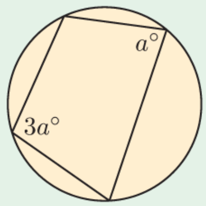
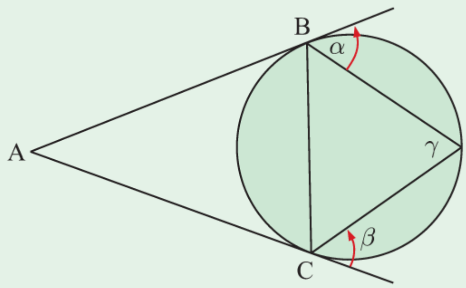


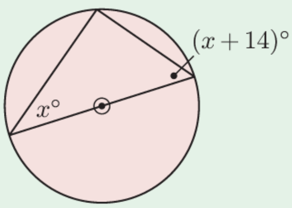
e


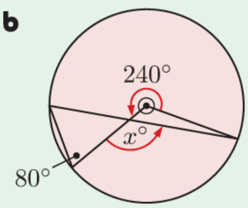
f


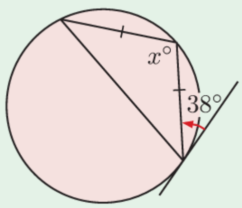
2 [AB] and [AC] are tangents to the circle.
Find an equation connecting α, β, and γ.



3 Find the value of x:

a


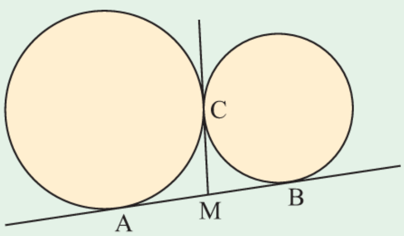
b


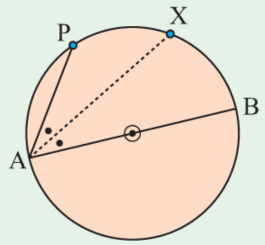
c


4 [AB] and [CM] are common tangents to two touching circles. Show that:

a M is the midpoint of [AB]

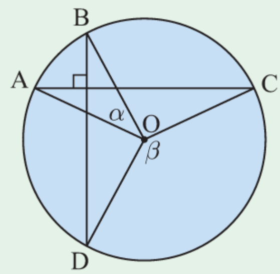
b $\mathrm{A\widehat{C}B}$ is a right angle.



5 

The circle alongside has diameter [AB], and P is another point on the circle. The angle bisector of $P\widehat{A}B$ meets the circle at X. Show that the tangent at X is parallel to [PB].

6 O is the centre of the circle alongside. The chords [AC] and [BD] are perpendicular.
Show that $\alpha + \beta = 180°$.
Hint: Join [BC].

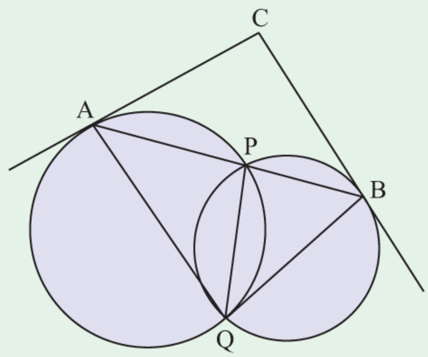


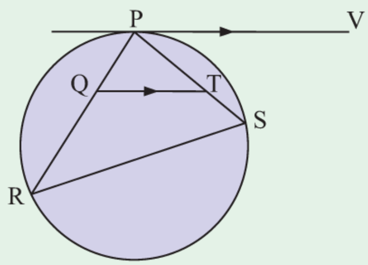
7 **a** Copy and complete: "*The angle between a tangent and a chord through the point of contact is equal to ......*"

b Two circles intersect at points P and Q. A line segment [APB] is drawn through P, and the tangents at A and B meet at C.

i Let $A\widehat{B}C = \alpha$ and $B\widehat{A}C = \beta$.
Write expressions for $P\widehat{Q}B$, $P\widehat{Q}A$, and $A\widehat{Q}B$ in terms of α and β.

ii Hence, show that ACBQ is a cyclic quadrilateral.

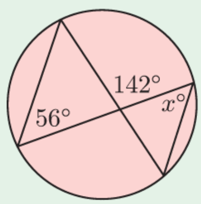


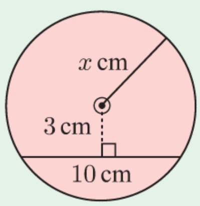
8 

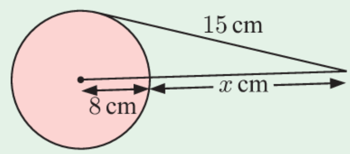
[PV] is a tangent to the circle, and [QT] is parallel to [PV]. Prove that QRST is a cyclic quadrilateral.

REVIEW SET 19B

1 Find the value of x:

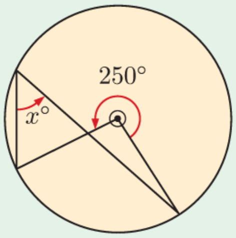
a 

b 

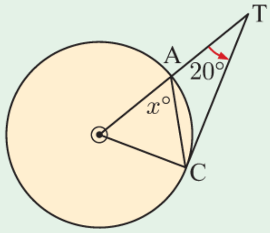
c 

2 Find the value of x:

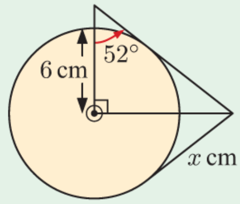
a



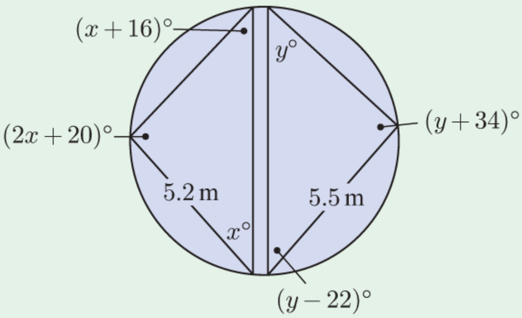
b



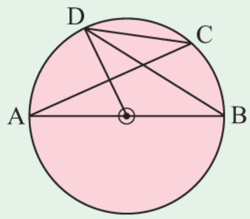
c



3 Find the length of the diameter of the circle below.

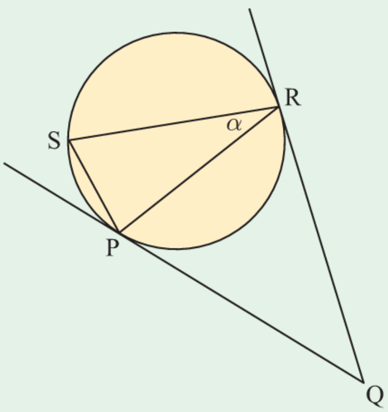


4 [AB] is the diameter of a circle with centre O.
[AC] and [BD] are any two chords.
Show that $\quad B\widehat{D}O = A\widehat{C}D$.



5 A, B, and C are three points on a circle. The bisector of angle BAC cuts [BC] at P, and the circle at Q. Prove that $\ A\widehat{P}C = A\widehat{B}Q$.

6



[QP] and [QR] are tangents to a circle. S is a point on the circle such that $P\widehat{S}R$ and $P\widehat{Q}R$ are equal, and both are double $P\widehat{R}S$.
Let $P\widehat{R}S$ be α.

a Find, in terms of α:

i $P\widehat{S}R$ **ii** $P\widehat{Q}R$
iii $P\widehat{R}Q$ **iv** $Q\widehat{P}R$

b Use triangle PQR to show that $\ \alpha = 30°$.

c Find the measure of $Q\widehat{R}S$.

d What can you conclude about [RS]?

7 [AB] is the diameter of a circle, and C and D are two other points on the circle. [AC] and [BD] meet at E, and [AD] and [BC] meet at F. Show that points C, D, E, and F form a cyclic quadrilateral.

8 In the figure alongside, [XT] and [XP] are tangents.
Prove that:

a BTXP is a cyclic quadrilateral

b [PT] bisects angle CTX.