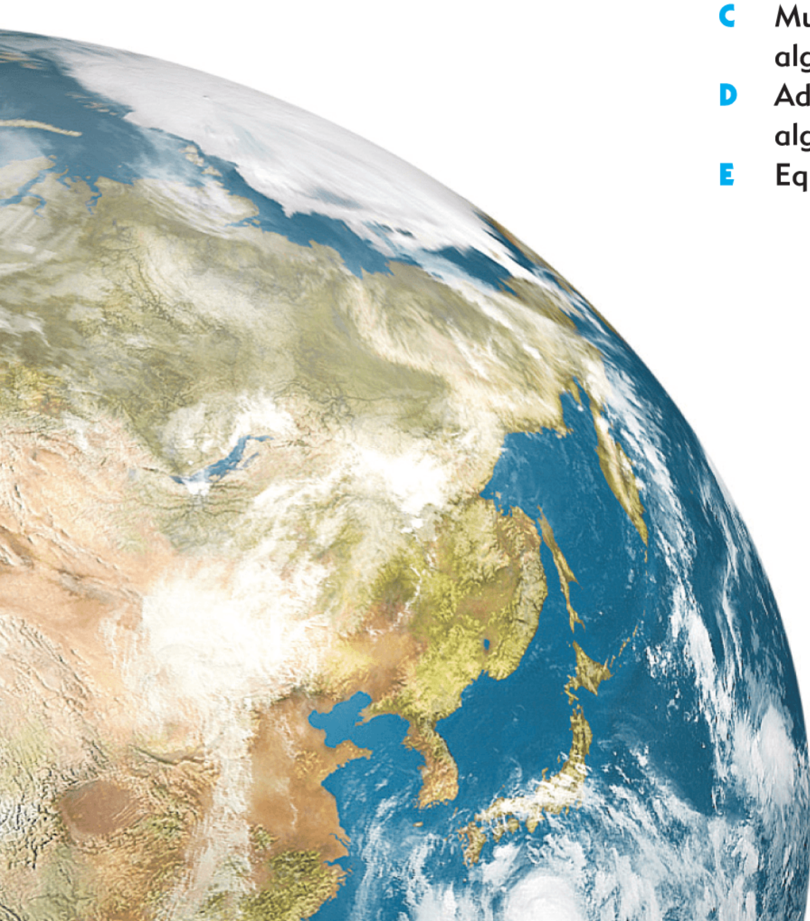
Chapter 10

Algebraic fractions

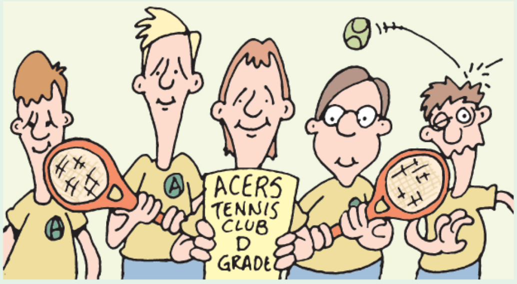


Contents:

OPENING PROBLEM

To participate in a tennis league, each team must pay \$150 in court fees, and a \$60 team registration fee. The court fees are shared equally between the players in the team, and the registration fee is shared equally between the players and the team coach.

Suppose a team has x players.



Things to think about:

a Can you write the amount that each player must pay as a single fraction involving x?

b How much must each player pay if the team has:

i 4 players **ii** 5 players?

Algebraic fractions are fractions which contain at least one variable or unknown.

The variable may be in the numerator, the denominator, or both the numerator and denominator.

For example, $\frac{x}{7}$, $\frac{-2}{5-y}$, and $\frac{x+2y}{1-y}$ are all algebraic fractions.

A EVALUATING ALGEBRAIC FRACTIONS

To **evaluate** an algebraic fraction, we replace the variables with their known values. We then give our answer in simplest form.

Example 1 Self Tutor

If $a = 2$, $b = -3$, and $c = -5$, evaluate:

a $\frac{a-b}{c}$ **b** $\frac{a-c-b}{b-a}$

a
$$\begin{aligned} &\frac{a-b}{c} \\ &= \frac{2-(-3)}{(-5)} \\ &= \frac{2+3}{-5} \\ &= \frac{5}{-5} \\ &= -1 \end{aligned}$$

b
$$\begin{aligned} &\frac{a-c-b}{b-a} \\ &= \frac{2-(-5)-(-3)}{(-3)-2} \\ &= \frac{2+5+3}{-3-2} \\ &= \frac{10}{-5} \\ &= -2 \end{aligned}$$

EXERCISE 10A

1 If $a = 3$, $b = 2$, and $c = 6$, evaluate:

a $\frac{c}{2}$ **b** $\frac{c}{a}$ **c** $\frac{a}{c}$ **d** $\frac{c}{b-a}$ **e** $\frac{a+c}{b}$

f $\frac{ab}{c}$ **g** $\frac{a^2}{b}$ **h** $\frac{c^2}{a}$ **i** $\frac{ab^2}{c}$ **j** $\frac{(ab)^2}{c}$

2 If $a = 2$, $b = -3$, and $c = -4$, evaluate:

a $\dfrac{c}{a}$ **b** $\dfrac{a}{c}$ **c** $\dfrac{-1}{b}$ **d** $\dfrac{c^2}{a}$ **e** $\dfrac{c}{a+b}$

f $\dfrac{a-c}{2b}$ **g** $\dfrac{b}{c-a}$ **h** $\dfrac{a-c}{a+c}$ **i** $\dfrac{c-a}{b^2}$ **j** $\dfrac{a^2}{c-b}$

B SIMPLIFYING ALGEBRAIC FRACTIONS

We have observed previously that number fractions can be simplified by cancelling common factors.

For example, $\dfrac{15}{35} = \dfrac{3 \times \cancel{5}^1}{7 \times \cancel{5}_1} = \dfrac{3}{7}$ where the common factor 5 is cancelled.

The same principle can be applied to algebraic fractions.

If the numerator and denominator of an algebraic fraction are both written in factored form and common factors are found, we can simplify by **cancelling the common factors**.

For example,
$$\begin{aligned} \frac{6bc}{3c} &= \frac{{}^2\cancel{6} \times b \times \cancel{c}^1}{{}_1\cancel{3} \times \cancel{c}_1} && \{3 \text{ and } c \text{ are common factors}\} \\ &= \frac{2b}{1} && \{\text{after cancellation}\} \\ &= 2b \end{aligned}$$

Fractions such as $\dfrac{3xy}{7z}$ cannot be simplified since the numerator and denominator do not have any common factors.

To simplify algebraic expressions:

- factorise the numerator and the denominator
- cancel any common factors
- simplify the result.

DISCUSSION

Discuss what is wrong with the cancellation $\dfrac{x + \cancel{4}^2}{\cancel{2}_1} = \dfrac{x+2}{1} = x + 2$.

Example 2

Self Tutor

Simplify: **a** $\dfrac{a^2}{2a}$ **b** $\dfrac{6a^2b}{3b}$ **c** $\dfrac{a+b}{a}$

a
$$\begin{aligned} &\frac{a^2}{2a} \\ &= \frac{a \times \cancel{a}^1}{2 \times \cancel{a}_1} \\ &= \frac{a}{2} \end{aligned}$$

b
$$\begin{aligned} &\frac{6a^2b}{3b} \\ &= \frac{{}^2\cancel{6} \times a \times a \times \cancel{b}^1}{{}_1\cancel{3} \times \cancel{b}_1} \\ &= \frac{2 \times a \times a}{1} \\ &= 2a^2 \end{aligned}$$

c $\dfrac{a+b}{a}$ cannot be simplified as $a + b$ is a sum, not a product.

EXERCISE 10B.1

1 Simplify:

a $\frac{2a}{4}$ **b** $\frac{4m}{2}$ **c** $\frac{6a}{a}$ **d** $\frac{6a}{2a}$ **e** $\frac{2a^2}{a}$

f $\frac{2x^3}{2x}$ **g** $\frac{2x^3}{x^2}$ **h** $\frac{2x^3}{x^3}$ **i** $\frac{2a^2}{4a^3}$ **j** $\frac{8m^2}{4m}$

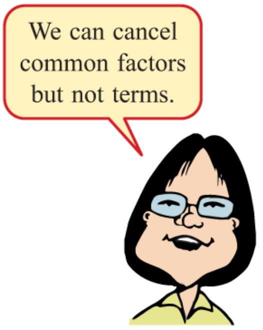
k $\frac{4a^2}{a^2}$ **l** $\frac{6t}{3t^2}$ **m** $\frac{4d^2}{2d}$ **n** $\frac{ab^2}{2ab}$ **o** $\frac{4ab^2}{6a^2b}$

2 Simplify if possible:

a $\frac{2t}{2}$ **b** $\frac{2+t}{2}$ **c** $\frac{xy}{x}$

d $\frac{x+y}{x}$ **e** $\frac{ac}{bc}$ **f** $\frac{a+c}{b+c}$

g $\frac{2a^2}{4a}$ **h** $\frac{5a}{9b}$ **i** $\frac{14c}{8d}$



Example 3 **Self Tutor**

Simplify: **a** $\frac{(-4b)^2}{2b}$ **b** $\frac{18}{3(c-1)}$

a $\frac{(-4b)^2}{2b}$

$= \frac{(-4b)\times(-4b)}{2\times b}$

$= \frac{{}^{8}\cancel{16}\times b\times \cancel{b}^{1}}{{}_{1}\cancel{2}\times \cancel{b}_{1}}$

$= 8b$

b $\frac{18}{3(c-1)}$

$= \frac{\cancel{18}^{6}}{{}_{1}\cancel{3}(c-1)}$

$= \frac{6}{c-1}$

3 Simplify:

a $\frac{(2a)^2}{a^2}$ **b** $\frac{(4n)^2}{8n}$ **c** $\frac{(-a)^2}{a}$ **d** $\frac{a^2}{(-a)^2}$

e $\frac{(-2a)^2}{4}$ **f** $\frac{(-3n)^2}{6n}$ **g** $\frac{2b}{(2b^2)^2}$ **h** $\frac{(3k^2)^2}{18a^3}$

4 Simplify:

a $\frac{4(x+5)}{2}$ **b** $\frac{2(n+5)}{12}$ **c** $\frac{7(b+2)}{14}$ **d** $\frac{6(k-2)}{8}$

e $\frac{15}{3(t-1)}$ **f** $\frac{10}{25(k+4)}$ **g** $\frac{4}{12(x-3)}$ **h** $\frac{20(p+4)}{12}$

Example 4 **Self Tutor**

Simplify: **a** $\dfrac{(2x+3)(x+4)}{5(2x+3)}$ **b** $\dfrac{12(x+4)^2}{3(x+4)}$

a $\dfrac{{}^{1}\cancel{(2x+3)}(x+4)}{5\cancel{(2x+3)}_{1}}$

$= \dfrac{(x+4)}{5}$

$= \dfrac{x+4}{5}$

b $\dfrac{12(x+4)^2}{3(x+4)}$

$= \dfrac{{}^{4}\cancel{12}(x+4)\cancel{(x+4)}^{1}}{{}_{1}\cancel{3}\cancel{(x+4)}_{1}}$

$= 4(x+4)$

5 Simplify:

a $\dfrac{(x+4)(x+2)}{9(x+4)}$ **b** $\dfrac{12(a-3)}{(a-3)(a+1)}$ **c** $\dfrac{(x+y)(x-y)}{3(x-y)}$

d $\dfrac{(x+y)^2}{x+y}$ **e** $\dfrac{2(x+2)}{(x+2)^2}$ **f** $\dfrac{(a+5)^2}{3(a+5)}$

g $\dfrac{2xy(x-y)}{6x(x-y)}$ **h** $\dfrac{5(y+2)(y-3)}{15(y+2)}$ **i** $\dfrac{x(x+1)(x+2)}{3x(x+2)}$

j $\dfrac{3(b-4)}{6(b-4)^2}$ **k** $\dfrac{8(p+q)^2}{12(p+q)}$ **l** $\dfrac{24(r-2)}{15(r-2)^2}$

m $\dfrac{x^2(x+2)}{x(x+2)(x-1)}$ **n** $\dfrac{(x+2)^2(x+1)}{4(x+2)}$ **o** $\dfrac{2(x+2)^2(x-1)^2}{8x(x+2)}$

FACTORISATION AND SIMPLIFICATION

It is often necessary to **factorise** either the numerator or denominator before simplification can take place. To do this we use the rules for factorisation that we have seen previously.

Example 5 **Self Tutor**

Simplify: **a** $\dfrac{3a+9}{3}$ **b** $\dfrac{4a+12}{8}$

a $\dfrac{3a+9}{3}$

$= \dfrac{{}^{1}\cancel{3}(a+3)}{{}_{1}\cancel{3}}$

$= a+3$

b $\dfrac{4a+12}{8}$

$= \dfrac{{}^{1}\cancel{4}(a+3)}{{}_{2}\cancel{8}}$

$= \dfrac{a+3}{2}$

EXERCISE 10B.2

1 Simplify by factorising:

a $\dfrac{2x+4}{2}$ **b** $\dfrac{3x-6}{3}$ **c** $\dfrac{3x+6}{6}$ **d** $\dfrac{4x-20}{8}$

e $\dfrac{4y+12}{12}$ **f** $\dfrac{6x-30}{4}$ **g** $\dfrac{ax+bx}{x}$ **h** $\dfrac{ax+bx}{cx+dx}$

2 Simplify, if possible:

a $\dfrac{4x+6}{6}$ **b** $\dfrac{4x+6}{5}$ **c** $\dfrac{6a-3}{2}$ **d** $\dfrac{6a-3}{3}$

e $\dfrac{6a+2}{4}$ **f** $\dfrac{3b+9}{2}$ **g** $\dfrac{3b+9}{6}$ **h** $\dfrac{8b-12}{6}$

Example 6 Self Tutor

Simplify by factorising:

a $\dfrac{ab+ac}{b+c}$ **b** $\dfrac{6x^2-6xy}{3x-3y}$

a $\dfrac{ab+ac}{b+c}$

$= \dfrac{a(b+c)}{b+c}$ ← HCF is a

$= \dfrac{a\cancel{(b+c)}^{1}}{\cancel{(b+c)}_{1}}$

$= a$

b $\dfrac{6x^2-6xy}{3x-3y}$

$= \dfrac{6x(x-y)}{3(x-y)}$ ← HCF is $6x$ ← HCF is 3

$= \dfrac{{}^{2}\cancel{6}\times x\times \cancel{(x-y)}^{1}}{{}_{1}\cancel{3}\times \cancel{(x-y)}_{1}}$

$= 2x$

3 Simplify by factorising:

a $\dfrac{3x+6}{4x+8}$ **b** $\dfrac{5x-15}{3x-9}$ **c** $\dfrac{ax+bx}{a+b}$ **d** $\dfrac{16x-8}{20x-10}$

e $\dfrac{a+b}{ay+by}$ **f** $\dfrac{ax+bx}{ay+by}$ **g** $\dfrac{4x^2+8x}{x+2}$ **h** $\dfrac{3x^2+9x}{x+3}$

i $\dfrac{5x^2-5xy}{7x-7y}$ **j** $\dfrac{9b^2-9ab}{12b-12a}$ **k** $\dfrac{6x^2-18x}{9x-27}$ **l** $\dfrac{6a+6b}{8a^3+4a^2b}$

Example 7 Self Tutor

Simplify: **a** $\dfrac{6a-6b}{b-a}$ **b** $\dfrac{xy^2-xy}{1-y}$

a $\dfrac{6a-6b}{b-a}$

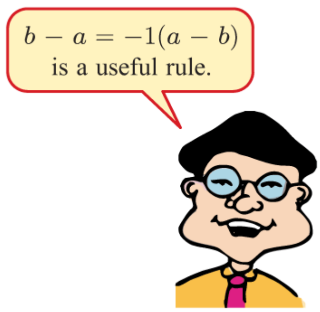
$= \dfrac{6\cancel{(a-b)}^{1}}{-1\cancel{(a-b)}_{1}}$

$= -6$

b $\dfrac{xy^2-xy}{1-y}$

$= \dfrac{xy\cancel{(y-1)}^{1}}{-1\cancel{(y-1)}_{1}}$

$= -xy$



4 Simplify:

a $\dfrac{2x-2y}{y-x}$ **b** $\dfrac{3x-3y}{2y-2x}$ **c** $\dfrac{m-n}{n-m}$ **d** $\dfrac{r-2s}{4s-2r}$

e $\dfrac{3r-6s}{2s-r}$ **f** $\dfrac{2x-2}{x-x^2}$ **g** $\dfrac{ab^2-ab}{2-2b}$ **h** $\dfrac{4x^2-4x}{2-2x}$

Example 8 **Self Tutor**

Simplify: **a** $\dfrac{x^2-1}{x^2+3x+2}$ **b** $\dfrac{6x^2+10x-4}{18x^2+3x-3}$

a $\dfrac{x^2-1}{x^2+3x+2}$

$= \dfrac{(x-1)\cancel{(x+1)}^{1}}{(x+2)\cancel{(x+1)}_{1}}$

$= \dfrac{x-1}{x+2}$

b $\dfrac{6x^2+10x-4}{18x^2+3x-3}$

$= \dfrac{2(3x^2+5x-2)}{3(6x^2+x-1)}$

$= \dfrac{2\cancel{(3x-1)}^{1}(x+2)}{3\cancel{(3x-1)}_{1}(2x+1)}$

$= \dfrac{2(x+2)}{3(2x+1)}$

5 Simplify:

a $\dfrac{x^2-1}{x-1}$ **b** $\dfrac{x^2-1}{x+1}$ **c** $\dfrac{x^2-1}{1-x}$ **d** $\dfrac{x+2}{x^2-4}$

e $\dfrac{a^2-b^2}{a+b}$ **f** $\dfrac{a^2-b^2}{b-a}$ **g** $\dfrac{2x+2}{x^2-1}$ **h** $\dfrac{9-x^2}{3x-x^2}$

i $\dfrac{3x^2-3y^2}{2xy-2y^2}$ **j** $\dfrac{2b^2-2a^2}{a^2-ab}$ **k** $\dfrac{4xy-y^2}{16x^2-y^2}$ **l** $\dfrac{4x(x-4)}{16-x^2}$

6 Simplify:

a $\dfrac{x^2-x-2}{x-2}$ **b** $\dfrac{x+3}{x^2-2x-15}$ **c** $\dfrac{2x^2+2x}{x^2-4x-5}$

d $\dfrac{x^2-4}{x^2+4x+4}$ **e** $\dfrac{x^2-x-12}{x^2-5x+4}$ **f** $\dfrac{x^2+2x+1}{1-x^2}$

g $\dfrac{x^2-x-20}{x^2+7x+12}$ **h** $\dfrac{2x^2+5x+2}{2x^2+7x+3}$ **i** $\dfrac{3x^2+7x+2}{6x^2-x-1}$

j $\dfrac{8x^2+2x-1}{4x^2-5x+1}$ **k** $\dfrac{12x^2-5x-3}{6x^2+5x+1}$ **l** $\dfrac{15x^2+17x-4}{5x^2+9x-2}$

PUZZLE

Click on the icon to load a dynamic puzzle for algebraic fractions.

PUZZLE

C MULTIPLYING AND DIVIDING ALGEBRAIC FRACTIONS

Variables are used in algebraic fractions to represent unknown numbers. We can treat algebraic fractions in the same way that we treat numerical fractions, since they are in fact *representing* numerical fractions.

The rules for multiplying and dividing algebraic fractions are identical to those used with numerical fractions.

MULTIPLICATION

To **multiply** two or more fractions, we multiply the numerators to form the new numerator, and we multiply the denominators to form the new denominator.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

We can then cancel any common factors, and write our answer in simplest form.

Example 9 — Self Tutor

Simplify: **a** $\dfrac{3}{m} \times \dfrac{m}{6}$ **b** $\dfrac{3}{m} \times m^2$

a
$$\frac{3}{m} \times \frac{m}{6}$$
$$= \frac{{}^{1}\cancel{3} \times \cancel{m}^{1}}{{}_{1}\cancel{m} \times \cancel{6}_{2}}$$
$$= \frac{1}{2}$$

b
$$\frac{3}{m} \times m^2$$
$$= \frac{3}{m} \times \frac{m^2}{1}$$
$$= \frac{3 \times m^2}{m \times 1}$$
$$= \frac{3 \times m \times \cancel{m}^{1}}{\cancel{m}_{1}}$$
$$= 3m$$

DIVISION

To **divide** by a fraction, we multiply by its **reciprocal**. The reciprocal is obtained by swapping the numerator and denominator.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Example 10 — Self Tutor

Simplify: **a** $\dfrac{4}{n} \div \dfrac{2}{n^2}$ **b** $\dfrac{3}{a} \div 2$

a
$$\frac{4}{n} \div \frac{2}{n^2}$$
$$= \frac{4}{n} \times \frac{n^2}{2}$$
$$= \frac{4 \times n^2}{n \times 2}$$
$$= \frac{{}^{2}\cancel{4} \times n \times \cancel{n}^{1}}{{}_{1}\cancel{n} \times \cancel{2}_{1}}$$
$$= 2n$$

b
$$\frac{3}{a} \div 2$$
$$= \frac{3}{a} \times \frac{1}{2}$$
$$= \frac{3 \times 1}{a \times 2}$$
$$= \frac{3}{2a}$$

The reciprocal of $2 = \frac{2}{1}$ is $\frac{1}{2}$.

EXERCISE 10C

1 Simplify:

a $\dfrac{x}{2} \times \dfrac{y}{5}$ **b** $\dfrac{a}{2} \times \dfrac{3}{a}$ **c** $\dfrac{a}{2} \times a$ **d** $\dfrac{a}{4} \times \dfrac{2}{3a}$

e $\dfrac{c}{5} \times \dfrac{1}{c}$ **f** $\dfrac{c}{5} \times \dfrac{c}{2}$ **g** $\dfrac{a}{b} \times \dfrac{c}{d}$ **h** $\dfrac{a}{b} \times \dfrac{b}{a}$

i $\dfrac{1}{m^2} \times \dfrac{m}{2}$ **j** $\dfrac{m}{2} \times \dfrac{4}{m}$ **k** $\dfrac{a}{x} \times \dfrac{x}{b}$ **l** $m \times \dfrac{4}{m}$

m $\dfrac{3}{m^2} \times m$ **n** $\left(\dfrac{a}{b}\right)^2$ **o** $\left(\dfrac{2}{x}\right)^2$ **p** $\dfrac{1}{a} \times \dfrac{a}{b} \times \dfrac{b}{c}$

2 Simplify:

a $\dfrac{a}{2} \div \dfrac{a}{3}$ **b** $\dfrac{2}{a} \div \dfrac{2}{3}$ **c** $\dfrac{3}{4} \div \dfrac{4}{x}$ **d** $\dfrac{3}{x} \div \dfrac{4}{x}$

e $\dfrac{2}{n} \div \dfrac{1}{n}$ **f** $\dfrac{c}{5} \div 5$ **g** $\dfrac{c}{5} \div c$ **h** $m \div \dfrac{2}{m}$

i $m \div \dfrac{m}{2}$ **j** $1 \div \dfrac{m}{n}$ **k** $\dfrac{3}{g} \div 4$ **l** $\dfrac{3}{g} \div \dfrac{9}{g^2}$

m $\dfrac{4}{x} \div \dfrac{x^2}{2}$ **n** $\dfrac{2}{x} \div \dfrac{6}{x^3}$ **o** $\dfrac{a}{b} \div \dfrac{a^2}{b}$ **p** $\dfrac{a^2}{5} \div \dfrac{a}{3}$

Example 11 Self Tutor

Simplify:

a $\dfrac{y^2-y}{y-2} \times \dfrac{3y-6}{4y-4}$ **b** $\dfrac{5m-20}{4} \div \dfrac{2m-8}{3}$

a
$$\begin{aligned} &\frac{y^2-y}{y-2} \times \frac{3y-6}{4y-4} \\ &= \frac{y\cancel{(y-1)}^{1}}{\cancel{y-2}_{1}} \times \frac{3\cancel{(y-2)}^{1}}{4\cancel{(y-1)}_{1}} \\ &= \frac{3y}{4} \end{aligned}$$

b
$$\begin{aligned} &\frac{5m-20}{4} \div \frac{2m-8}{3} \\ &= \frac{5m-20}{4} \times \frac{3}{2m-8} \\ &= \frac{5\cancel{(m-4)}^{1}}{4} \times \frac{3}{2\cancel{(m-4)}_{1}} \\ &= \frac{15}{8} \end{aligned}$$

3 Simplify:

a $\dfrac{x^2+3x}{x-2} \times \dfrac{5}{2x+6}$ **b** $\dfrac{t-5}{t^2+t} \times \dfrac{4t+4}{3t-15}$ **c** $\dfrac{4a-28}{a} \div \dfrac{a-7}{5}$

d $\dfrac{6k-2}{k+2} \times \dfrac{2k^2+4k}{9k-3}$ **e** $\dfrac{x^2-2x}{x+5} \div \dfrac{8-4x}{2x+10}$ **f** $\dfrac{m^2-4m}{10m-2} \div \dfrac{m^2-16}{3m+12}$

D ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

The rules for addition and subtraction of algebraic fractions are identical to those used with numerical fractions.

To **add** two or more fractions, we obtain the *lowest common denominator* and then add the resulting numerators. $\quad \dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$

To **subtract** two or more fractions, we obtain the *lowest common denominator* and then subtract the resulting numerators. $\quad \dfrac{a}{c} - \dfrac{d}{c} = \dfrac{a-d}{c}$

To find the lowest common denominator of numerical fractions, we look for the **lowest common multiple of the denominators**.

For example:
- when finding $\frac{3}{2}+\frac{5}{3}$, the lowest common denominator is 6
- when finding $\frac{5}{6}+\frac{4}{9}$, the lowest common denominator is 18.

The same method is used when there are variables in the denominator.

For example:
- when finding $\frac{2}{a}+\frac{3}{b}$, the lowest common denominator is ab
- when finding $\frac{2}{x}+\frac{4}{5x}$, the lowest common denominator is $5x$
- when finding $\frac{5}{6x}+\frac{4}{9y}$, the lowest common denominator is $18xy$.

Example 12 Self Tutor

Simplify: **a** $\frac{x}{2}+\frac{3x}{4}$ **b** $\frac{a}{3}-\frac{2a}{5}$

a
$$\begin{aligned}&\frac{x}{2}+\frac{3x}{4} \quad \{\text{LCD}=4\}\\&=\frac{x\times 2}{2\times 2}+\frac{3x}{4}\\&=\frac{2x}{4}+\frac{3x}{4}\\&=\frac{2x+3x}{4}\\&=\frac{5x}{4}\end{aligned}$$

b
$$\begin{aligned}&\frac{a}{3}-\frac{2a}{5} \quad \{\text{LCD}=15\}\\&=\frac{a\times 5}{3\times 5}-\frac{2a\times 3}{5\times 3}\\&=\frac{5a}{15}-\frac{6a}{15}\\&=\frac{5a-6a}{15}\\&=-\frac{a}{15}\end{aligned}$$

EXERCISE 10D

1 Simplify by writing as a single fraction:

a $\frac{a}{2}+\frac{a}{3}$ **b** $\frac{b}{5}-\frac{b}{10}$ **c** $\frac{c}{4}+\frac{3c}{2}$

d $\frac{d}{2}-\frac{3}{5}$ **e** $\frac{5}{8}+\frac{x}{12}$ **f** $\frac{x}{7}-\frac{x}{2}$

g $\frac{a}{3}+\frac{b}{4}$ **h** $\frac{t}{3}-\frac{5t}{9}$ **i** $\frac{m}{7}+\frac{2m}{21}$

j $\frac{5d}{6}-\frac{d}{3}$ **k** $\frac{3p}{5}-\frac{2p}{7}$ **l** $\frac{2t}{9}+\frac{4t}{15}$

m $\frac{7k}{8}-\frac{11k}{18}$ **n** $\frac{m}{2}+\frac{m}{3}+\frac{m}{6}$ **o** $\frac{a}{2}-\frac{a}{3}+\frac{a}{4}$

p $\frac{x}{4}-\frac{x}{3}+\frac{x}{6}$ **q** $\frac{z}{2}-\frac{z}{5}+\frac{z}{4}$ **r** $2q-\frac{q}{3}+\frac{2q}{7}$

Example 13 Self Tutor

Simplify: **a** $\dfrac{4}{a}+\dfrac{3}{b}$ **b** $\dfrac{5}{x}-\dfrac{4}{3x}$

a $\dfrac{4}{a}+\dfrac{3}{b}$ {LCD $= ab$}

$= \dfrac{4\times b}{a\times b}+\dfrac{3\times a}{b\times a}$

$= \dfrac{4b}{ab}+\dfrac{3a}{ab}$

$= \dfrac{4b+3a}{ab}$

b $\dfrac{5}{x}-\dfrac{4}{3x}$ {LCD $= 3x$}

$= \dfrac{5\times 3}{x\times 3}-\dfrac{4}{3x}$

$= \dfrac{15}{3x}-\dfrac{4}{3x}$

$= \dfrac{15-4}{3x}$

$= \dfrac{11}{3x}$

2 Simplify:

a $\dfrac{7}{a}+\dfrac{3}{b}$ **b** $\dfrac{3}{a}+\dfrac{2}{c}$ **c** $\dfrac{4}{a}+\dfrac{5}{d}$ **d** $\dfrac{2a}{m}-\dfrac{a}{n}$

e $\dfrac{a}{x}+\dfrac{b}{2x}$ **f** $\dfrac{3}{a}-\dfrac{1}{2a}$ **g** $\dfrac{4}{x}-\dfrac{1}{xy}$ **h** $\dfrac{5}{x}+\dfrac{6}{5x}$

i $\dfrac{11}{3z}-\dfrac{3}{4z}$ **j** $\dfrac{a}{b}+\dfrac{c}{d}$ **k** $\dfrac{3}{a}+\dfrac{a}{2}$ **l** $\dfrac{x}{y}+\dfrac{2}{3}$

m $\dfrac{8}{p}-\dfrac{2}{5}$ **n** $\dfrac{x}{6y}+\dfrac{2x}{9y}$ **o** $\dfrac{1}{8t}-\dfrac{3}{5t}$ **p** $\dfrac{5}{2x}+\dfrac{3}{x^2}$

Example 14 Self Tutor

Simplify: **a** $\dfrac{b}{3}+1$ **b** $\dfrac{a}{4}-a$

a $\dfrac{b}{3}+1$

$= \dfrac{b}{3}+\dfrac{3}{3}$

$= \dfrac{b+3}{3}$

b $\dfrac{a}{4}-a$

$= \dfrac{a}{4}-\dfrac{a\times 4}{1\times 4}$

$= \dfrac{a}{4}-\dfrac{4a}{4}$

$= \dfrac{-3a}{4}$

$= -\dfrac{3a}{4}$

3 Simplify by writing as a single fraction:

a $\dfrac{x}{2}+1$ **b** $\dfrac{y}{3}-1$ **c** $\dfrac{a}{2}+a$ **d** $\dfrac{b}{4}-3$

e $\dfrac{x}{2}-4$ **f** $2+\dfrac{a}{3}$ **g** $x-\dfrac{x}{5}$ **h** $2+\dfrac{1}{x}$

i $5-\dfrac{2}{x}$ **j** $a+\dfrac{2}{a}$ **k** $\dfrac{3}{b}+b$ **l** $\dfrac{1}{x^2}-2x$

Example 15 **Self Tutor**

Write as a single fraction: **a** $\dfrac{x}{6}+\dfrac{x-2}{3}$ **b** $\dfrac{x+1}{2}-\dfrac{x-2}{3}$

a
$$\begin{aligned}&\frac{x}{6}+\frac{x-2}{3} \quad \{\text{LCD}=6\}\\
&=\frac{x}{6}+\frac{2}{2}\left(\frac{x-2}{3}\right)\\
&=\frac{x}{6}+\frac{2(x-2)}{6}\\
&=\frac{x+2(x-2)}{6}\\
&=\frac{x+2x-4}{6}\\
&=\frac{3x-4}{6}\end{aligned}$$

b
$$\begin{aligned}&\frac{x+1}{2}-\frac{x-2}{3} \quad \{\text{LCD}=6\}\\
&=\frac{3}{3}\left(\frac{x+1}{2}\right)-\frac{2}{2}\left(\frac{x-2}{3}\right)\\
&=\frac{3(x+1)}{6}-\frac{2(x-2)}{6}\\
&=\frac{3(x+1)-2(x-2)}{6}\\
&=\frac{3x+3-2x+4}{6}\\
&=\frac{x+7}{6}\end{aligned}$$

4 Write as a single fraction, and hence simplify:

a $\dfrac{x}{2}+\dfrac{x+1}{3}$ **b** $\dfrac{x-1}{4}-\dfrac{x}{2}$ **c** $\dfrac{2x}{3}+\dfrac{x+3}{4}$

d $\dfrac{x+1}{2}+\dfrac{x-1}{3}$ **e** $\dfrac{x-1}{3}+\dfrac{1-2x}{4}$ **f** $\dfrac{2x+3}{2}+\dfrac{2x-3}{3}$

g $\dfrac{x}{3}+\dfrac{x+1}{4}$ **h** $\dfrac{3x+2}{4}+\dfrac{x}{2}$ **i** $\dfrac{x}{6}+\dfrac{3x-1}{5}$

j $\dfrac{a+b}{3}+\dfrac{b-a}{2}$ **k** $\dfrac{x+1}{5}+\dfrac{2x-1}{4}$ **l** $\dfrac{x+1}{7}+\dfrac{3-x}{2}$

m $\dfrac{x}{6}-\dfrac{2-x}{5}$ **n** $\dfrac{2x-1}{5}-\dfrac{x}{4}$ **o** $\dfrac{x}{8}-\dfrac{1-x}{4}$

p $\dfrac{x}{5}-\dfrac{2-x}{10}$ **q** $\dfrac{x-1}{5}-\dfrac{2x-7}{3}$ **r** $\dfrac{1-3x}{4}-\dfrac{2x+1}{3}$

Example 16 **Self Tutor**

Write as a single fraction: **a** $\dfrac{1}{x}+\dfrac{2}{x-1}$ **b** $\dfrac{2}{x-1}-\dfrac{3}{x+1}$

a
$$\begin{aligned}&\frac{1}{x}+\frac{2}{x-1} \quad \{\text{LCD}=x(x-1)\}\\
&=\frac{1}{x}\left(\frac{x-1}{x-1}\right)+\left(\frac{2}{x-1}\right)\frac{x}{x}\\
&=\frac{1(x-1)+2x}{x(x-1)}\\
&=\frac{x-1+2x}{x(x-1)}\\
&=\frac{3x-1}{x(x-1)}\end{aligned}$$

b
$$\begin{aligned}&\frac{2}{x-1}-\frac{3}{x+1} \quad \{\text{LCD}=(x-1)(x+1)\}\\
&=\left(\frac{2}{x-1}\right)\left(\frac{x+1}{x+1}\right)-\left(\frac{3}{x+1}\right)\left(\frac{x-1}{x-1}\right)\\
&=\frac{2(x+1)-3(x-1)}{(x-1)(x+1)}\\
&=\frac{2x+2-3x+3}{(x-1)(x+1)}\\
&=\frac{-x+5}{(x-1)(x+1)} \quad \text{or} \quad \frac{5-x}{(x-1)(x+1)}\end{aligned}$$

5 Simplify:

a $\frac{3}{x}+\frac{4}{x+1}$ **b** $\frac{5}{x+2}-\frac{3}{x}$ **c** $\frac{4}{x+1}-\frac{3}{x-1}$

d $3+\frac{1}{x+2}$ **e** $\frac{1}{x}+\frac{4}{x-4}$ **f** $\frac{2}{x+3}-4$

g $\frac{x+1}{x-1}+\frac{x}{x+1}$ **h** $\frac{5}{x}+\frac{6}{x-2}$ **i** $\frac{2}{x+2}-\frac{4}{x+1}$

j $\frac{x}{x-1}+\frac{4}{2x+1}$ **k** $\frac{5}{x-1}+\frac{x-2}{x+3}$ **l** $\frac{x}{x+5}-\frac{x}{x-3}$

m $\frac{1}{x}+\frac{1}{x+1}+\frac{1}{x+2}$ **n** $\frac{2}{x}-\frac{5}{x+3}+\frac{3}{x-4}$ **o** $\frac{x}{x-1}-\frac{1}{x}+\frac{x}{x+1}$

6 Answer the **Opening Problem** on page **208**.

7 Write as a single fraction:

a $\frac{2}{x(x+1)}+\frac{1}{x+1}$ **b** $\frac{2x}{x-3}+\frac{4}{(x+2)(x-3)}$

c $\frac{3}{(x-2)(x+3)}+\frac{x}{x+3}$ **d** $\frac{x+5}{x-2}-\frac{63}{(x-2)(x+7)}$

8 Simplify:

a $\dfrac{\frac{x}{x-2}-3}{x-3}$ **b** $\dfrac{\frac{3x}{x+4}-1}{x-2}$ **c** $\dfrac{\frac{x^2}{x+2}-1}{x+1}$

d $\dfrac{\frac{x^2}{2-x}+9}{x-3}$ **e** $\dfrac{\frac{1}{x^2}-\frac{1}{4}}{x-2}$ **f** $\dfrac{\frac{x-3}{x^2}-\frac{1}{16}}{x-4}$

9 Simplify each of the following expressions. Hence write down a value of x for which the expression is zero, and the values of x for which it is undefined.

a $\frac{2x}{x+1}-\frac{4}{(x-1)(x+1)}$ **b** $\frac{6}{(x+2)(x+5)}+\frac{x}{x+2}$

E EQUATIONS WITH ALGEBRAIC FRACTIONS

To solve equations involving algebraic fractions, we:

- write all fractions with the same **lowest common denominator (LCD)**, and then
- **equate numerators**.

Example 17 **Self Tutor**

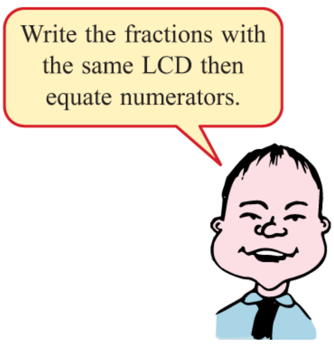
Solve for x: $\frac{6}{x}=\frac{2}{3}$

$\frac{6}{x}=\frac{2}{3}$ {LCD $=3x$}

$\therefore \frac{3\times 6}{3\times x}=\frac{2\times x}{3\times x}$ {to achieve a common denominator}

$\therefore 18=2x$ {equating numerators}

$\therefore x=9$ {dividing both sides by 2}



Example 18 **Self Tutor**

Solve for x: $\dfrac{5}{x+2} = \dfrac{2}{x-1}$

$$\dfrac{5}{x+2} = \dfrac{2}{x-1} \quad \{\text{LCD} = (x+2)(x-1)\}$$

$$\therefore \dfrac{5 \times (x-1)}{(x+2) \times (x-1)} = \dfrac{2 \times (x+2)}{(x-1) \times (x+2)} \quad \{\text{to achieve a common denominator}\}$$

$$\therefore 5(x-1) = 2(x+2) \quad \{\text{equating numerators}\}$$

$$\therefore 5x - 5 = 2x + 4 \quad \{\text{expanding brackets}\}$$

$$\therefore 3x - 5 = 4 \quad \{\text{subtracting } 2x \text{ from both sides}\}$$

$$\therefore 3x = 9 \quad \{\text{adding 5 to both sides}\}$$

$$\therefore x = 3 \quad \{\text{dividing both sides by 3}\}$$

EXERCISE 10E

1 Solve for x:

a $\dfrac{3}{x} = \dfrac{1}{5}$ **b** $\dfrac{3}{x} = \dfrac{2}{3}$ **c** $\dfrac{2}{7} = \dfrac{5}{x}$ **d** $\dfrac{1}{2x} = \dfrac{4}{3}$

e $\dfrac{7}{3x} = -\dfrac{4}{5}$ **f** $\dfrac{x-3}{x} = \dfrac{3}{5}$ **g** $\dfrac{x+5}{2x} = -\dfrac{8}{5}$ **h** $\dfrac{x-1}{4x} = \dfrac{5}{6}$

2 Solve for x:

a $\dfrac{2}{x} = \dfrac{8}{x+6}$ **b** $\dfrac{3}{x} = \dfrac{5}{x-6}$ **c** $\dfrac{1}{x+1} = \dfrac{7}{x-1}$

d $\dfrac{10}{x-3} = \dfrac{7}{x+6}$ **e** $\dfrac{8}{2x+3} = \dfrac{9}{x-1}$ **f** $\dfrac{5}{3x-2} = \dfrac{4}{x-3}$

REVIEW SET 10A

1 If $p = 5$, $q = -3$, and $r = 6$, evaluate:

a $\dfrac{r}{q}$ **b** $\dfrac{p-q}{p+q}$ **c** $\dfrac{\sqrt{p^2-16}}{r-q}$ **d** $\dfrac{p+2q-2r}{r^2-p^2}$

2 Simplify:

a $\dfrac{(2t)^2}{6t}$ **b** $\dfrac{16a+8b}{6a+3b}$ **c** $\dfrac{x(x-4)}{3(x-4)}$ **d** $\dfrac{8}{4x+8}$

3 Simplify:

a $\dfrac{2x+6}{x^2-9}$ **b** $\dfrac{x^2+4x+4}{x^2+2x}$ **c** $\dfrac{3x^2-6x}{3x^2-5x-2}$

4 Simplify:

a $\dfrac{2a-2b}{b-a}$ **b** $\dfrac{5x-15}{3x-x^2}$ **c** $\dfrac{16-x^2}{2x-8}$

5 Simplify:

a $\dfrac{a}{b} \times \dfrac{b}{3}$ **b** $\dfrac{a}{b} \div \dfrac{b}{3}$ **c** $\dfrac{a}{b} + \dfrac{b}{3}$ **d** $\dfrac{a}{b} - \dfrac{b}{3}$

6 Simplify:

a $\dfrac{7x-14}{x} \times \dfrac{3}{x-2}$ **b** $\dfrac{t^2-3t}{6t+6} \times \dfrac{t+1}{4t-12}$

7 Simplify:

a $\dfrac{9}{n} \div 6$ **b** $\dfrac{7}{3x-6} \div \dfrac{x+5}{x^2-2x}$

8 Write as a single fraction:

a $\dfrac{2x}{3} + \dfrac{x}{4}$ **b** $2 + \dfrac{x}{7}$ **c** $\dfrac{x}{4} - 1$ **d** $\dfrac{x}{2} + \dfrac{x}{4} - \dfrac{x}{3}$

9 Simplify:

a $\dfrac{x}{3} + \dfrac{x-1}{4}$ **b** $\dfrac{x+2}{3} - \dfrac{2-x}{6}$ **c** $\dfrac{2x+1}{5} - \dfrac{x-1}{10}$

10 Simplify:

a $\dfrac{1}{x+1} + \dfrac{2}{x-2}$ **b** $\dfrac{5}{x-1} - \dfrac{4}{x+1}$ **c** $\dfrac{1}{x^2} + \dfrac{1}{x+1}$

11 Solve for x: $\dfrac{6}{x} = \dfrac{5}{11-x}$

12 **a** Write as a single fraction: **i** $a - \dfrac{9}{a}$ **ii** $1 - \dfrac{a}{3}$

b Hence simplify $\left(a - \dfrac{9}{a}\right) \div \left(1 - \dfrac{a}{3}\right)$.

c Evaluate $\left(a - \dfrac{9}{a}\right) \div \left(1 - \dfrac{a}{3}\right)$ for:

i $a = 1$ **ii** $a = 3$ **iii** $a = 5$

REVIEW SET 10B

1 If $m = -4$, $n = 3$, and $p = 6$, evaluate:

a $\dfrac{p}{m+n}$ **b** $\dfrac{p-2n}{m+n}$ **c** $\dfrac{p-m}{\sqrt{m^2+n^2}}$

2 Simplify:

a $\dfrac{(3x)^2}{6x^3}$ **b** $\dfrac{3a+6b}{3}$ **c** $\dfrac{(x+2)^2}{x^2+2x}$

3 Simplify:

a $\dfrac{a+b}{3b+3a}$ **b** $\dfrac{2x^2-8}{x+2}$ **c** $\dfrac{x^2-6x+9}{4x-12}$

4 Simplify:

a $\dfrac{m}{n} \times \dfrac{2}{n}$ **b** $\dfrac{m}{n} \div \dfrac{2}{n}$ **c** $m^2 \div \dfrac{n}{m}$

5 Simplify:

a $\dfrac{3}{x} + \dfrac{5}{2x}$ **b** $\dfrac{6}{y} - \dfrac{a}{b}$ **c** $\dfrac{8}{3x} + \dfrac{1}{4x}$

6 Simplify:

a $\frac{3x}{7} - \frac{x}{14}$ **b** $\frac{4}{3x} + \frac{3}{x^2}$

7 Write as a single fraction:

a $5 + \frac{x}{2}$ **b** $3 - \frac{y}{x}$ **c** $1 + \frac{x}{2} + \frac{y}{3}$

8 Simplify:

a $\frac{y^2 - 5y}{y + 2} \times \frac{3}{2y - 10}$ **b** $\frac{9 - 3x}{4x + 16} \div \frac{x^2 - 3x}{x + 4}$

9 Simplify:

a $\frac{x}{4} - \frac{2 - x}{8}$ **b** $\frac{x + 5}{2} + \frac{2x + 1}{5}$ **c** $\frac{3 - x}{6} - \frac{2x}{9}$

10 Simplify:

a $\frac{2}{x - 1} - \frac{3}{x + 2}$ **b** $\frac{1}{x - 1} - \frac{2}{x^2}$ **c** $\frac{x}{x + 2} - \frac{2}{x} + \frac{x}{x + 3}$

11 **a** Evaluate $(x + y) \div \left(\frac{1}{x} + \frac{1}{y}\right)$ for:

i $x = 3,\ y = 4$ **ii** $x = 5,\ y = 10$

b In terms of x and y, what do you suspect $(x + y) \div \left(\frac{1}{x} + \frac{1}{y}\right)$ simplifies to?

c Prove your answer to **b** is correct by first writing $\frac{1}{x} + \frac{1}{y}$ as a single fraction.

d Hence, state the value of $\frac{41}{\frac{1}{21} + \frac{1}{20}}$.

12 Solve for x: $\frac{4}{x - 1} + \frac{x + 4}{x} = 1$